

Internal Assessment Test – II-Oct 2019

Sub:	Transform calculus ,Fourier series and Numerical techniques	Code:	18MAT31
Date:	12/ 10 /2019	Duration:	90 mins
		Max Marks:	50
		Sem:	3
		Branch:	ALL(REG)
Question 1 is compulsory and Answer any six from question 2 to question 8 .			
ALL BRANCHES (REGULAR)			

1. Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$, use Milne's method to find $y(0.4)$ and with the help of the data given below.

$y(0.1) = 0.995$	$y(0.2) = 0.9801$	$y(0.3) = 0.956$
$y'(0.1) = -0.0995$	$y'(0.2) = -0.1960$	$y'(0.3) = -0.2867$

2. Using Taylor's series method, find the value of y at $x = 0.1$ and 0.2 from $y' = x^2y - 1$, $y(0) = 1$. Consider terms upto fourth degree.

3. Using modified Euler's method, find an approximate value of y at $x = 0.2$ given that $y' = x + y$ and $y(0) = 1$, taking $h = 0.1$. Carry two iterations in each stage

Marks	OBE	
	CO	RBT
[8]	CO4	L3
[7]	CO4	L3
[7]	CO4	L3

R. Rev

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R. Rev

4. Applying classical 4th order Runge-Kutta method, find an approximate value of y for $x = 0.1$ given that $y' = x + y^2$ and $y(0) = 1$. Choose h suitably.
5. Given $y' = x^2(1 + y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$ by Adams-Bashforth method.
6. Using 4th order Runge-Kutta method, solve $y'' = xy'^2 - y^2$ for $x = 0.2$, given the initial conditions $x = 0$, $y = 1$, $y' = 0$ and $h = 0.2$
7. Find the Laplace transform of $f(t) = 3^t + t \sin t + \left(\frac{\cos(bt) - \cos(at)}{t} \right)$.
8. Find the Laplace transform of $f(t) = te^{-2t} \cos 3t + (4t^2 + 5) + \sinh 5t$

[7]	CO4	L3
[7]	CO4	L3
[7]	CO4	L3
[7]	CO1	L3
[7]	CO1	L3

R. Rev

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[7]	CO1	L3

R. Rev

Subject: Transfinite Calculus & Numerical Techniques

Subject Code: 18MAT31

Question 4 is compulsory and answer any 5 more questions.

1. Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$ Use Milne's method to find $y(0.4)$ with the help of data given below

$y(0.1) = 0.995$	$y(0.2) = 0.985$	$y(0.3) = 0.956$
$y'(0.1) = -0.0995$	$y'(0.2) = 0.1960$	$y'(0.3) = -0.2867$

Sol. $\lambda^2 = -(x^2 + 1)$

$z_1(0) = -1$, $z_1(0.1) = -0.985$, $z_1(0.2) = -0.941$

$z_1(0.3) = -0.87$

Milne's predictor formula

$$y_4^{(p)} = y_0 + \frac{4h}{3} (2z_1 - 2z_2 + 2z_3)$$

$$y_4^{(p)} = 0.995 + \frac{4(0.1)}{3} [2(-0.0995) - (-0.1960) + 2(-0.2867)]$$

$$y_4^{(p)} = 0.9231$$

$$z_4^{(p)} = 0 + \frac{4(0.1)}{3} [2(-0.985) - (-0.941) + 2(-0.87)]$$

$$z_4^{(p)} = -0.3692$$

$$z_4' = -0.7854$$

$$\boxed{y_4^{(1)} = 0.9235}$$

$$z_4^{(1)} = -0.3692$$

2. Using Taylor's series method, find the value of y at $x=0.1$ and 0.2 from $y' = x^2y - 1$, $y(0) = 1$ consider terms upto fourth degree.

Sol: $y' = x^2y - 1$, $y(0) = 1$.

$$y'' = 2xy + x^2y' \quad (y'')_0 = 0$$

$$y''' = 2y + 4xy' + x^2y'' \quad (y''')_0 = 2$$

$$y'''' = 6y' + 6xy'' + x^2y''' \quad (y'''')_0 = -6$$

$$y(x) = y_0 + x y'(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \dots$$

$$y(x) = 1 + x(-1) + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(-6) + \dots$$

$$y(x) = 1 - x + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

$y(0.1) = 0.90033 \quad \text{and} \quad y(0.2) = 0.80227$

3. Using modified Euler's method, find an approximate value of y at $x=0.2$ given that $y' = x+y$ and $y(0) = 1$ take $h=0.1$ carry two iterations in each stage.

