USN

CMRIT

CMR INSTITUTE OF TECHNOLOGY, BENGALURU.

ACCREDITED WITH A+ GRADE BY NAAC

Internal Assessment Test II – October 2019

Sub:	Calculus and Linear Algebra				Sub Code:	18MAT11	
Date:	24/10/2019	Duration:	90 mins	Max Marks: 50	Sem / Sec:	I / I,J,K,L,M,N,O	

Question 1 is compulsory and answer any SIX questions from the rest.

		MARKS	OBE CO	RBT
1.	(a) If $u = x + 3y^2 - z^3$, $v = 4x^2yz$ and $w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,-1,0).	[08]	CO2	L3
	(b) Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{\left(\frac{1}{x}\right)}$	[07]	CO2	L3
2.	Find the evolute of the parabola $y^2 = 4ax$	[07]	CO2	L3
3.	Obtain the Maclaurin's expansion of $\log(\sec x)$ upto **fourth** degree term.	[07]	CO2	L3
4.	A rectangular box, open at the top, is to have a volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.			

Scanned by CamScanner

5. If $u = f(\frac{x}{y}, \frac{y}{z}, \frac{z}{x})$,then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$ [07] CO2 L3

6. Find the extreme values of $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. [07] CO3 L3

7. Solve: $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ [07] CO3 L3

8. Solve : $(4xy + 3y^2 - x)dx + x(x + 2y)\,dy = 0$ [07] CO2 L3

Scanned by CamScanner

Chemistry Cycle Internal Assessment Test – II October 19

Sub: Calculus and Linear Algebra – 18MAT11

1 (a) If $u = x + 3y^2 - z^3$, $v = 4x^2yz$ and $w = 2z^2 - xy$. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$

» $u = x+3y^2-z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 6y & -3z^2 \\ 8xyz & 4x^2z & 4x^2y \\ -y & -x & 4z \end{vmatrix}$$

at $(1,-1,0)$

$$\therefore \frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} 1 & -6 & 0 \\ 0 & 0 & -4 \\ 1 & -1 & 0 \end{vmatrix}$$

on expanding $1(0-4) + 6(0+4) + 0 = 20$

1 (b) $K = \lim_{x\to 0} \left(\frac{a^x+b^x}{2}\right)^{1/x}$ --- 1^∞

Taking 'log' both sides

$$\log K = \lim_{x\to 0} \frac{\log\left(\frac{a^x+b^x}{2}\right)}{x} \quad \text{---} \ \frac{0}{0}$$

By Applying L-Hospital law.

$$\log k = \lim_{x\to 0} \frac{\frac{\cancel{2}}{a^x+b^x}\left[a^x\log a + b^x \log b\right]}{\cancel{2}}$$

$$= \frac{\cancel{2}}{a^0+b^0} \frac{a^0\log a + b^0\log b}{\cancel{2}}$$

$$\log k = \frac{\log a + \log b}{2} = \log(ab)^{1/2}$$

$$\Rightarrow \boxed{k = (ab)^{1/2}}$$

2. i) we shall consider the parametric equation of $y^2 = 4ax$, $x = at^2$, $y = 2at$

$$\therefore \frac{dx}{dt} = 2at \quad ; \quad \frac{dy}{dt} \; at$$

$$y_1 = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{t}$$

$$y_2 = \frac{d^2y}{dx^2} = -\frac{1}{t^2}\frac{dt}{dx} = -\frac{1}{2at^3}$$

we have $\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2)$ and $\bar{y} = y + \frac{(1+y_1^2)}{y_2}$

being the coordinates of centre of curvature

$$\bar{x} = at^2 - \frac{1/t}{-1/2at^3}\left(1+\frac{1}{t^2}\right) = 3at^2+2a \quad \text{--- (1)}$$

Scanned by CamScanner

Next $\bar{y} = 2at + \dfrac{\left(1+\frac{1}{t^2}\right)}{\left(-\frac{1}{2at^3}\right)} = -2at^3$ —(2)

we shall eliminate t from (1) and (2)

From (1) $t^2 = \dfrac{\bar{x}-2a}{3a}$

Raising to the power 3/2 on both sides of the above we have,

$(t^2)^{3/2} = \left(\dfrac{\bar{x}-2a}{3a}\right)^{3/2}$ or $t^3 = \left(\dfrac{\bar{x}-2a}{3a}\right)^{3/2}$

