CBCS SCHEME

USN 1 C R 1 6 M E 0 2 0 **15ME73**

Seventh Semester B.E. Degree Examination, Dec.2019/Jan.2020

Control Engineering

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With a block diagram differentiate open loop and closed loop system. **(08 Marks)**

b. Discuss the main requirements of an ideal control system. **(08 Marks)**

OR

2 Explain following types of controller with block diagram and state its characteristics.

(i) Proportional
(ii) Proportional plus derivative
(iii) Integral
(iv) Proportional plus integral **(16 Marks)**

Module-2

3 a. Obtain the transfer function for an armature controlled D.C motor, which relates output angular displacement (Q) with input voltage (e). **(08 Marks)**

b. A thermometer is dipped in a vessel containing liquid at a constant temperature of θ_1. thermometer has a thermal capacitance for storing heat as C and thermal resistance to limit heat flow as R. If the temperature indicated by thermometer is θ_r, obtain the transfer function of the system. **(08 Marks)**

OR

4 a. Obtain the overall transfer function of the block diagram shown in Fig.Q4(a) by reduction technique. **(10 Marks)**

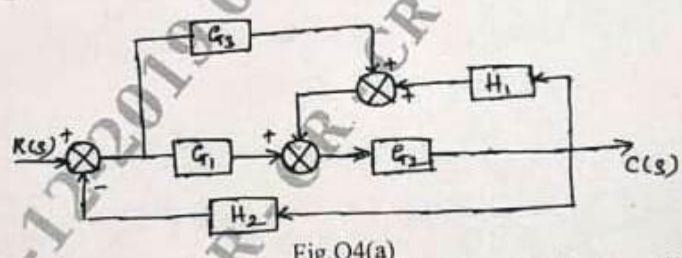


Fig.Q4(a)

b. Discuss Mason's gain formula and define the following terms used in signal flow graphs.
(i) Node (ii) Branch gain (iii) Forward path (iv) Path gain (v) Feedback loop
(vi) Self loop **(06 Marks)**

Module-3

5 Obtain the expressions for Peak time, Rise time, Maximum overshoot and settling time for a second order control system in terms of damping factor and nature frequency. **(16 Marks)**

OR

6 Sketch the root locus of unity feedback system whose forward path transfer function is

$$G(s) = \frac{k}{s(s^2 + 5s + 6)}$$

Determine the range of k for the system to be stable. **(16 Marks)**

Module-4

7 Draw the Bode plot for the following transfer function and determine gain margin and phase margin.

$$G(s)H(s) = \frac{10.5}{(s + 0.2)(s + 0.8)(s + 10)}$$

(16 Marks)

OR

8 Using Nyquist criterion, investigate the stability of a system whose open loop transfer function is $G(s)H(s) = \dfrac{k}{(s+1)(s+2)(s+3)}$ **(16 Marks)**

Module-5

9 Obtain the transfer functions of the following types of compensators:

(i) Lag compensator

(ii) Lead compensator **(16 Marks)**

OR

10 a. Explain the following :

(i) Kalman's test of controllability

(ii) Kalman's test of observability **(06 Marks)**

b. Determine the controllability and observability of the systems represented by

$$\dot{x} = \begin{bmatrix} -3 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 2 & 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(10 Marks)

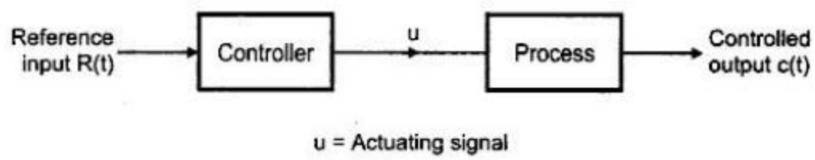
* * * * *

Ans 1 a)

OPEN LOOP SYSTEM

Definition : A system in which output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system, is called as Open Loop System.

In a broad manner it can be represented as in Fig.



Reference input [R(t)] is applied to the controller which generates the actuating signal (u) required to control the process which is to be controlled. Process is giving out the necessary desired controlled output C(t).

Advantages :

1) Such systems are simple in construction.
2) Very much convenient when output is difficult to measure.
3) Such systems are easy from maintenance point of view.
4) Generally these are not troubled with the problems of stability.
5) Such systems are simple to design and hence economical.

Disadvantages :

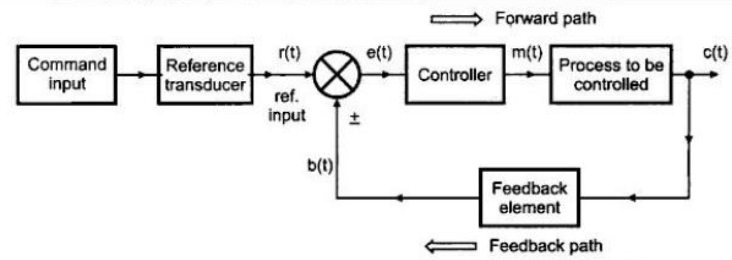
1) Such systems are inaccurate and unreliable because accuracy of such systems are totally dependent on the accurate precalibration of the controller.
2) Such systems give inaccurate results if there are variations in the external environment i.e. cannot sense environmental changes.
3) Similarly they cannot sense internal disturbances in the system, after the controller stage.
4) To maintain the quality and accuracy, recalibration of the controller is necessary, time to time.

CLOSED LOOP SYSTEM

Definition : A system in which the controlling action or input is somehow dependent on the output or changes in output is called as closed loop system.

To have dependence of input on the output, such system uses the feedback property.

Closed loop system can be represented as in Fig.



r(t) = Reference Input   e(t) = Error signal

c(t) = Controlled output   m(t) = Manipulated signal   b(t) = Feedback signal

Advantages :

1) Accuracy of such system is always very high because controller modifies and manipulates the actuating signal such that error in the system will be zero.

2) Such system senses environmental changes, as well as internal disturbances and accordingly modifies the error.

3) In such system, there is reduced effect of nonlinearities and distortions.

4) Bandwidth of such system i.e. operating frequency zone for such system is very high.

Disadvantages :

1) Such systems are complicated and time consuming from design point of view and hence costlier.

2) Due to feedback, system tries to correct the error time to time. Tendency to overcorrect the error may cause oscillations without bound in the system. Hence system has to be designed taking into consideration problems of instability due to feedback.