Sol: $x_0 = 0$, $y_0 = 1$, $h = 0.1$, $x_1 = x_0 + h = x_1 = 0 + 0.1$
 $y(0.1) = 1$.

Sol: $y_0 = 1$, $x_0 = 0$, $h = 0.1$

Stage I $y_1^{(1)} = y_0 + h f(x_0, y_0) = 1 + 0.1 [1] = 1.1$

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$= 1 + \frac{0.1}{2} [1 + 0.1 + 1.1] = 1.11$$

$y_1^{(1)} = 1.11$

$$y_1^{(2)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(2)})]$$

$$= 1 + 0.1/2 [1 + 0.1 + 2.1] = 1.1105$$

$y_1^{(2)} = 1.1105$

Stage II $x_1 = 0.1$, $x_2 = 0.2$ $y(0.2) = 1$

$$y_1^{(0)} = y_0 + h f(x_0, y_0) = 1 + (0.2) \cdot 1 = 1.2315$$

modified Euler's formula.

$$y_1^{(1)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(0)})]$$

$$\boxed{y_1^{(1)} = 1.2426}$$

$$y_1^{(2)} = 1.244$$

$$\boxed{y_1^{(3)} = 1.2431}$$

4. Applying classical 4th order Runge-Kutta method, find an approximate value of y for $x = 0.1$ given that

$$y' = x + y^2 \text{ and } y(0) = 1, \text{ Choose } h \text{ suitably.}$$

Sol: Given $y_0 = 1$, $x_0 = 0$, let $h = 0.1$
 $y(0.1) = ?$

$$y' = x + y^2$$

$$k_1 = hf(x_0, y_0) = 0.1 f(0, 1) = 0.1$$

$$k_2 = hf(x_0 + h/2, y_0 + k_1/2) = 0.1 f(0.05, 1.05) = 0.11525$$

$$k_3 = hf(x_0 + h/2, y_0 + k_2/2) = 0.1 f(0.05, 1.0576) = 0.11685$$

$$k_3 = 0.11685$$

$$k_4 = hf(x_0 + h, y_0 + k_3) = 0.1 f(0.1, 1.11685) = 0.1347$$

$$\boxed{k_4 = 0.1347}$$

$$\begin{aligned} \therefore y(0.1) &= y_0 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \\ &= 1 + \frac{0.1}{6} (1 + 2 \times 0.11525 + 2 \times 0.11685 + 0.1347) \\ &= 1.1164 \end{aligned}$$

5. Given $y' = x^2(1+y)$ and $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$
 $y(1.3) = 1.979$ Evaluate $y(1.4)$ by Adams Bashforth

Soln
 Given $y' = \frac{dy}{dx} = f(x, y) = x^2(1+y)$

x	y	$y' = x^2(1+y)$
$x_0 = 1$	$y_0 = 1$	$y'_0 = x_0^2(1+y_0) = 2$
$x_1 = 1.1$	$y_1 = 1.233$	$y'_1 = x_1^2(1+y_1) = 2.7019$
$x_2 = 1.2$	$y_2 = 1.548$	$y'_2 = 3.6691$
$x_3 = 1.3$	$y_3 = 1.979$	$y'_3 = 5.0345$
$x_4 = 1.4$	$y_4 = ?$	$y'_4 =$

$$y_4^{(0)} = y_3 + \frac{h}{24} [55y'_3 - 59y'_2 + 37y'_1 - 9y'_0]$$

$$y_4^{(0)} = 1.979 + \frac{0.1}{24} [55(5.0345) - 59(3.6691) + 37(2.7019) - 9(2)]$$

$$\boxed{y_4^{(0)} = 2.5722}$$

$$y_4' = x_4^2(1+y_4^{(0)}) = 7.0015$$

$$y_4^{(1)} = y_3 + \frac{h}{24} [9y_4' + 19y'_3 - 5y'_2 + y'_1]$$

$$y_4^{(1)} = 1.979 + \frac{0.1}{24} [9(7.0015) + 19(5.0345) - 5(3.6691) + 2.7019]$$

$$\boxed{y_4^{(1)} = 2.5749}$$

$$y_4' = x_4^2(1+y_4^{(1)}) = 7.0068$$

$$\boxed{y_4^{(2)} = 2.575}$$

6. Using 4th order Runge-Kutta method solve $y'' = xy'^2 - y^2$ for $x=0.2$ given the initial conditions $x=0, y=1, y'(0)=0, h=0.2$

