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CMRIT
CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
ACCREDITED WITH A+ GRADE BY NAAC
CELEBRATING 25 YEARS

Internal Assessment Test 2 – Nov. 2020

| Sub: | Discrete Mathematical Structures | | | | | Sub Code: | 18CS36 | Branch: | CS & IS |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 05/11/2020 | Duration: | 90 minutes | Max Marks: | 50 | Sem /Sec: | III A, B & C | | OBE |

| | | MARKS | CO | RBT |
|---|---|---|---|---|
| 1 | If a tree has 2020 vertices, what is the sum of the degrees of the vertices?<br>(a) 2020 (b) 2019 (c) 4038 (d) 4083 | [01] | CO5 | L1 |
| 2 | In a tree with r+s vertices, if r vertices are pendant vertices and s vertices have degree 4 each, which of the following is true?<br>(a) 2r=s-2 (b) r=s-2 (c) s=r-2 (d) 2s=r-2 | [01] | CO5 | L1 |
| 3 | A classroom contains 25 microcomputers that must be connected to a wall socket that has 4 outlets. Connections are made by using extension cords that have 4 outlets each. Find the least number of cords needed to get this computer set up for the class.<br>(a) 10 (b) 8 (c) 6 (d) 4 | [02] | CO5 | L2 |
| 4 | Given the prefix code a:111, b:z, c:xy00, d=1101, e:10, the value of x, y, z are respectively given by<br>(a) 1, 1, 0 (b) 1, 1, 1 (c) 1, 0, 1 (d) 0, 1, 0 | [01] | CO5 | L1 |
| 5 | Obtain an optimal prefix code for the message PROPOSAL ACCEPTED. Indicate the code for the message. | [05] | CO5 | L3 |
| 6 | If the truth value of the statement p->(q->r) is 0 then the truth values of p, q, r are respectively given by<br>(a) 1, 0, 1 (b) 0, 0, 1 (c) 1, 1, 0 (d) 1, 0, 0 | [01] | CO1 | L1 |
| 7 | For any propositions p, q, r, the compound propositions<br>$[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$ is a<br>(a) Tautology (b) Contradiction (c) Contingency (d) None | [03] | CO1 | L3 |
| 8 | For any propositions p, q, r, the compound proposition $[(p \to p) \wedge (p \to \sim q)]$ is logically equivalent to<br>(a) p (b) q (c)~p (d) ~q | [03] | CO1 | L3 |
| 9 | Prove the logical equivalence $(p \to q) \wedge [\sim q \wedge (r \vee \sim q)] \Leftrightarrow \sim (q \vee p)$ | [05] | CO1 | L3 |
| 10 | The dual of $p \leftrightarrow q$ is<br>(a) $p \leftrightarrow q$ (b) $(\sim p \vee q) \wedge (\sim q \vee p)$ (c) $(\sim p \wedge q) \vee (\sim q \wedge p)$ (d) None | [02] | CO1 | L2 |
| 11 | Check whether the following argument is valid or not.<br>$p \to r$<br>$r \to s$<br>$t \vee \sim s$<br>$\sim t \vee u$<br>$\sim u$<br>———<br>$\therefore \sim p$ | [05] | CO1 | L3 |

12 Let $p(x) = x^2 - 7x + 10 = 0, q(x) = x^2 - 2x - 3 = 0, r(x < 0)$. Set of all integers is the universe. The truth or falsity of the following statements respectively are: [03] CO1 L2

(i) $\forall x, p(x) \to \sim r(x)$ (ii) $\forall x, q(x) \to r(x)$ (iii) $\exists x, p(x) \to r(x)$

13 The negation of the statement "if x is a real number where $x^2 > 16 then x < -4$ *or* $x > 4$ is: [03] CO1 L2

(a) If x is a real number where $x^2 < 16$ then $x > -4$ and $x < 4$.
(b) For some real number x, $x^2 > 16$ and $x \geq -4$ and $x \leq 4$.
(c) For some real number x, $x^2 > 16$ and $x > -4$ or $x < 4$
(d) If x is a real number where $x \leq 16$ then $x \geq -4$ or $x \leq 4$.

