IAT-2 PHYSICS SCHEME

1.A FORCED OSCILLATIONS

Forced oscillations are produced when an external oscillating force is applied to the particle subject to SHM. The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces. Vibrations of tuning fork placed on a resonating box make the walls of the box and the air inside oscillate.

Let $F = F_o \cos\omega_f t$ be the oscillating applied force

The equation of motion is given by

$$F = ma = -kx - bv + F_o \cos\omega_f t$$

$$m\frac{d^2x}{dt^2} + b.\frac{dx}{dt} + kx = F_o \cos\omega_f t \quad \text{.......(1)}$$ (1 mark)

This equation is non homogeneous. The complimentary function is given by

$$x_c(t) = C \cos\omega_f t + D\sin\omega_f t \quad \text{.............(2)}$$

This can also be expressed as $x_c(t) = A\cos(\omega_o t - \theta)$ where

$$A = \sqrt{C^2 + D^2} \quad \tan\theta = D/C \quad \text{......(3)}$$

$$\dot{x}_c(t) = -C\omega_f \sin\omega_f t + D\omega_f \cos\omega_f t$$

$$\ddot{x}_c(t) = -C\omega^2{}_f \cos\omega_f t - D\omega^2{}_f \sin\omega_f t$$

Substituting in (1)

$$\left[(k - m\omega_f{}^2)C + Db\omega_f\right]\cos\omega_f t + \left[-bC\omega_f + (k - m\omega_f{}^2)D\right]\sin\omega_f t = F_o\cos\omega_f t$$

By equating coefficients of the sine and cosine terms on both sides

$$(k - m\omega_f{}^2)C + Db\omega_f = F_o$$

$$-bC\omega_f + (k - m\omega_f{}^2)D = 0$$

(2 marks)

Solving for C and D

$$C = F_o\frac{k - m\omega_f{}^2}{(k - m\omega_f{}^2)^2 + b^2\omega_f{}^2}$$

$$D = F_o\frac{b\omega_f}{(k - m\omega_f{}^2)^2 + b^2\omega_f{}^2}$$

Substituting for $\omega_o = \sqrt{k/m}$

$$C = F_o\frac{m(\omega_o^2 - \omega_f{}^2)}{m^2(\omega_o^2 - \omega_f{}^2)^2 + b^2\omega_f{}^2}$$

$$D = F_o\frac{b\omega_f}{m^2(\omega_o^2 - \omega_f{}^2)^2 + b^2\omega_f{}^2}$$

The general solution is

$$x(t) = F_o\frac{m(\omega_o^2 - \omega_f{}^2)}{m^2(\omega_o^2 - \omega_f{}^2)^2 + b^2\omega_f{}^2}\cos\omega_f t + F_o\frac{b\omega_f}{m^2(\omega_o^2 - \omega_f{}^2)^2 + b^2\omega_f{}^2}\sin\omega_f t$$ (2 marks)

Using (3), the solution can also be expressed as

$$x(t) = A\cos(\omega_o t - \theta)$$

where amplitude $A = \sqrt{C^2 + D^2}$ and

phase $\theta = \tan^{-1}D/C = \tan^{-1}\left(\frac{b\omega_f}{m(\omega_o^2 - \omega_f^2)}\right)$ (2 marks)

1.B. Uses: (1+1+1)

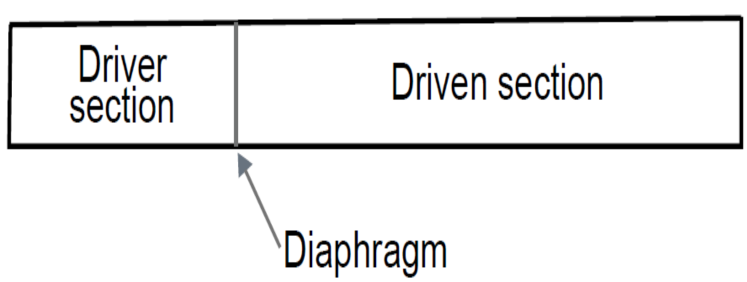
- Aerodynamics – hypersonic shock tunnels, scramjet engines.
- High temperature chemical kinetics – ignition delay
- Rejuvenating depleted bore wells
- Material studies – effect of sudden impact pressure, blast protection materials
- Investigation of traumatic brain injuries
- Needle-less drug delivery
- Wood preservation