Sol: $\frac{d^2y}{dx^2} = x(\frac{dy}{dx})^2 - y^2$

$$\frac{dy}{dx} = z \Rightarrow \frac{d^2y}{dx^2} = \frac{dz}{dx}$$

$$\frac{dz}{dx} = xz^2 - y^2 \quad \text{with } y=1, z=0 \text{ at } x=0$$

We have the system of Equations $\frac{dy}{dx} = z, \frac{dz}{dx} = xz^2 - y^2$

Let $f(x, y, z) = z, \quad g(x, y, z) = xz^2 - y^2, \quad x_0 = 0, y_0 = 1, z_0 = 0, h = 0.2$

$$K_1 = hf(x_0, y_0, z_0) = 0.2 f(0, 1, 0) = 0$$

$$L_1 = 0.2 [0(0)^2 - 1^2] = -0.2$$

$$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}, z_0 + \frac{L_1}{2}) = 0.2 f(0.1, 1, -0.1) = 0.2(-0.1) = -0.02$$

$$L_2 = (0.2) [(0.1)(-0.1)^2 - 1^2] = -0.1998$$

$$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}, z_0 + \frac{L_2}{2})$$

$$K_3 = (0.2) f(0.1, 0.99, -0.0999) = -0.01998$$

-0.0199

$$L_3 = (0.2) [(0.1) [-0.0999]^2 - (0.99)^2] = -0.1952$$

$$K_4 = hf(x_0 + h, y_0 + K_3, z_0 + L_3) = (0.2) f(0.2, 0.98002, -0.1958)$$

$K_4 = 0.03916$

$$L_4 = (0.2) [(0.2) (-0.1958)^2 - (0.98002)^2] = -0.19055$$

$$y(x_0 + h) = y_0 + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4) = 1 + \frac{1}{6} [0 + 2(-0.02) + 2(-0.01998) - 0.03916] = 0.9801$$

7. Find the Laplace transform of $f(t) = 3t + t \sin t + \frac{\cos(bt) - \cos(at)}{t}$

Sol, $L\{f(t)\} = L\{3t\} + L\{t \sin t\} + L\left\{\frac{\cos(bt) - \cos(at)}{t}\right\}$

$$= L\{e^{t \log 3}\} + (-1) \frac{d}{ds} \left(\frac{1}{s^2+1}\right) + \int_s^\infty L\{\cos(bt)\} - L\{\cos(at)\} ds$$

$$= \frac{1}{s - \log 3} + \frac{2s}{(s^2+1)^2} + \int_s^\infty \frac{s}{s^2+b^2} - \frac{s}{s^2+a^2} ds$$

$$= \frac{1}{s - \log 3} + \frac{2s}{(s^2+1)^2} + \frac{1}{2} \left[\log(s^2+b^2) - \log(s^2+a^2) \right]_s^\infty$$

$$= \frac{1}{s - \log 3} + \frac{2s}{(s^2+1)^2} + \log \sqrt{\frac{s^2+a^2}{s^2+b^2}}$$

8. Find the Laplace transform of

$$f(t) = t e^{-2t} \cos 3t + 4t^2 + 5 + \sinh 5t$$

$$L\{t e^{-2t} \cos 3t\} \Rightarrow f(t) = \cos 3t \Rightarrow L\{f(t)\} = \frac{s}{s^2+3^2}$$

$$L\{t \cos 3t\} = (-1) \frac{d}{ds} \left[\frac{s}{s^2+3^2} \right] = (-1) \left[\frac{(s^2+3^2)(1) - s(2s)}{(s^2+3^2)^2} \right]$$

$$L\{t \cos 3t\} = \frac{s^2-9}{(s^2+3^2)^2}$$

$$L\{t e^{-2t} \cos 3t\} = \frac{(s+2)^2-9}{[(s+2)^2+3^2]^2} =$$

$$L\{4t^2\} = \frac{4(2!)}{s^3} = \frac{8}{s^3}$$

$$L\{5\} = \frac{5}{s}$$

$$L\{\sinh 5t\} = \frac{5}{s^2-25}$$

$$\therefore L\{f(t)\} = \frac{(s+2)^2-9}{(s+2)^2+3^2} + \frac{8}{s^3} + \frac{5}{s} + \frac{5}{s^2-25}$$