14 The universe is the set of all non-zero integers. The truth value of the following statements respectively are: [03] CO1 L2

(i) $\exists x, \forall y, [xy = 1]$ (ii) $\exists x, \exists y, [(3x - y = 17) \wedge (2x + 4y = 3)]$

(a) 0, 0 (b) 0, 1 (c) 1, 0 (d) 1, 1

15 Test the validity of the following argument: Some intelligent boys are lazy. Ravi is an intelligent boy. Therefore, Ravi is lazy. [02] CO1 L1

(a) Valid (b) Invalid

16 Write the following in symbolic form: None of my friends are perfect. [02] CO1 L1

(a) $\exists x, F(x) \wedge \sim P(x)$ (b) $\exists x, \sim F(x) \wedge P(x)$

© $\exists x, \sim F(x) \wedge \sim P(x)$ (d) $\sim \exists x, F(x) \wedge P(x)$

17 P and Q are two logical propositions. Which of the following are equivalent? [03] CO1 L2

(a) $P \vee \sim Q$ (b) $\sim(\sim P \wedge Q)$ (c) $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

(d) $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

18 Is the following true or false? [03] CO1 L1

$\{[P \vee (Q \vee R)] \wedge \sim Q\} \Rightarrow (P \vee R)$

(a) True (b) False

19 "If a triangle is not isosceles then it is not equilateral" is equivalent to [02] CO1 L1

(a) If a triangle is not equilateral then it is not isosceles.
(b) If a triangle is isosceles then it is equilateral.
(c) A triangle is isosceles and it is not equilateral.
(d) If a triangle is equilateral then it is isosceles.

| Course Outcomes | | Modules covered | PO1 | PO2 | PO3 | PO4 | PO5 | PO6 | PO7 | PO8 | PO9 | PO10 | PO11 | PO12 | PSO1 | PSO2 | PSO3 | PSO4 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| CO 1 | Examine the correctness of an argument using propositional and predicate logic and truth table. | 1 | 2 | - | - | - | - | - | - | - | - | - | - | - | 1 | - | - | - |
| CO 2 | Solve problems using counting techniques and combinatorics in the context of discrete probabilities. | 1 | 1 | - | - | - | - | - | - | - | - | - | - | - | 1 | - | - | - |
| CO 3 | Solve problems involving relations and functions and their properties. | 1 | 2 | - | - | - | - | - | - | - | - | - | - | - | 1 | - | 1 | 1 |
| CO 4 | Construct proofs using direct proof, proof by contradiction, and proof by cases and mathematical induction. | .75 | 2 | 2 | - | - | - | - | - | - | - | - | - | - | 1 | - | 1 | 1 |
| CO 5 | Explain and differentiate graphs and trees. | 1 | 1 | - | - | 2 | - | - | - | - | - | - | - | - | 1 | 2 | 1 | 1 |
| CO 6 | Solve problems involving recurrence relations. | .25 | 2 | - | 1 | 2 | - | - | - | - | - | - | - | - | 1 | - | 1 | 1 |

| COGNITIVE LEVEL | REVISED BLOOMS TAXONOMY KEYWORDS |
|---|---|
| L1 | List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc. |
| L2 | summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend |
| L3 | Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover. |
| L4 | Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer. |
| L5 | Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize. |

| PROGRAM OUTCOMES (PO), PROGRAM SPECIFIC OUTCOMES (PSO) | | | | CORRELATION LEVELS | |
|---|---|---|---|---|---|
| PO1 | Engineering knowledge | PO7 | Environment and sustainability | 0 | No Correlation |
| PO2 | Problem analysis | PO8 | Ethics | 1 | Slight/Low |
| PO3 | Design/development of solutions | PO9 | Individual and team work | 2 | Moderate/ Medium |
| PO4 | Conduct investigations of complex problems | PO10 | Communication | 3 | Substantial/ High |
| PO5 | Modern tool usage | PO11 | Project management and finance | | |
| PO6 | The Engineer and society | PO12 | Life-long learning | | |
| PSO1 | Develop applications using different stacks of web and programming technologies. | | | | |
| PSO2 | Develop secured and distributed applications on a network. | | | | |
| PSO3 | Apply software engineering methods to design, develop, test and manage software systems. | | | | |
| PSO4 | Develop intelligent applications for business and industry. | | | | |

