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Internal Assesment Test – II-Oct 2019

Sub:	Transform calculus ,Fourier series and Numerical techniques							Code:	18MAT31
Date:	12/ 10 /2019	Duration:	90 mins	Max Marks:	50	Sem:	3	Branch:	**ALL(REG)**

Question 1 is compulsory and Answer any six from question 2 to question 8 .

ALL BRANCHES (REGULAR)

	Marks	OBE CO	OBE RBT
1. Given $y''+xy'+y=0$, y(0) = 1, y′(0) = 0, use Milne's method to find y(0.4) and with the help of the data given below.	[8]	CO4	L3
2. Using Taylor's series method, find the value of y at x = 0.1 and 0.2 from $y' = x^2y - 1$, y(0) = 1.Consider terms upto fourth degree.	[7]	CO4	L3
3. Using modified Euler's method, find an approximate value of y at x = 0.2 given that y′= x + y and y(0) = 1, taking h = 0.1. Carry two iterations in each stage	[7]	CO4	L3

Data for question 1:

y(0.1) = 0.995	y(0.2) = 0.9801	y(0.3) = 0.956
y′(0.1) = - 0.0995	y′(0.2) = - 0.1960	y′(0.3) = - 0.2867

Q.Rev

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Q.Rev 2

4.	Applying classical 4^{th} order Runge-Kutta method, find an approximate value of y for $x = 0.1$ given that $y' = x + y^2$ and $y(0) = 1$. Choose h suitably.	[7]	CO4	L3
5.	Given $y' = x^2(1 + y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by Adams-Bashforth method.	[7]	CO4	L3
6.	Using 4^{th} order Runge-Kutta method, solve $y'' = xy'^2 - y^2$ for $x = 0.2$, given the initial conditions $x = 0$, $y = 1$, $y' = 0$ and $h = 0.2$	[7]	CO4	L3
7.	Find the Laplace transform of $f(t) = 3^t + t \sin t + \left(\dfrac{\cos(bt) - \cos(at)}{t}\right)$.	[7]	CO1	L3
8.	Find the Laplace transform of $f(t) = te^{-2t} \cos 3t + (4t^2+5) + \sinh 5t$	[7]	CO1	L3

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Internal Assessment Test II - Oct 2019

Subject: Transform Calculus & Numerical Techniques

Subject Code: 18MAT31

Question 1 is compulsory and answer any Six question

1. given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$ Use milne's method to find $y(0.4)$ with the help of data given below

$y(0.1) = 0.995$	$y(0.2) = 0.9801$	$y(0.3) = 0.956$
$y'(0.1) = -0.0995$	$y'(0.2) = 0.1960$	$y'(0.3) = -0.2867$

Sol.

$z' = -(xz + y)$

$z'(0) = -1$, $z'(0.1) = -0.985$, $z'(0.2) = -0.941$

$z'(0.3) = -0.87$

milne's predictor formula

$$y_4^{(p)} = y_0 + \frac{4h}{3}(2z_1 - z_2 + 2z_3)$$

$$y_4^{(p)} = 0.995 + \frac{4(0.1)}{3}\left[2(-0.0995) - (-0.196) + 2(-0.2867)\right]$$

$$y_4^{(p)} = 0.9231$$

$$z_4^{(p)} = 0 + \frac{4(0.1)}{3}\left(2(-0.985) - (-0.94) + 2(-0.87)\right)$$

$$z_4^{(p)} = -0.3692$$

$$z_4' = -0.7254$$

$$\boxed{y_4^{(c)} = 0.9230}$$

$$z_4^{(c)} = -0.3692$$

2. Using Taylor's series method, find the value of y at $x=0.1$ and 0.2 from $y' = x^2y - 1$, $y(0)=1$ consider terms upto fourth degree.

Soln: $y' = x^2y - 1$, $y(0) = 1$.

$y'' = 2xy + x^2y'$ $\quad (y'')_0 = 0$

$y''' = 2y + 4xy' + x^2y''$ $\quad (y''')_0 = 2$.

$y'''' = 6y' + 6xy'' + x^2y'''$ $\quad (y^{iv})_0 = -6$

$$y(x) = y_0 + x\,y'(0) + \frac{x^2}{2!}y''(0) + \frac{x^3}{3!}y'''(0) \ldots$$

$$y(x) = 1 + x(-1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(-6) \ldots$$

$$y(x) = 1 - x + \frac{x^3}{3} - \frac{x^4}{4} \ldots$$

$$\boxed{y(0.1) = 0.90033 \quad \text{and} \quad y(0.2) = 0.80227}$$

3. Using modified Euler's method, find an approximate value of y at $x=0.2$ given that $y' = x+y$ and $y(0)=1$ take $h=0.1$ carry two iterations in each stage.