	Open Loop		Closed Loop
1)	Any change in output has no effect on the input i.e. feedback does not exists.	1)	Changes in output, affects the input which is possible by use of feedback.
2)	Output measurement is not required for operation of system.	2)	Output measurement is necessary.
3)	Feedback element is absent.	3)	Feedback element is present.
4)	Error detector is absent.	4)	Error detector is necessary.
5)	It is inaccurate and unreliable.	5)	Highly accurate and reliable.
6)	Highly sensitive to the disturbances.	6)	Less sensitive to the disturbances.
7)	Highly sensitive to the environmental changes.	7)	Less sensitive to the environmental changes.
8)	Bandwidth is small.	8)	Bandwidth is large.
9)	Simple to construct and cheap.	9)	Complicated to design and hence costly.
10)	Generally are stable in nature.	10)	Stability is the major consideration while designing
11)	Highly affected by nonlinearities.	11)	Reduced effect of nonlinearities.

Ans 1 b)

REQUIREMENTS OF AN IDEAL CONTROL SYSTEM

An ideal system of control is that which makes the controlling function easy, effective and smooth.

Speed

The control system's response should be swift. This is necessary to ensure that the process variables do not stray away from the set point which can affect the process.

Damping

The number of oscillations should be as low as possible. Oscillations can cause disturbances in the system. The damping of the system should be optimum.

Accuracy

Accuracy refers to the ability of the system to detect even the smallest deviation and initiate a corrective response. The sensors and the error sensing algorithms should have a high resolution.

Noise

Noise refers to undesirable interferences in the signal of the transducer. There are many reasons for noise. Electromagnetic Interference (EMI) is one of the reasons. A good Control system should be able to filter the noise and process only the principal signal.

Stability

Any control system should be stable about the set point. It should maintain the process variable at the set point. It should not react to changes in the surroundings, temperature or any other external parameter. It should react only to the principal input signal.

Bandwidth

Bandwidth is dependent on the frequency of operation. The Bandwidth should be as large as possible for a good frequency response of the system.

Ans 2)

PROPORTIONAL CONTROLLERS

For **Proportional controllers** to be used :

1. Deviation should not be large; it means there should be less deviation between the input and output.
2. Deviation should not be sudden.

As the name suggests in a proportional controller the output (also called the actuating signal) is directly proportional to the error signal. Now let us analyze proportional controller mathematically. As we know in proportional controller output is directly proportional to error signal, writing this mathematically we have,

$$A(t) \propto e(t)$$

Removing the sign of proportionality we have,

$$A(t) = K_p \times e(t)$$

Where, K_p is proportional constant also known as controller gain.
It is recommended that K_p should be kept greater than unity. If the value of K_p is greater than unity (>1), then it will amplify the error signal and thus the amplified error signal can be detected easily.

Advantages of Proportional Controller

1. Proportional controller helps in reducing the steady state error, thus makes the system more stable.
2. Slow response of the over damped system can be made faster with the help of these controllers.

Disadvantages of Proportional Controller

1. Due to presence of these controllers, we get some offsets in the system.
2. Proportional controllers also increase the maximum overshoot of the system.

PROPORTIONAL DERIVATIVE CONTROLLER (PD)

As the name suggests it is a combination of proportional and a derivative controller the output (also called the actuating signal) is equals to the summation of proportional and derivative of the error signal. Writing this mathematically we have,

$$A(t) \propto \frac{de(t)}{dt} + A(t) \propto e(t)$$

Removing the sign of proportionality we have,

$$A(t) = K_d \frac{de(t)}{dt} + K_p e(t)$$

Advantages

1. Overall stability of system improves
2. Capable of handling processes with time lag
3. Reduces settling time by improving damping and reducing overshoot

Disadvantages

1. Not suited for fast responding systems which are usually lightly damped or initially unstable.
2. Amplifies noise at higher frequencies which result in improper handling of actuators.
3. Does not eliminate steady state error

INTEGRAL CONTROLLERS

In **integral controllers** the output (also called the actuating signal) is directly proportional to the integral of the error signal.

$$A(t) \propto \int_0^t e(t)dt$$

Removing the sign of proportionality we have

$$A(t) = K_i \times \int_0^t e(t)dt$$

Where, K_i is integral constant also known as controller gain. Integral controller is also known as reset controller.

Advantages of Integral Controller

Due to their unique ability they can return the controlled variable back to the exact set point following a disturbance and are known as reset controllers.

Disadvantages of Integral Controller

It tends to make the system unstable because it responds slowly towards the produced error.

PROPORTIONAL INTEGRAL CONTROLLER (PI)

As the name suggests it is a combination of proportional and an integral controller the output (also called the actuating signal) is equal to the summation of proportional and integral of the error signal. Writing this mathematically we have,

$$A(t) = K_i \int_0^t e(t)dt + K_p e(t)$$

Where, K_i and k_p proportional constant and integral constant respectively.

Advantages

1. Desired value can be achieved accurately.
2. Ease to apply for fast response processes as well as processes in which load change is large and frequent.
3. Removes steady state error.

Disadvantages

1. The speed of response of system becomes sluggish due to the addition of integral term.
2. During start-up of a batch process, the integral action causes an overshoot.

Ans 3 a)

Assumptions :

(i) Flux is directly proportional to current through field winding,

$$\phi_m = K_f I_f = \text{constant}$$

(ii) Torque produced is proportional to product of flux and armature current.

$$T = K'_m \phi I_a$$

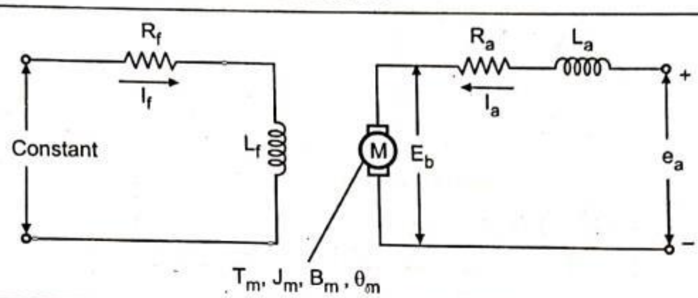
$$T = K'_m K_f I_f I_a$$

(iii) Back e.m.f. is directly proportional to shaft velocity ω_m, as flux ϕ is constant.