2.A

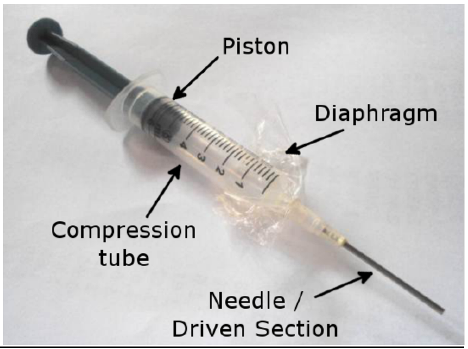
Reddy shock tube: (3 marks)

A shock tube is a device used to study the changes in pressure & temperature which occur due to the propagation of a shock wave. A shock wave may be generated by an explosion caused by the buildup of high pressure which causes diaphragm to burst.

It is a hand driven open ended shock tube. It was conceived with a medical syringe. A plastic sheet placed between the plastic syringe part and the needle part constitutes the diaphragm.



- A high pressure (driver) and a low pressure (driven) side separated by a diaphragm.
- When diaphragm ruptures, a shock wave is formed that propagates along the driven section.
- Shock strength is decided by driver to driven pressure ratio, and type of gases used.

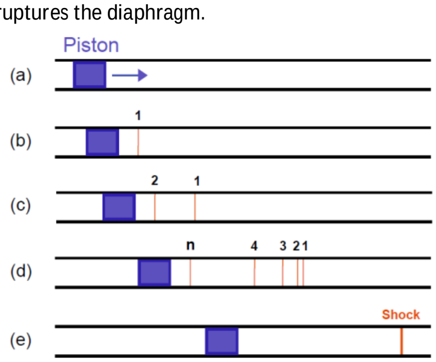


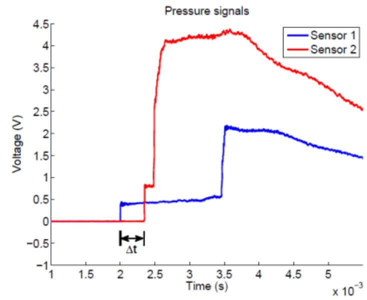
Working: (4 marks)

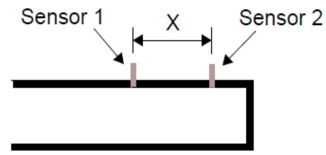
- The piston is initially at rest and accelerated to final velocity V in a short time t.
- The piston compresses the air in the compression tube. At high pressure, the diaphragm ruptures and the shock wave is set up. For a shock wave to form, $V_{piston} > V_{sound}$.

Formation of shock wave:

As the piston gains speed, compression waves are set up. Such compression waves increase in number. As the piston travels a distance, all the compression waves coalesce and a single shock wave is formed. This wave ruptures the diaphragm.







$$U_s = \frac{X}{\Delta t}$$

Mach number $M = \frac{V_{Shock}}{V_{Sound}}$

2.B.(1 mark formula +1 subst.+1 final answer)

$$A = \frac{\frac{F_o}{m}}{\sqrt{\left(\omega_o^2 - \omega_f^2\right)^2 + 0}} \quad \because b = 0$$

$$A = 0.521m$$

3A.

Expression for Fermi Level in Intrinsic Semiconductor

Electron density in conduction band is given by

$$n_e = 2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}}$$ (1 mark)

Hole density in valence band may be obtained from the result

$$n_h = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$ (1 mark)

For an intrinsic semiconductor, $n_e = n_h$

$$2\left(\frac{2\pi m_e^* kt}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_C - E_F}{kT}} = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{\frac{3}{2}} e^{-\frac{E_F - E_V}{kT}}$$

$$\left(\frac{m_e^*}{m_h^*}\right)^{\frac{3}{2}} = e^{\frac{-E_f + E_v + E_c - E_f}{kT}}$$

$$\frac{3}{2}\ln\left(\frac{m_e^*}{m_h^*}\right) = \frac{-2E_f + E_v + E_c}{kT}$$ (2 marks)

$$E_f = \frac{E_v + E_c}{2} - \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right)$$