# IAT-2, DMS (SOLUTION), 2020

(1.) $n = 2020$, $\therefore$ no. of edges $= 2020 - 1 = 2019$

$\therefore$ The sum of the degree of the vertices $= 2 \times 2019$
$= 4038$

(2.) By Handshaking property

$$(r \times 1) + 4s = 2(r + s - 1)$$

$$r + 4s = 2r + 2s - 2$$

$$\Rightarrow \quad r = 2s + 2 \quad \Rightarrow \quad 2s = r - 2$$

(3.) $m = 4$, $p = 25$

$\therefore$ No. of internal vertices is

$$q = \frac{p-1}{m-1} = 8$$

$\therefore$ No. of extension cords is $q - 1 = 7$.

(4.) $x = 1$, $y = 1$, $z = 0$, otherwise for the other choices the given code won't be a prefix code.

(8.) $[(p \to q) \wedge (p \to \neg q)] \equiv (\neg p \vee q) \wedge (\neg p \vee \neg q)$

$\equiv \neg p \vee (q \wedge \neg q)$ (Distributive law)

$\equiv \neg p \vee F$ (Inverse law)

$\equiv \neg p$ (Identity)

(9.) $(p \to q) \wedge [\neg q \wedge (r \vee \neg q)] \equiv (\neg p \vee q) \wedge [\neg q]$ (Absorption law)

$\equiv (\neg p \wedge \neg q) \vee (q \wedge \neg q)$ (Distributive)

$\equiv (\neg p \wedge \neg q) \vee F$ (Inverse)

$\equiv \neg (p \vee q) \vee F$ (De-Morgan's law)

$\equiv \neg (q \vee p)$ (Identity & commutative)

(10.) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p) \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

Its dual is: $(\neg p \wedge q) \vee (\neg q \wedge p)$

(11.) $p \to r$ $\quad$ $\Rightarrow$ $p \to s$ (Syllogism in 1st & 2nd)

$r \to s$ $\quad$ $s \to t$ (commutative & $\underline{p \to q \equiv \neg p \vee q}$)

$t \vee \neg s$ $\quad$ $t \to u$ ( " )

$\neg t \vee u$ $\quad$ $\neg u$

$\neg u$

$\Rightarrow$ $p \to t$ (Syllogism in 1st & 2nd)

$t \to u$

$\neg u$

$\Rightarrow$ $p \to u$ (Syllo. in 1st & 2nd)

$\neg u$

$\therefore \neg p$ (Modus Tollens)

(12.) $\forall x, p(x) \to \neg r(x)$ $\quad$ T $\quad$ ($\because x = 2, 5$)

$\forall x, q(x) \to r(x)$ $\quad$ F $\quad$ ($\because x = 3$)

$\exists x, p(x) \to r(x)$ $\quad$ F $\quad$ ($\because x = 2, 5$ both are positive)

(13.) $p(x): x^2 > 16$, $\quad q(x): x < -4$, $\quad r(x): x > 4$

Symbolic form is

$$\forall x \in R, p \to (q \vee r)$$

The negation is

$$\exists x \in R, \neg\{p(x) \to q(x) \vee r(x)\}$$

$$\equiv \exists x \in R, \neg(\neg p(x) \vee (q(x) \vee r(x))) \quad \because p \to q \equiv \neg p \vee q$$

$$\equiv \exists x \in R, p(x) \wedge \neg q(x) \wedge \neg r(x)$$ {De Morgan's & double neg.

(14.) (i) $\exists x \exists y, [xy = 1]$, T, $(x = 1, y = 1)$

(ii) $\exists x \exists y [(3x - y = 17) \wedge (2x + 4y = 3)]$, F

$$x = \frac{71}{14} \notin \mathbb{Z}$$

(15.) Sym. form $\exists x, p(x) \wedge q(x)$

$p(a)$

$\therefore q(a)$

Invalid, as for first premise $x$ can be other than $a$.