Soln. $x_0 = 0$, $y_0 = 1$ $\quad h = 0.1$. $\quad x_1 = x_0 + h = x_1 = 0+0.1$

$y(0.1) = 1$.

Soln. $y_0 = 1$, $x_0 = 0$ $\quad h = 0.1$

Stage I $\quad y_1^{(0)} = y_0 + h f(x_0, y_0) = 1 + 0.1(1) = 1.1$

$$y_1^{(1)} = y_0 + h/2\,[f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$= 1 + \frac{0.1}{2}[1 + 0.1 + 1.1] = 1.11$$

$$\boxed{y_1^{(1)} = 1.11}$$

$$y_1^{(2)} = y_0 + h/2\,[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$= 1 + 0.1/2\,[1 + 0.1 + 1.11] = 1.1105 \qquad \boxed{y_1^{(2)} = 1.1105}$$

Stage II $x_1 = 0.1$, $x_2 = 0.2$ $y(0.2) = ?$

$$y_1^{(0)} = y_0 + h f(x_0, y_0) = 1 + (0.2)1 = 1.2315$$

modified Euler's formula

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$\boxed{y_1^{(1)} = 1.2426}$$

$$y_1^{(2)} = 1.244$$

$$\boxed{y_1^{(3)} = 1.2431}$$

4. Applying classical 4th order Runge-Kutta method, find an approximate value of y for $x = 0.1$ given that $y' = x + y^2$ and $y(0) = 1$, choose h suitably.

Soln. given $y_0 = 1$ $x_0 = 0$, let $h = 0.1$

$y(0.1) = ?$

$$y' = x + y^2$$

$$k_1 = hf(x_0 y_0) = 0.1\, f(0,1) = 0.1$$

$$k_2 = hf(x_0 + h/2,\ y_0 + k_1/2) = 0.1 f(0.05, 1.05)$$

$$= 0.11525$$

$$k_3 = hf(x_0 + h/2,\ y_0 + k_2/2) = 0.1\, f(0.05;\ 1.0576)$$

$$k_3 = 0.11685$$

$$k_4 = hf(x_0 + h,\ y_0 + k_3) = 0.1 f(0.1,\ 1.11685)$$

$$\boxed{k_4 = 0.1347}$$

$$\therefore\ y(0.1) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{0.1 + (2 \times 0.11525) + (2 \times 0.11605) + 0.1342}{6}$$

$$\boxed{y(0.1) = 1.1164}$$

5. given $y' = x^2(1+y)$ and $y(1)=1$, $y(1.1)=1.233$, $y(1.2)=1.548$, $y(1.3)=1.979$ Evaluate $y(1.4)$ by adams Bash fith

Sol

given $y' = \frac{dy}{dx} = f(x,y) = x^2(1+y)$

x	y	$y' = x^2(1+y)$
$x_0=1$	$y_0=1$	$y_0' = x_0(1+y_0)=2$
$x_1=1.1$	$y_1=1.233$	$y_1' = x_1^2(1+y_1)=2.7019$
$x_2=1.2$	$y_2=1.546$	$y_2' = 3.6691$
$x_3=1.3$	$y_3=1.979$	$y_3' = 5.0345$
$x_4=1.4$	$y_4=$?	$y_4' =$

$$y_4^{(0)} = y_3 + \frac{h}{24}\left[55y_3' - 59y_2' + 37y_1' - 9y_0'\right]$$

$$y_4^{(0)} = 1.979 + \frac{0.1}{24}\left[55(5.0345) - 59(3.6691) + 37(2.7019) - 9(2)\right]$$

$$\boxed{y_4^{(0)} = 2.5722}$$

$$y_4' = x_4^2(1+y_4^{(0)}) = 7.0015$$

$$y_4^{(1)} = y_3 + \frac{h}{24}\left[9y_4' + 19y_3' - 5y_2' + y_1'\right]$$

$$y_4^{(1)} = 1.979 + \frac{0.1}{24}\left[9(7.0015) + 19(5.0345) - 5(3.6691) + 2.7091\right]$$

$$\boxed{y_4^{(1)} = 2.5749.}$$

$$y_4' = x_4^2(1+y_4^{(1)}) = 7.0068$$

$$\boxed{y_4^{(1)} = 2.5751}$$

6. Using 4th order Runge-Kutta method solve $y'' = xy'^2 - y^2$ for $x = 0,2$ given the initial conditions $x=0, y=1, y'(0)=0$, $h=0,2$