For Intrinsic semiconductor, $m^*(e) = m^*(h)$

$$E_f = \frac{E_v + E_C}{2} = E_g / 2$$ (2 marks)

3.B. **(1 mark formula +1 subst.+2 final answer)**

$$\rho = \frac{1}{\sigma} = \frac{1}{ne\mu} = 29.76\Omega m$$

$$R_H = \frac{1}{ne} = -4.17 m^2 C^{-1}$$

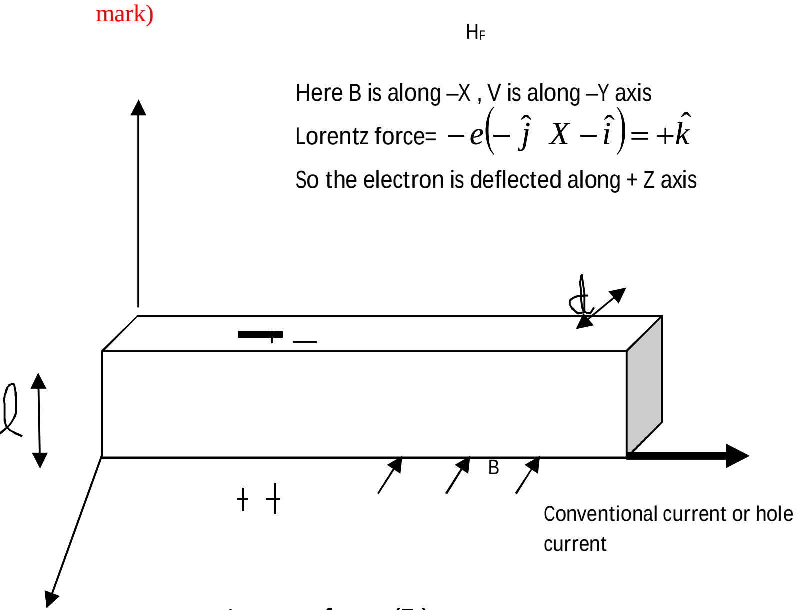
4.A

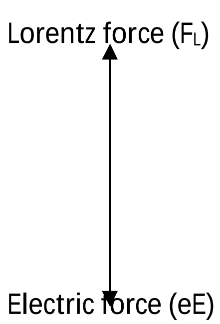
Hall effect: When a conductor carrying current is placed in transverse magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field. (1 mark)

Here B is along –X , V is along –Y axis

Lorentz force= $-e\left(-\hat{j}\ X - \hat{i}\right) = +\hat{k}$

So the electron is deflected along + Z axis





Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H-Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up. (2 marks)

Expression for Hall Coefficient:

At equilibrium, Lorentz force is equal to force due to applied electric field

$$Bev = -e E_H$$

Hall Field $E_H = Bv$

$$Current\ density\ J = -n_e ev$$

$$v = \frac{J}{n_e e}$$ (2 marks)

$$E_H = B\frac{-J}{n_e e}$$

Hence $\frac{E_H}{JB} = -\frac{1}{n_e e} = R_H$

$$V_H = E_H .l = -R_H JBl$$

(2 marks)

4.B. **To show that energy levels below Fermi energy are completely occupied:**

For E **<** E_F, at T = 0,

f (E) = $\frac{1}{e^{(\frac{E-E_F}{kT})}+1} = 1$ (1 mark)

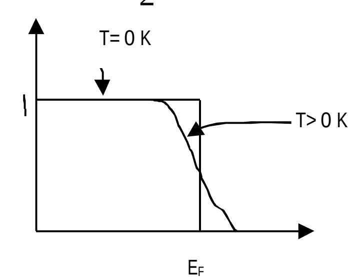
To show that energy levels above Fermi energy are empty:

For E > E_F , at T=0

f (E) = $\frac{1}{e^{(\frac{E-E_F}{kT})}+1} = 0$

At ordinary temperatures, for E = E_F,

f(E) **=** $\frac{1}{2}$ (2 marks)



5.A.

CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms (α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

where N is the number of atoms per unit volume

α is the polrisability of the atom

ε_r is the relative permittivity of the medium

ε_0 is the permittivity of free space. (1 mark)

Proof:

If there are N atoms per unit volume, the electric dipole moment per unit volume – known as polarization is given by

$$P = N\alpha E_i$$

By the definition of polarization P, it can be shown that

P = $\varepsilon_0 E_a(\varepsilon_r - 1) = N\alpha E_i$

$$\varepsilon_0\varepsilon_r E_a - \varepsilon_0 E_a = N\alpha E_i$$

$$\varepsilon_r = 1 + \frac{N\alpha E_i}{\varepsilon_0 E_a}$$ ....(1) (2 marks)

The internal field at an atom in a cubic structure (γ =1/3) is of the form

$$E_i = E_a + \frac{p}{3\varepsilon_0} = E_a + \frac{N\alpha E_i}{3\varepsilon_0}$$

$$\frac{E_i}{E_a} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\varepsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E_a}$ in equation (1)

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}\left[\frac{1}{\left(1 - \frac{N\alpha}{3\varepsilon_0}\right)}\right] = \frac{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right] + \frac{N\alpha\varepsilon_0}{\varepsilon_0}}{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right]}$$

$$= \frac{1 + \frac{2}{3}\left(\frac{N\alpha}{\varepsilon_0}\right)}{1 - \frac{1}{3}\left[\frac{N\alpha}{\varepsilon_0}\right]}$$

(2 marks)

$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{\frac{1 + (2/3)\frac{N\varepsilon}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} - 1}{\frac{1 - (2/3)\frac{N\alpha}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} + 2} = \frac{N\alpha}{3\varepsilon_0}$$

(2 marks)

5.B **(1 mark formula +1 mark subst.+1 final answer)**

$\alpha_e = \varepsilon_0(\varepsilon_r - 1)/N = 3.4\times10^{-40}$

$\varepsilon_r = 1.001$

6A.

EXPRESSION FOR FERMI ENERGY

From Fermi –Dirac theory

$$n = \int_0^{E_F} g(E).f(E).dE = \int_0^{E_F} \frac{4\pi(2m)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}} dE.1$$

$$= \frac{4\pi(2m)^{3/2}}{h^3}\frac{E_F^{\frac{3}{2}}}{\frac{3}{2}}$$

$$E_F^{\frac{3}{2}} = \frac{h^3 3n}{8\pi(2m)^{\frac{3}{2}}}$$

(4 marks)

$$E_F = \frac{h^2}{8m}\left[\frac{3n}{\pi}\right]^{\frac{2}{3}}$$ (1 mark)

Success of quantum theory: (2 marks)

1. Specific heat:

Classical theory predicted high values of specific heat for metals on the basis of the assumption that all the conduction electrons are capable of absorbing the heat energy as per Maxwell - Boltzmann distribution i.e.,

$C_V = \frac{3}{2}RT$

But according to the quantum theory, only those electrons occupying energy levels close to Fermi energy (E_F) are capable of absorbing heat energy to get excited to higher energy levels. Thus only a small percentage of electrons are capable of receiving the thermal energy and specific heat value becomes small.

It can be shown that $C_V = 10^{-4}R$.

This is in conformity with the experimental values.

2. **Temperature dependence of electrical conductivity.**

According to classical free electron theory,

Electrical conductivity $\propto \frac{1}{\sqrt{Temperature}}$

Where as from quantum theory

Electrical conductivity

$$\propto \frac{1}{\text{collisional area of cross section of lattice atoms}} \alpha \frac{1}{\text{vibrational energy}} \alpha \frac{1}{\text{Temperature}}$$

This is in agreement with experimental values.

3**. Dependence of electrical conductivity on electron concentration:**

According to classical theory,

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto n$$

But it has been experimentally found that Zinc which is having higher electron concentration than copper has lower Electrical conductivity.

According to quantum free electron theory, Electrical conductivity $\sigma = \frac{ne^2}{m}\left(\frac{\lambda}{V_F}\right)$ where V_F is the Fermi velocity.

Zinc possesses lesser conductivity because it has higher Fermi velocity.