(17.) $(p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$\equiv [p \wedge (q \vee \sim q)] \vee (\sim p \wedge \sim q)$ (Distributive)

$\equiv (p \wedge T) \vee (\sim p \wedge \sim q)$ (Inverse)

$\equiv p \vee (\sim p \wedge \sim q)$ (Identity)

$= (p \vee \sim p) \wedge (p \vee \sim q)$ (Distributive)

$\equiv T \wedge (p \vee \sim q)$ (Inverse)

$\equiv p \vee \sim q$ (Identity)

& $\sim(\sim p \wedge q) \equiv p \vee \sim q$ (De-Morgan's Law)

(18.)

| p ① | q | r | $q \vee r$ ② | $\neg q$ | ① $\vee$ ② ③ | ③ $\wedge \neg q$ | $p \vee r$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 → | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 → | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 → | 1 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |

Whenever $\{p \vee (q \vee r)\} \wedge \neg q$ is 1, $p \vee r$ is 1.

$\therefore \{p \vee (q \vee r)\} \wedge \neg q \Rightarrow p \vee r$

(7.)

| p | q | r | $p \vee q$ ① | $p \to r$ ② | $q \to r$ ③ | ② $\wedge$ ③ ④ | ① $\wedge$ ④ ⑤ | ⑤ $\to r$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 |

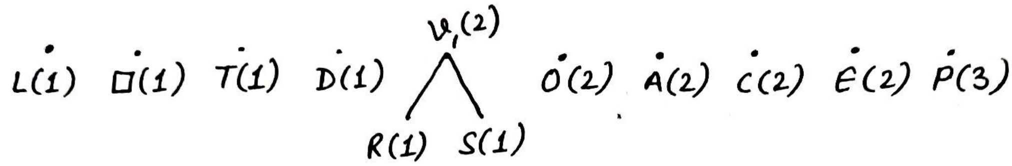
Since the given compound proposition is alway 1 irrespective of what the truth values of its components are.

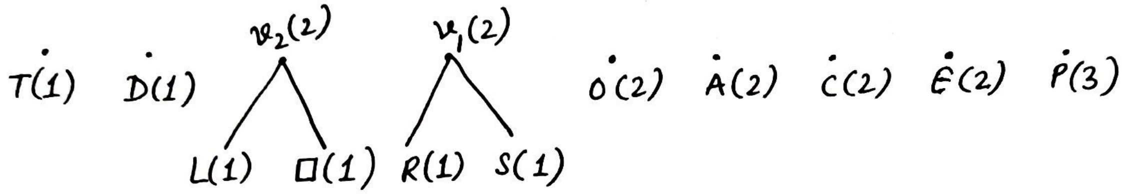
5. PROPOSAL ACCEPTED

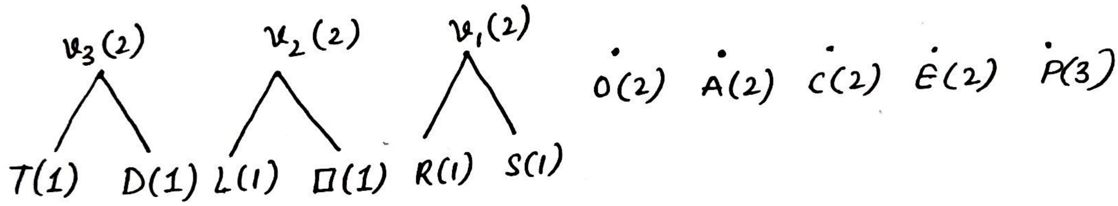
P: 3; R: 1; O: 2; S: 1; A: 2; L: 1; □: 1; C: 2, E: 2, T: 1, D: 1