Soln:
$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$$

$$\frac{dy}{dx} = z \Rightarrow \frac{d^2y}{dx^2} = \frac{dz}{dx}$$

$$\frac{dz}{dx} = xz^2 - y^2 \quad \text{with } y=1,\ z=0 \text{ at } x=0$$

We have the system of Equations $\frac{dy}{dx} = z,\ \frac{dz}{dx} = xz^2 - y^2$

Let $f(x,y,z) = z$, $g(x,y,z) = xz^2 - y^2$, $x_0 = 0$, $y_0 = 1$
$z_0 = 0$, $h = 0,2$

$K_1 = hf(x_0, y_0, z_0) = 0,2\, f(0,1,0) = 0$

$l_1 = 0,2\,[0(0)^2 - 1^2) = -0,2$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{l_1}{2}) = 0,2\, f(0,1, 1, -0,1)$
$= 0,2\,(-0,1) = -0,02$

$l_2 = (0,2)\,[(0,1)(-0,1)^2 - 1^2) = -0,1998$

$K_3 = hf[x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{l_2}{2}]$

$K_3 = (0,2)\, f(0.1, 0.99, -0,0999) = -\cancel{0,01998}$ -0.0199

$l_3 = (0,2)\,[(0,1)\,[-0,0999)^2 - (0,99)^2] = -0,1962$

$K_4 = hf(x_0 + h, y_0 + K_3, z_0 + l_3) = (0,2)\, f(0,2, 0,98002, -0,1958)$

$\boxed{K_4 = -0,03916}$

$l_4 = (0,2)\,[(0,2)(-0,1956)^2 - (0,98002)^2] = -0,19055$

$y(x_0+h) = y_0 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 1 + \frac{1}{6}[0 + 2(-0,02) + 2(-0.01998) - 0,03916] = \boxed{0,9801}$

7. Find the laplace transform of $f(t) = 3^t + t\sin t + \frac{\cos(bt) - \cos(at)}{t}$

Sol, $L[f(t)] = L[3^t] + L[t\sin t] + L\left[\frac{\cos(bt) - \cos(at)}{t}\right]$

$$L[e^{t\log 3}] + (-1)^1 \frac{d^1}{ds^1}\left(\frac{1}{s^2+1}\right) + \int_s^\infty L[\cos bt] - L[\cos at]\,ds$$

$$= \frac{1}{s - \log 3} + \frac{2s}{(s^2+1)^2} + \int_s^\infty \frac{s}{s^2+b^2} - \frac{s}{(s^2+a^2)}\,ds.$$

$$= \frac{1}{s^2 - \log 3} + \frac{2s}{(s^2+1)^2} + \frac{1}{2}\left[\log(s^2+b^2) - \log(s^2+a^2)\right]_s^\infty$$

$$= \frac{1}{s - \log 3} + \frac{2s}{(s^2+1)^2} + \log\sqrt{\frac{s^2+a^2}{s^2+b^2}}.$$

8. Find the laplace transform of

$f(t) = t\,e^{-2t}\cos 3t + 4t^2 + 5 + \sinh 5t$

$L[t e^{-2t}\cos 3t] \Rightarrow f(t) = \cos 3t \Rightarrow L[f(t)] = \frac{s}{s^2+3^2}.$

$$L[t\cos 3t] = (-1)^1 \frac{d^1}{ds^1}\left[\frac{s}{s^2+3^2}\right] = (-1)\left[\frac{(s^2+3^2)(1) - s(2s)}{(s^2+3^2)^2}\right]$$

$$L[t\cos 3t] = \frac{s^2-9}{(s^2+3^2)^2},$$

$$L[t e^{-2t}\cos 3t] = \frac{(s+2)^2 - 9}{[(s+2)^2+3^2]^2} =$$

$$L[4t^2] = \frac{4 \cdot 2!}{s^3} = \frac{8}{s^3}$$

$$L[5] = \frac{5}{s}$$

$$L[\sinh 5t] = \frac{5}{s^2-25}.$$

$$\therefore \boxed{L[f(t)] = \frac{(s+2)^2-9}{[(s+2)^2+3^2]^2} + \frac{8}{s^3} + \frac{5}{s} + \frac{5}{s^2-25}}$$