$$\text{as} \quad \omega_m = \frac{d\theta(t)}{dt}$$

$$E_b = K_b \omega_m(s) = K_b s \theta_m(s)$$

Apply Kirchhoff's law to armature circuit :



$$e_a = E_b + I_a(R_a) + L_a \frac{di_a}{dt}$$

Take Laplace transform,

$$\therefore \quad E_a(s) = E_b(s) + I_a(s)[R_a + sL_a]$$

$$\therefore \quad I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + sL_a}$$

$$I_a(s) = \frac{E_a(s) - K_b s\theta_m(s)}{R_a + sL_a}$$

Now

$$T_m = K'_m K_f I_f I_a$$

$$T_m = K'_m K_f I_f \left\{\frac{E_a - K_b s\theta_m(s)}{R_a + sL_a}\right\}$$

Also

$$T_m = \{J_m s^2 + sB_m\}\theta_m(s) \quad \text{... from equation (3)}$$

Equating equations of T_m,

$$\frac{K'_m K_f I_f E_a(s)}{(R_a + sL_a)} = \frac{K'_m K_f I_f K_b s\theta_m(s)}{(R_a + sL_a)} + (J_m s^2 + sB_m)\theta_m$$

$$\therefore \quad \frac{K'_m K_f I_f}{(R_a + sL_a)} E_a(s) = \left[\frac{K'_m K_f I_f K_b s}{(R_a + sL_a)} + J_m s^2 + sB_m\right]$$

$$\therefore \quad \frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{sR_a B_m(1+s\tau_m)(1+s\tau_a)}}{1+\dfrac{K_m}{sR_a B_m(1+s\tau_m)(1+s\tau_a)\cdot sK_b}}$$

$$= \frac{G(s)}{1+G(s)H(s)}$$

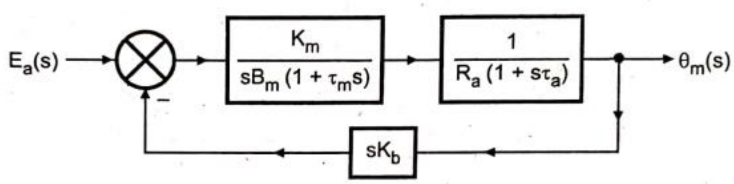
where $\tau_m = J_m/B_m$ and $\tau_a = \dfrac{L_a}{R_a}$

$$K_m = K'_m K_f$$

$$G(s) = \frac{K_m}{sR_a B_m(1+s\tau_m)(1+s\tau_a)}$$

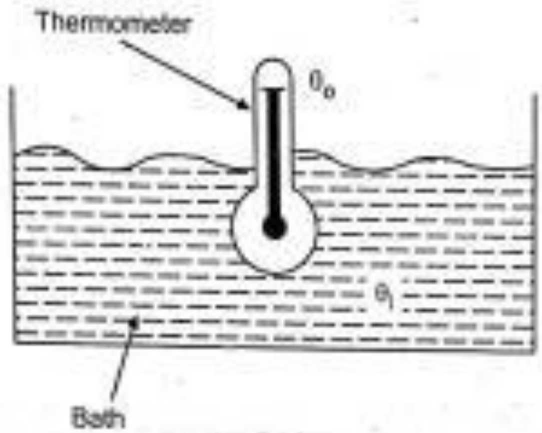
$$H(s) = sK_b$$

Therefore can be represented in its block diagram form as in Fig. 6.29



Ans 3 b)

Consider a thermometer placed in a water bath having temperature θ_i, as shown



θ_o is the temperature indicated by the thermameter. The rate of heat flow into the thermometer through its wall is,

$$\frac{dq}{dt} = \frac{\theta_i - \theta_o}{R}$$

where R = Thermal resistance of the thermometer wall

The indicated temperature, rises at a rate of

$$\frac{d\theta_o}{dt} = \frac{1}{C}\frac{dq}{dt}$$

where C is thermal capacity of the thermometer.

$$\therefore \quad \frac{d\theta_o}{dt} = \frac{1}{C}\cdot\left[\frac{\theta_i - \theta_o}{R}\right]$$

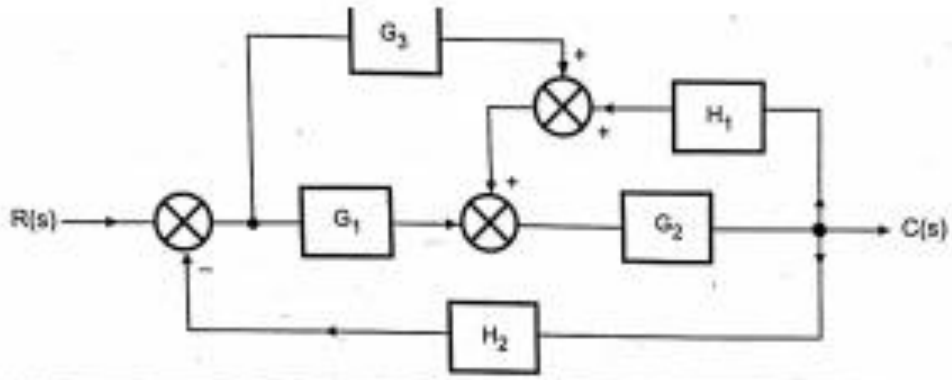
Taking Laplace of the equation,

$$\therefore \quad s\theta_o(s) = \frac{1}{RC}[\theta_i(s) - \theta_o(s)]$$

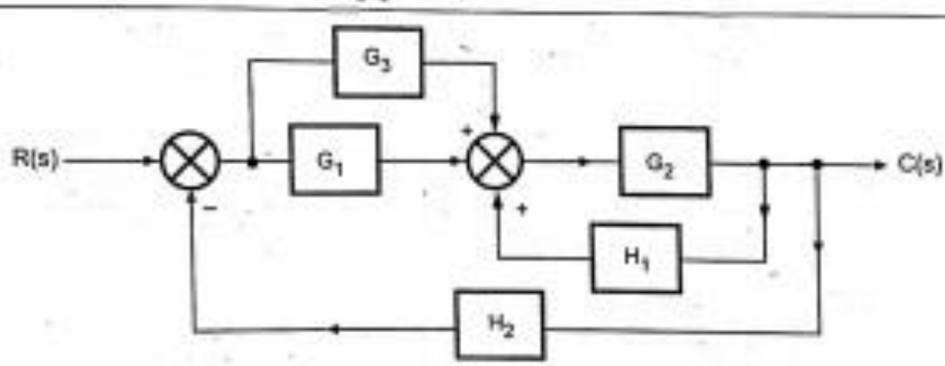
$$\therefore \quad \frac{\theta_o(s)}{\theta_i(s)} = \frac{1}{1 + sRC}$$

The time constant is RC.