Metal	n	σ
Cu	$8.45\times10^{28}/m^3$	$6\times10^{7}(\Omega m)^{-1}$
Zn	$13\times10^{28}/m^3$	$1\times10^{7}(\Omega m)^{-1}$

6.B. **(1 mark formula +1 subst.+1 final answer)**

$$f(E) = \frac{1}{e^{\left(\frac{E-E_F}{kT}\right)}+1} = 0.01$$

$$0.01 = \frac{1}{e^{\frac{(2.5-2)x1.6x10^{-19}}{1.38x10^{-23}XT}}+1}$$

Ans T = 1261 K

7A

<u>**Expression for condition for Numerical aperture**</u>

Consider a light ray falling in to the optical fibre at an angle of incidence θ_0 equal to acceptance angle. Let n_0 be the refractive index of the surrounding medium .

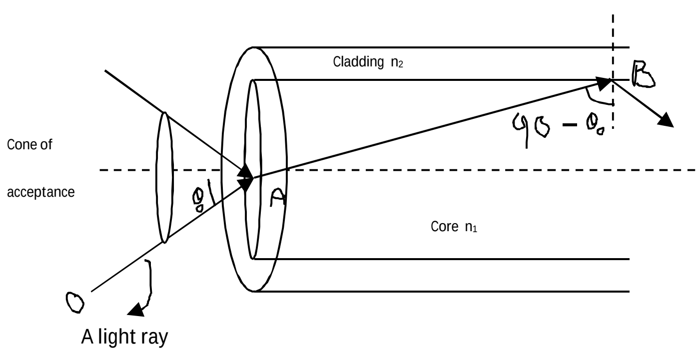
Let n_1 be the refractive index of the core.

Let n_2 be the refractive index of the cladding.

From Snell's Law:

For the ray OA $n_0 \sin\theta_0 = n_1 \sin\theta_1$ (2 marks)

$$= n_1(1-\cos^2\theta_1)^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$



For the ray AB

$$n_1 \sin(90 - \theta_1) = n_2 \sin 90$$

[here the angle of incidence is $(90 - \theta_1)$ for which angle of refraction is 90^0].

$n_1 \cos\theta_1 = n_2$ (2 marks)

Substituting for $\cos\theta_1$ in equation (1)

$$n_0 \sin\theta_0 = n_1\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

If the medium surrounding the fiber is air then $n_0 = 1$,

Numerical aperture = $\sin\theta_0 = \sqrt{n_1^2 - n_2^2}$ (2 marks)

The total internal reflection will take place only if the angle of incidence $\theta_i < \theta_0$

$\therefore \sin\theta_i < \sin\theta_0$

$$\mathbf{\sin\theta_i} < \sqrt{n_1^2 - n_2^2} \text{ (1 mark)}$$

This is the condition for propagation.

7.B **(1 mark formula +1 subst.+1 final answer)**

$$Attenuation = \frac{10}{L}\log\left(\frac{P_{in}}{P_{out}}\right)$$

$$2 = \frac{10}{3}\log\left[\frac{P_{in}}{P_{out}}\right]$$

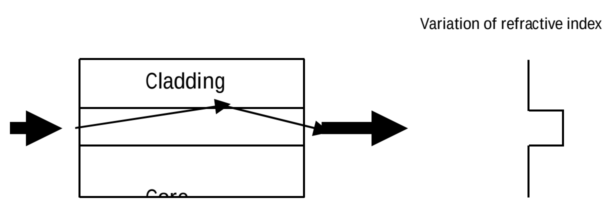
$$\frac{p_{in}}{p_{out}} = 3.98$$

$$\frac{p_{out}}{p_{in}} = 0.251$$

8A

1. Single mode fiber: (2 marks)

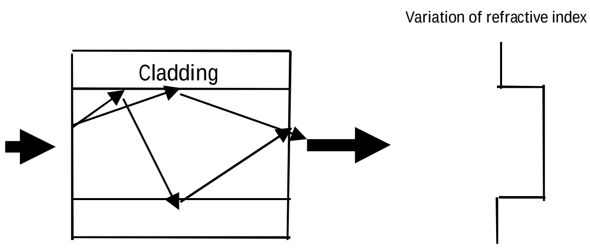
Core diameter is around 5-10 μm. The core is narrow and hence it can guide just a single mode.