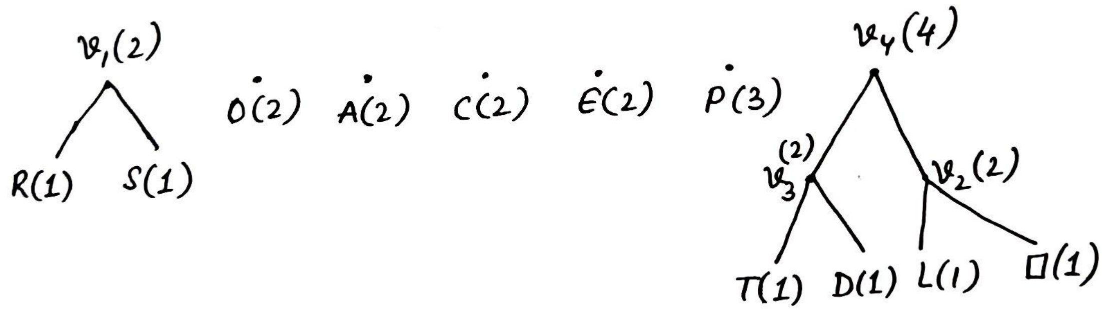
Arranging the letters with weights in non-decreasing order

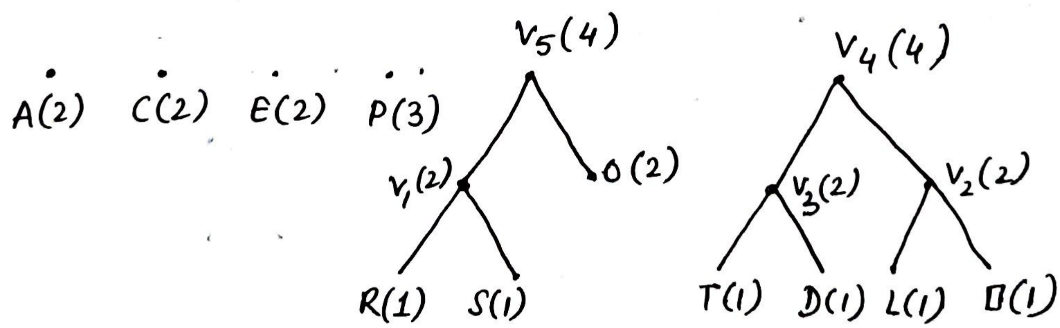
R(1) S(1) L(1) □(1) T(1) D(1) O(2) A(2) C(2) E(2) P(3)