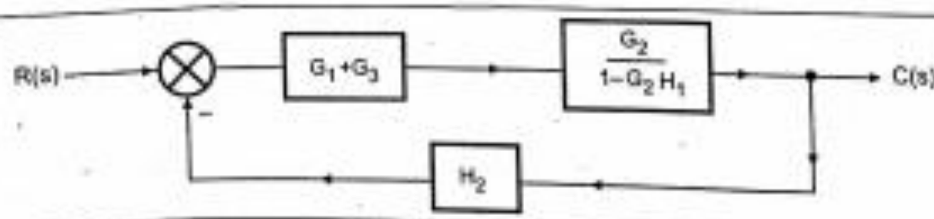
Ans 4 a)



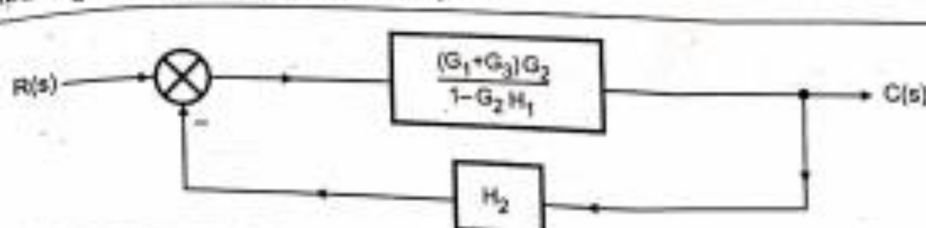
Sol. : Combining the two summing points,



The blocks G_1 and G_3 are in parallel while G_2 and H_1 forms a minor feedback loop.



Combining the blocks in the forward path,



Hence the overall transfer function is,

$$\frac{C(s)}{R(s)} = \frac{\dfrac{(G_1+G_3)G_2}{1-G_2H_1}}{1+\dfrac{(G_1+G_3)G_2H_2}{(1-G_2H_1)}} = \frac{(G_1+G_3)G_2}{1-G_2H_1+(G_1+G_3)G_2H_2}$$

$$= \frac{G_1G_2+G_3G_2}{1-G_2H_1+G_1G_2H_2+G_2G_3H_2}$$

Ans 4 b)

i) **Source Node :** The node having only outgoing branches is known as source or input node. e.g. x_0 is source node.

ii) **Sink Node :** The node having only incoming branches is known as sink or output node. e.g. x_5 is sink node.

iii) **Chain Node :** A node having incoming and outgoing branches is known as chain node. e.g. x_1, x_2, x_3 and x_4.

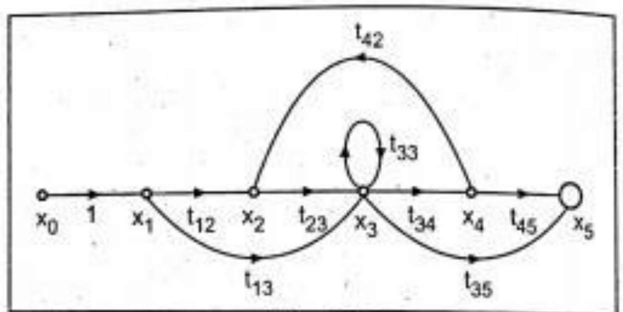


Fig. 5.6

iv) **Forward Path :** A path from the input to output node is defined as forward path.

e.g. $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ First forward path

$x_0 - x_1 - x_3 - x_4 - x_5$ Second forward path.

$x_0 - x_1 - x_3 - x_5$ Third forward path.

$x_0 - x_1 - x_2 - x_3 - x_5$ Fourth forward path.

v) **Feedback Loop :** A path which originates from a particular node and terminating at the same node, travelling through at least one other node, **without tracing any node twice** is called feedback loop. For example, $x_2 - x_3 - x_4 - x_2$.

vi) **Self Loop :** A feedback loop consisting of only one node is called self loop. i.e. t_{33} at x_3 is self loop. A self loop can not appear while defining a forward path or feedback loop as node containing it gets traced twice which is not allowed.

vii) **Path Gain :** The product of branch gains while going through a forward path is known as path gain. i.e. path gain for path $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ is,

$1 \times t_{12} \times t_{23} \times t_{34} \times t_{45}$. This can be also called **forward path gain.**

Ans 5)

Derivation of Peak Time T_p

Transient response of second order underdamped system is given by,

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

Where $$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

As at $t = T_p$, c(t) will achieve its maxima. According to Maxima theorem,

$$\left.\frac{dc(t)}{dt}\right|_{t=T_p} = 0$$

So differentiating c(t) w.r.t. `t' we can write,

i.e. $$-\frac{e^{-\xi\omega_n t}(-\xi\omega_n)\sin(\omega_d t+\theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_d \cos(\omega_d t+\theta) = 0$$

Substituting $\omega_d = \omega_n \sqrt{1-\xi^2}$

$$\frac{\xi\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t+\theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\omega_n \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \quad \xi \sin(\omega_d t+\theta) - \sqrt{1-\xi^2}\cos(\omega_d t+\theta) = 0$$

$$\therefore \quad \tan(\omega_d t+\theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

Now $$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$\therefore \quad \frac{\sqrt{1-\xi^2}}{\xi} = \tan\theta$$

$$\therefore \quad \tan(\omega_d t+\theta) = \tan\theta$$

From trignometric formula,

$$\tan(n\pi+\theta) = \tan\theta$$

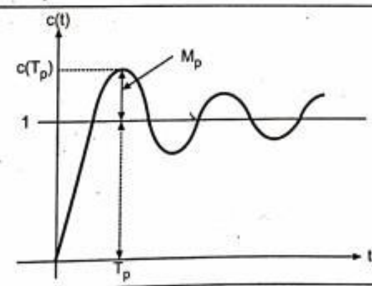
$$\therefore \quad \omega_d t = n\pi \qquad \text{where } n = 1, 2, 3$$

But T_p, time required for first peak overshoot. $\therefore$ $n = 1$

$$\omega_d T_p = \pi$$

$$\therefore \quad \boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec}}$$

Derivation of M_p



From the Fig. 7.27, $M_p = C(T_p) - 1$

$$M_p = \left\{1 - \frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)\right\} - 1$$

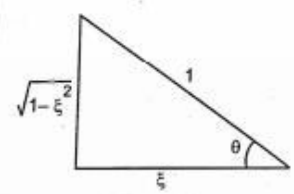
$\therefore$ $$M_p = -\frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)$$

But $T_p = \frac{\pi}{\omega_d}$, substituting above we get,

$\therefore$ $$M_p = \frac{-e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\pi + \theta)$$

Now, $\sin(\pi + \theta) = -\sin(\theta)$

$\therefore$ $$M_p = \frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin\theta$$



we know, $\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$ as shown in the Fig. 7.28.