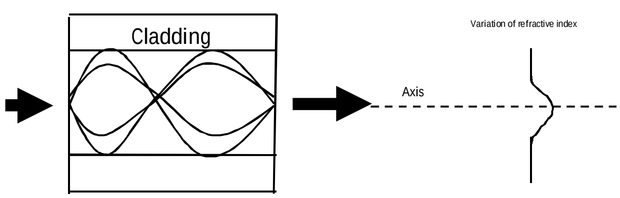
- No modal dispersion
- Difference between n_1 & n_2 is less.Critical angle is high. Low numerical aperture.
- Low Attenuation -0.35db/km
- Bandwidth -100GHz
- Preferred for short range

Step index multimode fibre : (2 marks)

- Here the diameter of core is larger so that large number of rays can propagate. Core diameter is around 50. µm.
- High modal dispersion
- Difference between n_1 & n_2 is high. Low Critical angle. Large numerical aperture.
- Losses high
- Bandwidth -500MHz
- Allows several modes to propagate
- Preferred for Long range



Graded index multimode fiber:
In this type, the refractive index decreases in the radially outward direction from the axis and becomes equal to that of the cladding at the interface. Modes travelling close to the axis move slower where as the modes close to the cladding move faster. As a result the delay between the modes is reduced. This reduces modal dispersion.

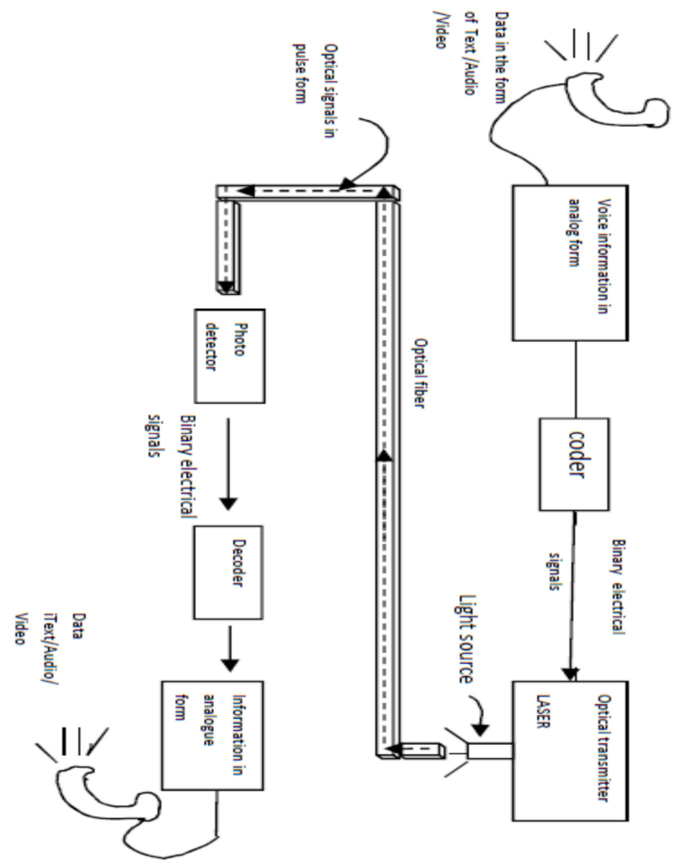


- Low modal dispersion
- High data carrying capacity.
- High cost
- Many modes propagate
- Bandwidth -10GHz

<u>Point to point communication system using optical fibers</u> (3 marks)

This system is represented through a block diagram as follows.

The information in the form of voice/ picture/text is converted to electrical signals through the transducers such as microphone/video camera. The analog signal is converted in to binary data with the help of coder. The binary data in the form of electrical pulses are converted in to pulses of optical power using Semiconductor Laser. This optical power is fed to the optical fiber. Only those modes within the angle of acceptance cone will be sustained for propagation by means of total internal reflection. At the receiving end of the fiber, the optical signal is fed in to a photo detector where the signal is converted to pulses of current by a photo diode. Decoder converts the sequence of binary data stream in to an analog signal . Loudspeaker/CRT screen provide information such as voice/ picture.



8.B (1 mark formula +1 subst.+1 final answer)

$$n_{water} . \sin \theta_A = \sqrt{n_{core}^2 - n_{cladding}^2}$$

$$\theta_A = 16.78^0$$