Using the expression of t^3 in the RHS of (2)

we have $\bar{y} = -2a\left(\dfrac{\bar{x}-2a}{3a}\right)^{3/2}$

squaring on both sides we get

$(\bar{y})^2 = 4a^2\left(\dfrac{\bar{x}-2a}{3a}\right)^3$ or $(\bar{y})^2 = \dfrac{4a^2}{27a^3}(\bar{x}-2a)^3$

i.e $27a(\bar{y})^2 = 4(\bar{x}-2a)^3$

Now by taking the locus of $(\bar{x},\bar{y})$ we obtain the required evolute.

Thus the evolute is given by

$$\boxed{27ay^2 = 4(x-2a)^3}$$

Scanned by CamScanner

(3) $y = \log(\sec x) \Rightarrow y(0) = 0$

$y_1 = \frac{1}{\sec x} \sec x \tan x = \tan x$

$y_1 = \tan x \Rightarrow y_1(0) = 0$

$y_2 = \sec^2 x = (1+\tan^2 x) = (1+y_1^2) \Rightarrow y_2(0) = 1$

$\therefore y_2 = (1+y_1^2)$

$y_3 = 2y_1y_2 \quad \therefore y_3(0) = 0$

$y_4 = 2(y_1y_3 + y_2^2) \quad \therefore y_4(0) = 2$

we know that

$y(x) = y(0) + x y_1(0) + \frac{x^2}{\lfloor 2} y_2(0) + \frac{x^3}{\lfloor 3} y_3(0) + \frac{x^4}{\lfloor 4} y_4(0)$

$= 0 + 0 + \frac{x^2}{2} \cdot 1 + 0 + \frac{x^4}{24} \times 2$

$\Rightarrow \log(\sec x) = \frac{x^2}{2} + \frac{x^4}{12}$

(4) $\Rightarrow$ Let V be its volume and S be its surface area.

By data $V = xyz = 32$

we know that the total surface area is

$2(xy+yz+zx)$

Since the box is open at the top the corresponding surface area $S = 2(xy+yz+zx) - xy = xy+2yz+2zx$

Scanned by CamScanner

we form the equations, $F_x = 0$, $F_y = 0$, $F_z = 0$

$\therefore$ $(y+2z) + \lambda yz = 0$ or $\lambda = -(y+2z)/yz$

$(x+2z) + \lambda xz = 0$ or $\lambda = -(x+2z)/xz$

$(2x+2y) + \lambda xy = 0$ or $\lambda = -(2y+2x)/xy$

Now $$\frac{-(y+2z)}{yz} = \frac{-(x+2z)}{xz} = \frac{-(2y+2x)}{xy}$$

From the first pair, $xy + 2xz = xy + 2yz$

$\Rightarrow$ $x = y$

From the second pair

$xy + 2yz = 2yz + 2xz$

$\Rightarrow$ $y = 2z$

That is $x = y = 2z$ $\Rightarrow$ $x = y$, $z = \frac{x}{2}$

Hence by substitution $y = x$, $z = x/2$ in $xyz = 32$

we get $x^3 = 64$ or $x = 4$

hence $x = y = 4$, $z = 2$

Thus the dimensions of rectangular box such that S is presumed to be minimum are.

$$\boxed{x = 4,\ y = 4,\ z = 2}$$

Scanned by CamScanner

(5)

$\Rightarrow$ Let $p = \frac{x}{y}$, $q = \frac{y}{z}$, $r = \frac{z}{x}$

so $u = f(p, q, r)$

i.e $u \longrightarrow (p, q, r) \longrightarrow (x, y, z)$

By definition of total differentiation

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial u}{\partial q} \cdot \frac{\partial q}{\partial x} + \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} \quad \text{--- (1)}$$

Here $p = \frac{x}{y} \Rightarrow \frac{\partial p}{\partial x} = \frac{1}{y}$

$q = \frac{y}{z} \Rightarrow \frac{\partial q}{\partial x} = 0$

$r = \frac{z}{x} \Rightarrow \frac{\partial r}{\partial x} = -\frac{z}{x^2}$

substituting in eq (1)

$$\frac{\partial u}{\partial x} = \frac{1}{y} \frac{\partial u}{\partial p} - \frac{z}{x^2} \frac{\partial u}{\partial r} \quad \text{--- (1)}$$

similarly

$$\frac{\partial u}{\partial y} = \frac{1}{z} \frac{\partial u}{\partial q} - \frac{x}{y^2} \frac{\partial u}{\partial p} \quad \text{--- (2)}$$

$$\frac{\partial u}{\partial z} = \frac{1}{x} \frac{\partial u}{\partial r} - \frac{y}{z^2} \frac{\partial u}{\partial q} \quad \text{--- (3)}$$

Taking L.H.S.