$\therefore$ $$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \text{ but hypotenuse} = \sqrt{(\sqrt{1-\xi^2})^2 + \xi^2} = 1$$

$\therefore$ $$\sin\theta = \sqrt{1-\xi^2}$$

$\therefore$ $$M_p = \frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \cdot \frac{\sqrt{1-\xi^2}}{1}$$

$$= e^{-\xi\omega_n T_p}$$

Substitute $T_p = \frac{\pi}{\omega_d}$

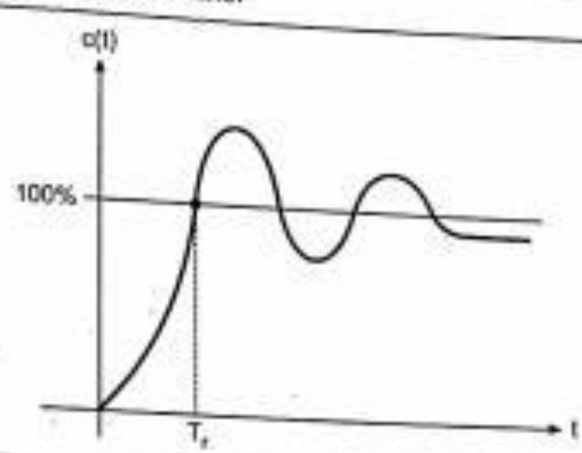
$\therefore$ $$M_p = e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$= e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

$\therefore$ $$\boxed{\% M_p = 100\, e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}}$$

Derivation of T_r

Time required by output to achieve 100% of its final value, starting from zero during the first attempt is the rise time.



i.e. $\{c(t)|_{t=T_r}\} = 1$ for unit step input

$$\therefore \quad 1 = 1 - \frac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}} \sin(\omega_d T_r + \theta)$$

$$\therefore \quad -\frac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}} \sin(\omega_d T_r + \theta) = 0$$

Equation will get satisfied only if,

$$\sin(\omega_d T_r + \theta) = 0$$

Trigonometrically this is true only if,

$$\omega_d T_r + \theta = n\pi \quad \text{where } n = 1, 2 \ldots$$

As we are interested in first attempt use $n = 1$

$$\therefore \quad \omega_d T_r + \theta = \pi$$

$$\therefore \quad \boxed{T_r = \frac{\pi - \theta}{\omega_d} \text{ sec}}$$

Derivation of T_s

The settling time T_s is the time required by the output to settle down within ± 2% of tolerance band. So T_s is the time when output becomes 98% of its final value and remains within the range of ± 2% as $t \to \infty$.

$$\therefore \quad c(t) \text{ at } (t = T_s) = 0.98$$

Now at $t = T_s$, the transient oscillatory term completely vanishes. The only term which controls the amplitude of the output within ± 2% is $e^{-\xi \omega_n t}$. Hence value of T_s is obtained considering only exponentially decaying envelope, neglecting all other terms.

$$\therefore \quad c(t) \text{ at } (t = T_s) = 1 - e^{-\xi \omega_n T_s}$$

$$\therefore \quad 0.98 = 1 - e^{-\xi \omega T_s}$$

$$\therefore \quad e^{-\xi \omega_n T_s} = 1 - 0.98 = 0.02$$

$$\therefore \quad -\xi \omega_n T_s = \ln (0.02)$$

$$= -3.912$$

$$\therefore \quad T_s = \frac{3.912}{\xi \omega_n}$$

In practice the settling time is assumed to be,

$$\boxed{T_s = \frac{4}{\xi \omega_n} \quad \text{for} \pm 2\% \text{ tolerance}}$$

Ans 6)

Sol. : ***Step 1 :*** P = 3, Z = 0, N = P = 3 branches

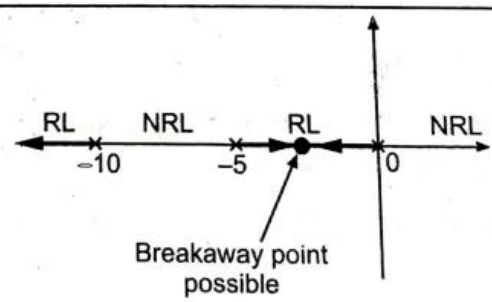
P – Z = 3 branches approaching to ∞

Starting points = 0, –5, –10 ... open loop poles.

Terminating points = ∞, ∞, ∞ ... no open loop zeros so ∞.

Step 2 : Sections of real axis

One breakaway point between 0 and – 5 exists.



Step 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)\,180^\circ}{P-Z}, \quad q = 0, 1, 2$$

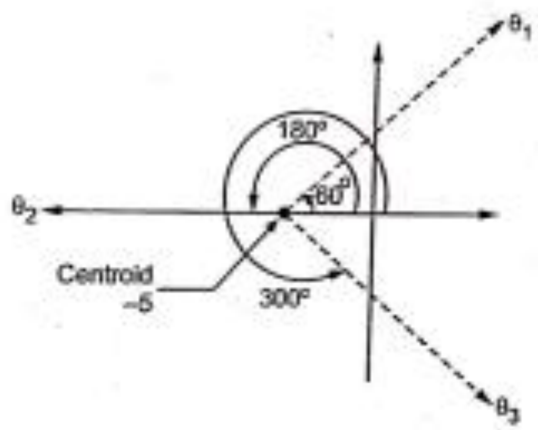
$$\theta_1 = \frac{180^\circ}{3} = 60^\circ, \quad \theta_2 = \frac{3\times 180^\circ}{3} = 180^\circ, \quad \theta_3 = \frac{5\times 180^\circ}{3} = 300^\circ$$

Three asymptotes required for 3 branches approaching to ∞.

Step 4 : Centroid

$$\sigma = \frac{\sum \text{R.P. of poles} - \sum \text{R.P. of zeros}}{P-Z} = \frac{0-5-10-0}{3}$$

$$= -5$$



Step 5 : Breakaway point

$$1 + G(s)\,H(s) = 0$$

$$1+\frac{K}{s(s+5)\,(s+10)} = 0$$

$$\therefore\ s^3+15s^2+50s+K = 0$$

$$\therefore\ K = -s^3-15s^2-50s \qquad \text{... (1)}$$

$$\therefore\ \frac{dK}{ds} = -3s^2-30s-50 = 0$$

$$\therefore\ s^2+10s+16.667 = 0$$

$$\therefore\ s = \frac{-10\pm\sqrt{(10)^2-4\times 16.67}}{2} = -2.113,\ -7.88$$

Thus s = – 2.113 is a valid breakaway point.

Substituting in (1),

$$K = -(-2.113)^3-15(-2.113)^2-50(-2.118) = +48.112$$

so $K = +48.112$ for $s = -2.113$

As K is positive, s = − 2.113 is valid breakaway point.