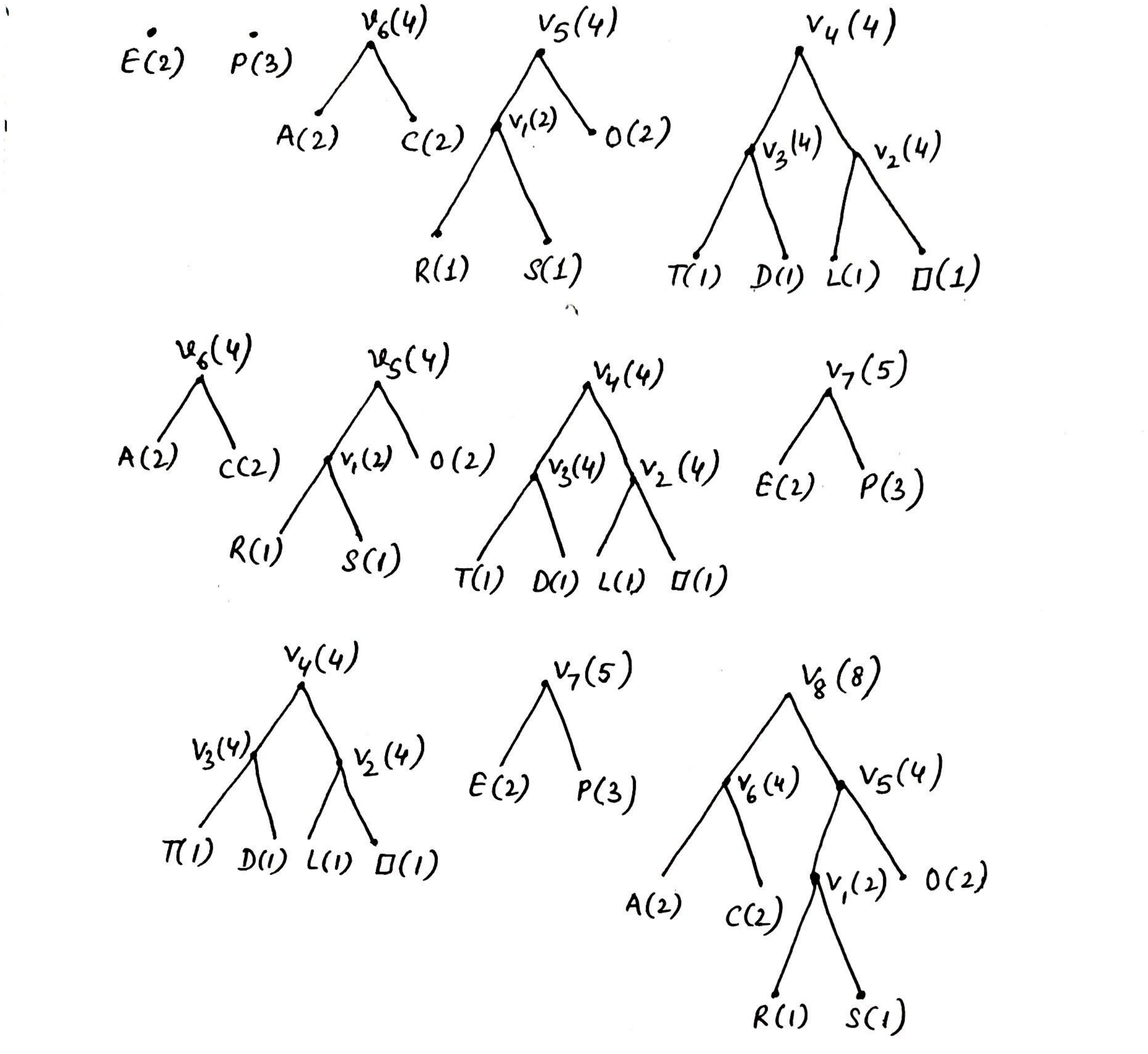


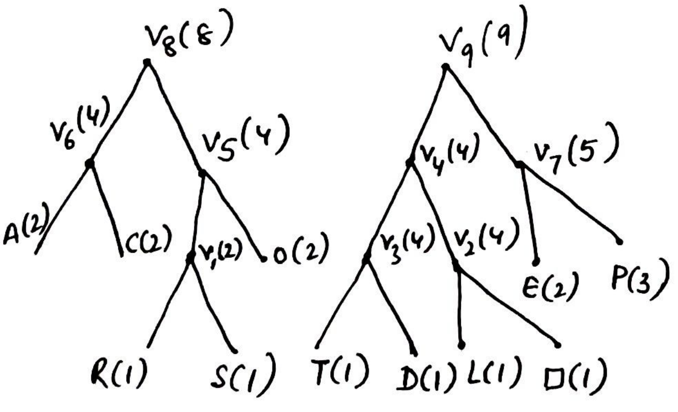



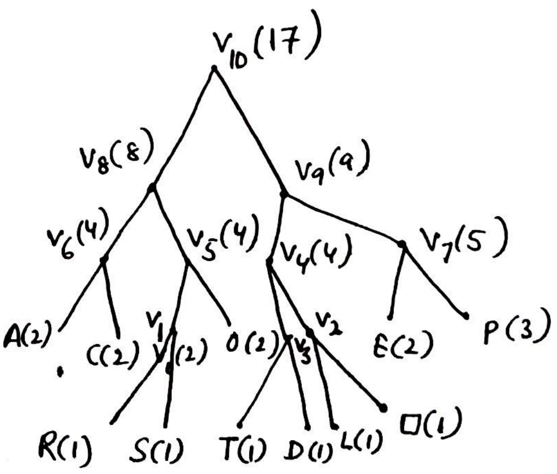






Codes fore

P : 111
R : 0100
O : 011
S : 0101
A : 000
L : 1010
□ : 1011
C : 001
E : 110
T : 1000
D : 1001

Code for the message is :

111010001111101101010100010101011000001001110 1111000110 1001