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$$

Scanned by CamScanner

⑥ $f(x,y) = x^3+y^3-3x-12y+20$

$f_x = 3x^2-3$, $f_y = 3y^2-12$

we shall find points (x,y) such that $f_x=0$ & $f_y=0$

i.e $3x^2-3=0$ and $3y^2-12=0$

or $x^2-1=0$ $y^2-4=0$

$x=\pm1$ $y=\pm2$

$\therefore$ $(1,2)$, $(1,-2)$, $(-1,2)$, $(-1,-2)$ are the stationary points.

Let $r=f_{xx}$, $s=f_{xy}$, $t=f_{yy}$

	$(1,2)$	$(1,-2)$	$(-1,2)$	$(-1,-2)$
$r=f_{xx}=6x$	$6>0$	6	-6	$-6<0$
$s=f_{xy}=0$	0	0	0	0
$t=f_{yy}=6y$	12	-12	12	-12
$rt-s^2$	$72>0$	$-72<0$	$-72<0$	$72>0$
Conclusion	Min pt	Saddle pt	Saddle pt.	Max pt.

Maximum value of $f(x,y)$ is,

$f(-1,-2) = 38$

Minimum value of $f(x,y)$ is

$f(1,2) = 2$

Thus $\boxed{\text{Maximum value is 32 and Minimum value is 2}}$

Scanned by CamScanner

7. Given

$$\frac{dy}{dx} + \frac{(y\cos x + \sin y + y)}{(\sin x + x\cos y + x)} = 0$$

$$\frac{dy}{dx} = -\frac{(y\cos x + \sin y + y)}{(\sin x + x\cos y + x)}$$

$$\Rightarrow (\sin x + x\cos y + x)\,dy = -(y\cos x + \sin y + y)\,dx$$

$$\Rightarrow (y\cos x + \sin y + y)\,dx + (x\cos y + \sin x + x)\,dy = 0$$

$\therefore$ $M = y\cos x + \sin y + y$, $N = x\cos y + \sin x + x$

$$\frac{\partial M}{\partial y} = \cos x + \cos y + 1 \qquad \frac{\partial N}{\partial x} = \cos y + \cos x + 1$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{— Exact d.e.}$$

solution

$$\int M(x,y)\,dx + \int N(y)\,dy = C$$

$$\int (y\cos x + \sin y + y)\,dx + \int 0\,dy = C$$

$$\Rightarrow \boxed{y\sin x + x\sin y + xy = c} \text{ - Required solution}$$

(8)

Let $M = 4xy + 3y^2 - x$ and $N = x(x+2y) = x^2 + 2xy$

$$\frac{\partial M}{\partial y} = (4x + 6y) \quad \text{and} \quad \frac{\partial N}{\partial x} = (2x + 2y)$$

(The equation is not exact)

Consider, $\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2x + 4y = 2(x + 2y)$ --- close to N.

Now $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2(\cancel{x+2y})}{x(\cancel{x+2y})} = \frac{2}{x} = f(x)$

Hence $e^{\int f(x)dx}$ is an integrating factor.

i.e $e^{\int f(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

Multiplying the given equation by x^2 we now have,

$M = 4x^3 + 3x^2y^2 - x^3$, $N = x^4 + 2x^3y$

$$\frac{\partial M}{\partial y} = 4x^3 + 6x^2y \quad \text{and} \quad \frac{\partial N}{\partial x} = 4x^3 + 6x^2y$$

Solution $\int M(x,y)\,dx + \int N(y)\,dy = c$

$\int (4x^3y + 3x^2y^2 - x^3)\,dx + \int 0\,dy = c$

Thus $\boxed{x^4y + x^3y^2 - \frac{x^4}{4} = c}$ - is the required solution.

Scanned by CamScanner