Step 6 : Intersection with negative real axis.

$$s^3 + 15s^2 + 50s + K = 0$$

s^3	1	50
s^2	15	K
s^1	$\frac{750 - K}{15}$	0
s^0	K	

From row of s^1, $750 - K = 0$

$$\therefore \quad K_{mar} = 750$$

$$A(s) = 15s^2 + K = 0$$

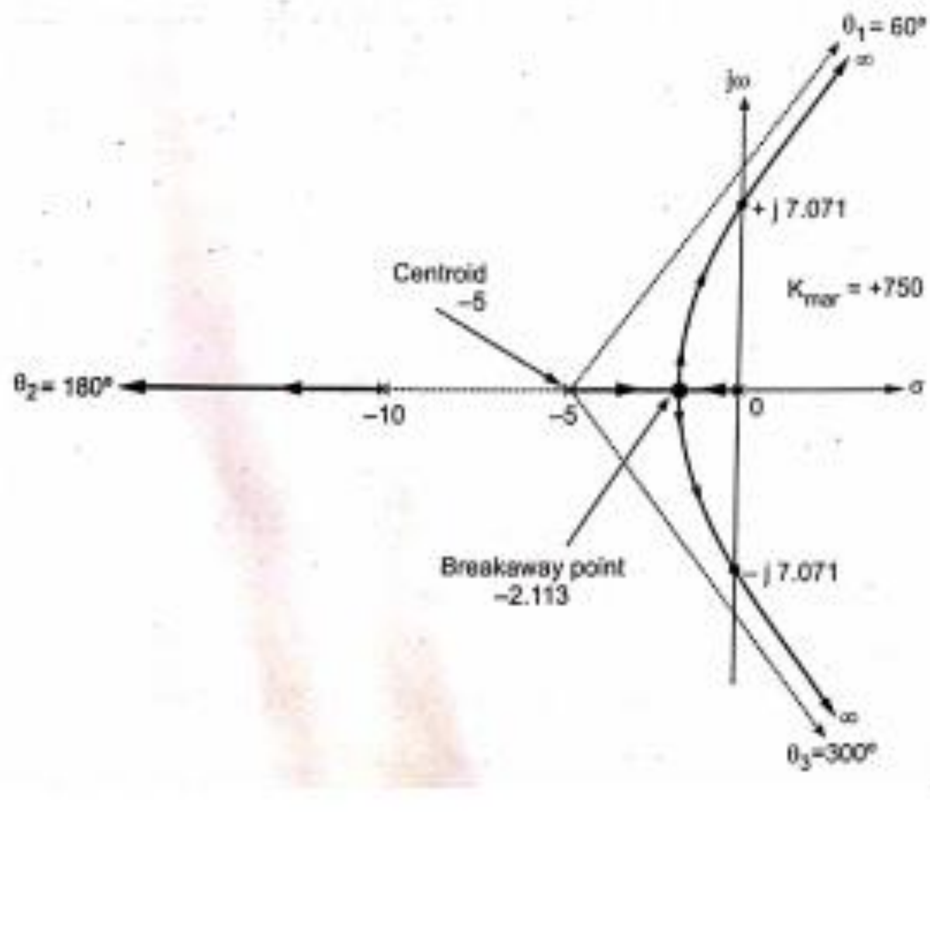
$$\therefore \quad 15s^2 + 750 = 0$$

$$\therefore \quad s^2 = -\frac{750}{15} = -50$$

$$\therefore \quad s = \pm j\sqrt{50} = \pm j\,7.071 \quad \text{... Intersection with imaginary axis}$$

Step 7 : No complex poles hence angles of departure not required. Root locus breaks at ±90°, at breakaway point.

Step 8 : The complete root locus is shown in the figure.



Step 9 : Comment on stability

For $0 < K < 750$, system is stable as entire root locus is in left half of s-plane

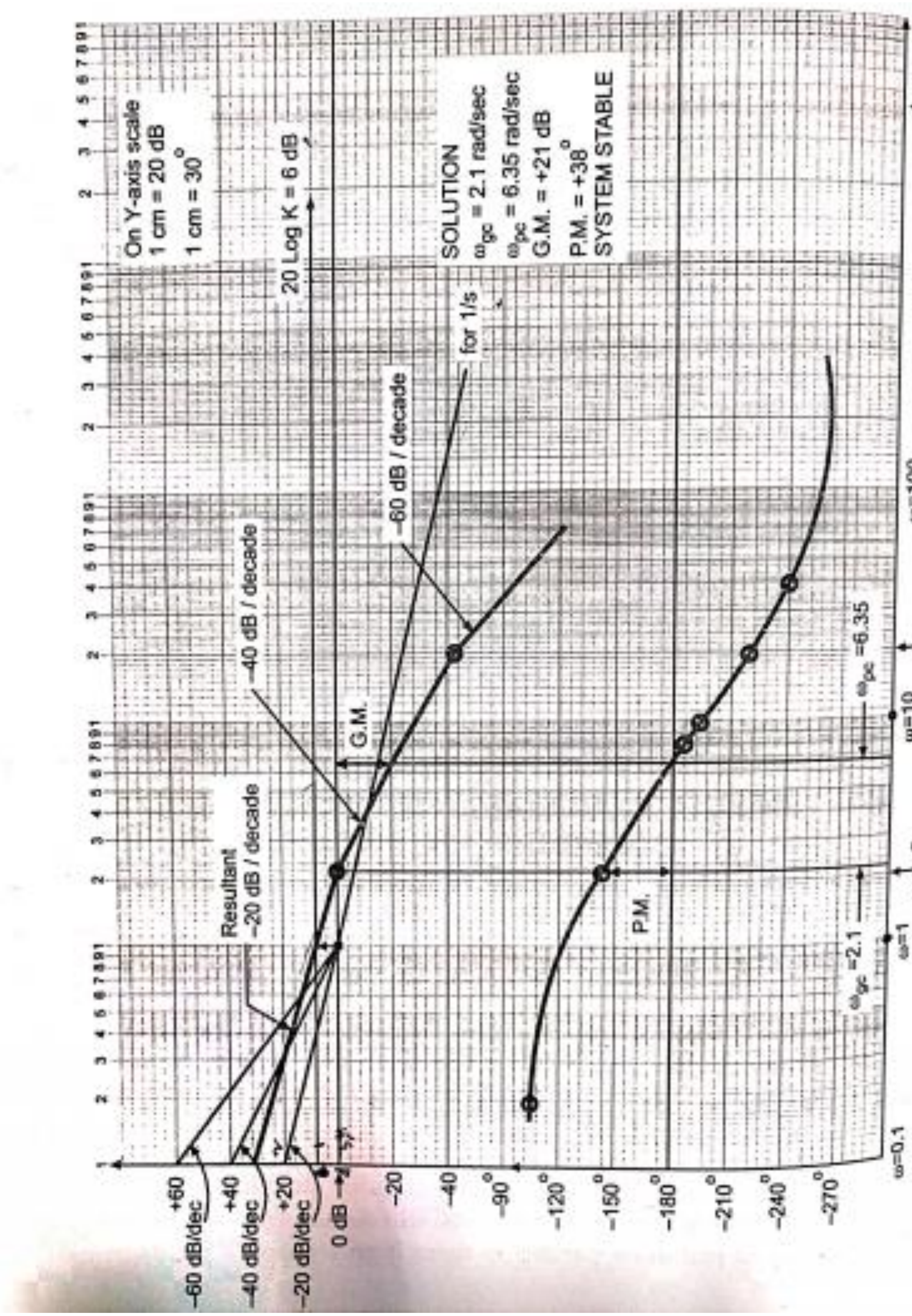
At $k = 750$, the system is marginally stable.

For $750\ K < \infty$, the system is unstable. This is because the dominant roots move in right half of s-plane.

Key Point : *Stability is predicted by the locations of the dominant roots. The dominant roots are those which are located closest to the imaginary axis. The branches starting from such roots which are dominant decide the stability.*

In this problem, branches starting from $s = 0$ and $s = -5$ are dominant root locus branches. The branch starting from $s = -10$ is not dominant as it moves away form imaginary axis to the left. So stability of system will get decided by root locus branches starting from $s = 0$ and -5 which are dominant. Using the same concept, a third order system having three roots can be approximated as a second order, considering only its dominant roots.

Ans 7



On Y-axis scale
1 cm = 20 dB
1 cm = 30°
20 Log K = 6 dB
SOLUTION
ωgc = 2.1 rad/sec
ωpc = 6.35 rad/sec
G.M. = +21 dB
P.M. = +38°
SYSTEM STABLE
for 1/s
-60 dB / decade
-40 dB / decade
Resultant
-20 dB / decade
G.M.
P.M.
-60 dB/dec
-40 dB/dec
-20 dB/dec
+60
+40
+20
0 dB
-20
-40
-90°
-120°
-150°
-180°
-210°
-240°
-270°
ω=0.1
ω=1
ω=10
ω=100
ωgc=2.1
ωc1=2
ωpc=6.35
ωc2=20
Log ω

Ans 8)

Sol. : *Step 1* : $P = 0$

Step 2 : $N = -P = 0$, the critical point $-1 + j0$ should not get encircled by Nyquist plot.

Step 3 : Pole at origin hence Nyquist path is,

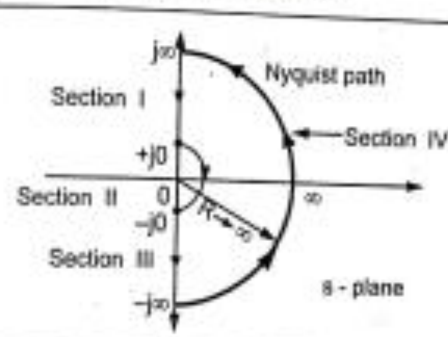


Fig. 13.28

Step 4 : $$G(j\omega)\,H(j\omega) = \frac{K}{j\omega(2+j\omega)(10+j\omega)}$$

$$M = |G(j\omega)\,H(j\omega)| = \frac{K}{\omega \times \sqrt{4+\omega^2} \times \sqrt{100+\omega^2}}$$

$$\phi = \frac{\tan^{-1}\left(\frac{0}{K}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega}{2}\right)\tan^{-1}\left(\frac{\omega}{10}\right)}$$

$$= -90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

Section I : $s = +j\infty$ to $s = +j0$ i.e. $\omega \to \infty$ to $\omega \to +0$

Starting point	$\omega \to \infty$	$0\angle -270^\circ$	$-90-(-270) = +180^\circ$ Anticlockwise rotation
Terminating point	$\omega \to +0$	$0\angle +90^\circ$	

Section II : $s = +j0$ to $s = -j0$ i.e. $\omega \to +0$ to $\omega \to -0$

Starting point	$\omega \to +0$	$\infty\angle -90^\circ$	$90-(-90) = +180^\circ$ Anticlockwise rotation
Terminating point	$\omega \to -0$	$\infty\angle +90^\circ$	

Section III is mirror image of section about real axis.

Section IV is an origin.

Step 5 : $$G(j\omega)\,H(j\omega) = \frac{K}{j\omega(10+j\omega)(2+j\omega)}$$

Rationalizing ,

$$G(j\omega)\,H(j\omega) = \frac{K(-j\omega)(10-j\omega)(2-j\omega)}{(j\omega)(-j\omega)(10+j\omega)(10-j\omega)(2+j\omega)(2-j\omega)}$$

$$\therefore \quad G(j\omega)\,H(j\omega) = \frac{-Kj\omega[20-12j\omega-\omega^2]}{\omega^2(4+\omega^2)(100+\omega^2)} = \frac{-12K\omega^2}{D} - \frac{Kj\omega(20-\omega^2)}{D}$$

where $D = \omega^2(4+\omega^2)(100+\omega^2)$

Equating imaginary part to zero,

$$\omega(20-\omega^2) = 0$$

i.e. $\omega^2 = 20$

$\therefore \quad \omega_{pc} = \sqrt{20}$

Substituting in real part,

$$\text{Point } Q = \frac{-12K\times 20}{20\times(20+4)\times(100+20)} = -\frac{K}{240}$$

Step 6 : The Nyquist Plot is,

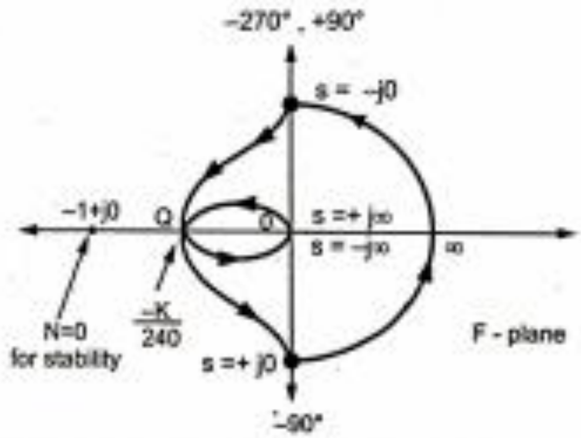


Fig. 13.29

Step 7 : Now for absolute stability, N = 0

i.e. it should be located on left side of point Q

i.e. $|OQ| < 1$

$\therefore \quad \left|-\frac{K}{240}\right| < 1$

$\therefore \quad K < 240$

So range of values of K for stability is

$$0 < K < 240$$

Ans 9)

Lead Compensator

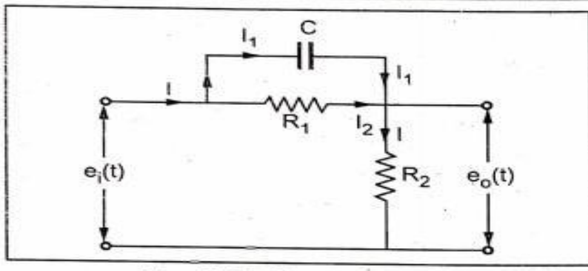


Fig. 14.4 Lead network

Consider an electrical network which is a lead compensating network, as shown in the Fig. 14.4.

Let us obtain the transfer function of such an electrical lead network. Assuming unloaded circuit and applying KCL for the output node we can write,

$$I_1 + I_2 = I$$

$$C\frac{d(e_i - e_o)}{dt} + \frac{1}{R_1}(e_i - e_o) = \frac{1}{R_2}e_o$$

Taking Laplace transform of the equation,

$$sC\,E_i(s) - sC\,E_o(s) + \frac{1}{R_1}E_i(s) - \frac{1}{R_1}E_o(s) = \frac{1}{R_2}E_o(s)$$

$$\therefore \quad E_i(s)\left[sC + \frac{1}{R_1}\right] = E_o(s)\left[sC + \frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{R_1R_2}{R_1 + R_2 + R_1R_2\,sC}\cdot\frac{1 + sCR_1}{R_1}$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{\left(s + \frac{1}{R_1C}\right)}{s + \frac{(R_1 + R_2)}{R_1R_2C}} = \frac{\left(s + \frac{1}{R_1C}\right)}{\left[s + \frac{1}{\left(\frac{R_2}{R_1 + R_2}\right)R_1C}\right]}$$

This is generally expressed as,

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}}$$

where $\quad T = R_1C \quad$ and $\quad \alpha = \frac{R_2}{R_1 + R_2} < 1$

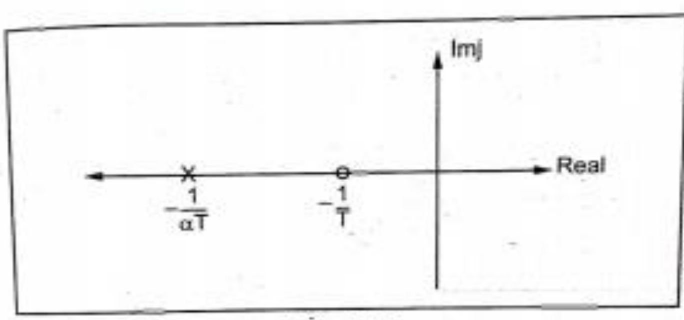


Fig. 14.5

The lead compensator has zero at $s = -\frac{1}{T}$ and a pole at $s = -\frac{1}{\alpha T}$.

As $0 < \alpha < 1$, the zero is always located to the right of the pole. The pole zero plot is shown in the Fig. 14.5. The minimum value of α is generally taken as 0.05.

Ans 10)

Lag Compensator

Consider an electrical network which is a lag compensating network, as shown, in the Fig. 14.11.

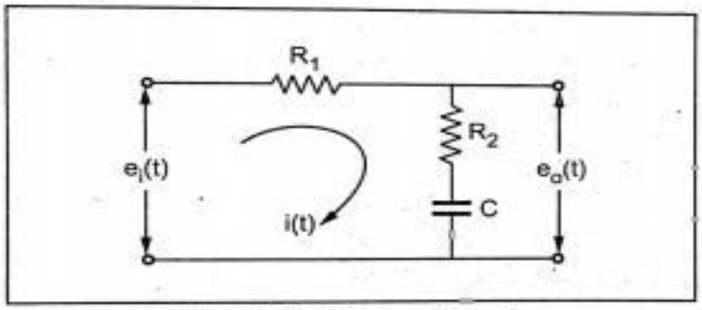


Fig. 14.11 Lag network

Let us obtain the transfer function of such an electrical lag network.

Assuming unloaded circuit and applying KVL to the loop we can write,

$$e_i(t) = i(t)\,R_1 + i(t)\,R_2 + \frac{1}{C}\int i(t)\,dt$$

Taking laplace transform of the equation,

$$E_i(s) = I(s)\left[R_1 + R_2 + \frac{1}{sC}\right] \quad \ldots(1)$$

Now the output equation is,

$$e_o(t) = i(t)\,R_2 + \frac{1}{C}\int i(t)\,dt$$

Taking laplace transform,

$$E_o(s) = I(s)\left[R_2 + \frac{1}{sC}\right] \quad \ldots(2)$$

Substituting I(s) from (2) in (1) we get,

$$E_i(s) = \frac{E_o(s)}{\left[R_2 + \frac{1}{sC}\right]}\left[R_1 + R_2 + \frac{1}{sC}\right]$$

$$\therefore \quad E_i(s) = \frac{E_o(s)[(R_1 + R_2)sC + 1]}{(1 + R_2 sC)}$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \frac{1 + sR_2C}{1 + s.(R_1 + R_2)C}$$

$$= \left(\frac{R_2}{R_1 + R_2}\right) \cdot \frac{s + \frac{1}{R_2C}}{s + \frac{1}{(R_1 + R_2)C}}$$

This is generally expressed as,

$$\boxed{\frac{E_o(s)}{E_i(s)} = \frac{1}{\beta} \cdot \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}} \qquad \text{...(3)}$$

where $\quad T = R_2C, \quad \beta = \frac{R_1 + R_2}{R_2} > 1$

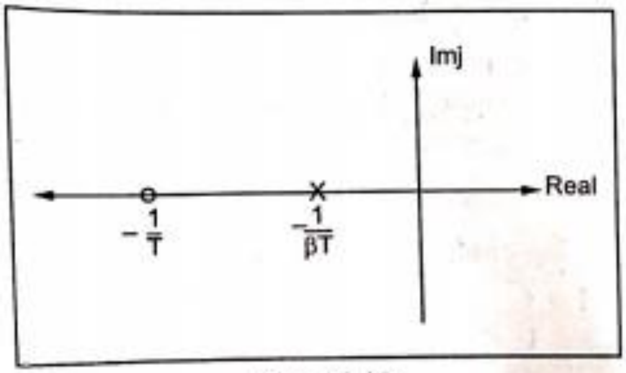


Fig. 14.12

The lag compensator has zero at $s = -\frac{1}{T}$ and a pole at $s = -\frac{1}{\beta T}$. As $\beta > 1$, the pole is always located to the right of the zero. The pole-zero plot is shown in the Fig. 14.12. Usually β is choosen greater than 10.