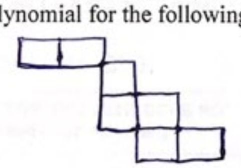
USN

CMRIT

Internal Assessment Test 3 – Dec. 2020

Sub:	Discrete Mathematical Structures					Sub Code:	18CS36	Branch:	CS & IS
Date:	16/12/2020	Duration:	90 minutes	Max Marks:	50	Sem / Sec:	III A, B & C		OBE

		MARKS	CO	RBT
1	Given p -->q , the following methods of proving it (i) Let q be false, then prove that p is false, (ii) Let p be true, then prove that q is true, (iii) Let ~q be true then prove that ~p is true, are known as (respectively): (a) Direct Method, Indirect Method, Proof by Contradiction (b) Indirect Method, Proof by Contradiction, Direct Method (c) Indirect Method, Direct Method, Proof by Contradiction (d) Proof by Contradiction, Direct Method, Indirect Method	[03]	CO4	L2
2	Can every integer x such that $61 < x < 80$ be written as a sum of two prime numbers? (Proof by Exhaustion) (a) Yes (b) No	[05]	CO4	L2
3	Are the following statements true or false? (i) The induction step without the basis step can constitute a proof by induction, (ii) For Induction step, the range for k is n > k. (a) T, T (b) T, F (c) F, F (d) F, T	[02]	CO2	L1
4	In how many ways can one distribute eight identical balls into four distinct containers so that (i) no container is left empty? (ii) the fourth container gets an odd number of balls? (a) 35, 48 (b) 38, 96 (c) 70, 35 (d) 35, 70	[05]	CO2	L3
5	How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? (a) 722 (b) 720 (c) 860 (c) 572	[05]	CO2	L3
6	Determine the coefficient of $x^{11}y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$. (a) 3920 (b) 5672 (c) 4320 (d) None	[05]	CO2	L3
7	A sequence $\{a_n\}$ is defined is recursively defined recursively by $a_1 = 4$, $a_n = a_{n-1} + 4$ for $n \geq 2$. Find a_n in explicit form. (a) a_n = 3+ [n(n+1)]/2 (b) 2+ [n(n-1)]/3 (c) 6+n(n+1) (d) 1+n(n-1)/7	[05]	CO2	L2
8	Find the rook polynomial for the following board.  (a) 1+7x + 15x^2 + 10x^3 + 2x^4 (b) 1+7x + 16x^2 + 13x^3 + 2x^4 © 1+8x + 15x^2 + 10x^3 + 2x^4 (d) 1+7x + 15x^2 + 11x^3 + 2x^4	[03]	CO2	L1
9	There are 15 pairs of children's socks in a box. Each pair is of a different colour. Suppose the right socks are distributed at random to n children, and thereafter the left socks are also to be distributed to them at random. Find the probability that (i) no child gets a matching pair, (ii) every child gets a matching pair, (iii) exactly one child gets a matching pair. (a) d14/15!, 1/15!, d14/15! (b) d1/14!, 2/15!, d13/15! © d15/15!, 1/15!, d14/15! (d) d15/15!, 1/15!, d15/15!	[03]	CO2	L2

10 A question paper contains 10 questions of which 7 are to be answered. In how many ways a student can select the 7 questions (i) if he can choose any seven? (ii) if he should select three questions from the first five and four questions from the last five? [03] CO2 L2

(a) 120, 50 (b) 50, 120 (c) 75, 120 (d) 49, 123

11 Find the coefficient of x^5y^2 in the expansion of (2x-3y)^7 [03] CO2 L1

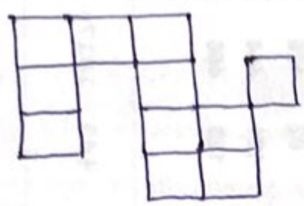
(a) 6048 (b) 6041 (c) 7521 (d) 5214

12 Obtain a recursive definition of the sequence $\{a_n\}$ where $a_n = 2 - (-1)^n$. [02] CO2 L2

(a) $a_1 = 3,\ a_{n+1} = a_n + 2$ for $n \geq 1$.
(b) $a_1 = 3,\ a_{n+1} = a_n - 2$ for $n \geq 1$.
© $a_1 = 3,\ a_{n+1} = a_n + 2(-1)^n$ for $n \geq 1$.

(d) None

13 What is the degree of the rook polynomial of the following board? [02] CO2 L1



(a) 3 (b) 4 (c) 5 (d) 2

14 In how many ways can you form a dancing couple from 4 boys and 4 girls so that no boy dances with his respective girlfriend? [02] CO2 L1

(a) 8 (b) 16 (c) 9 (d) 24

15 Determine the number of non negative integer solutions of the equation x1+x2+x3+x4 = 7. [03] CO2 L1

(a) 45 (b) 120 (c) 125 (d) 71

Course Outcomes		Modules covered	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO2	PSO3	PSO4
CO 1	Examine the correctness of an argument using propositional and predicate logic and truth table.	1	2	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-
CO 2	Solve problems using counting techniques and combinatorics in the context of discrete probabilities.	1	1	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-
CO 3	Solve problems involving relations and functions and their properties.	1	2	-	-	-	-	-	-	-	-	-	-	-	1	-	1	1
CO 4	Construct proofs using direct proof, proof by contradiction, and proof by cases and mathematical induction.	.75	2	2	-	-	-	-	-	-	-	-	-	-	1	-	1	1
CO 5	Explain and differentiate graphs and trees.	1	1	-	-	2	-	-	-	-	-	-	-	-	1	2	1	1
CO 6	Solve problems involving recurrence relations.	.25	2	-	1	2	-	-	-	-	-	-	-	-	1	-	1	1

COGNITIVE LEVEL	REVISED BLOOMS TAXONOMY KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PROGRAM OUTCOMES (PO), PROGRAM SPECIFIC OUTCOMES (PSO)				CORRELATION LEVELS	
PO1	Engineering knowledge	PO7	Environment and sustainability	0	No Correlation
PO2	Problem analysis	PO8	Ethics	1	Slight/Low
PO3	Design/development of solutions	PO9	Individual and team work	2	Moderate/ Medium
PO4	Conduct investigations of complex problems	PO10	Communication	3	Substantial/ High
PO5	Modern tool usage	PO11	Project management and finance		
PO6	The Engineer and society	PO12	Life-long learning		
PSO1	Develop applications using different stacks of web and programming technologies.				
PSO2	Develop secured and distributed applications on a network.				
PSO3	Apply software engineering methods to design, develop, test and manage software systems.				
PSO4	Develop intelligent applications for business and industry.				

IAT-3: Solution

DMS – 18CS36

1. (i) ~~Method of~~ Proof by Contradiction
(ii) Direct Method
(iii) Indirect Method

2. $S = \{62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 79\}$

$62 = 31 + 31$ $68 =$
$63 = 61 + 2$ $69 =$
$64 = 59 + 5$ $70 =$
$65 =$ $71 =$
$67 =$

65 can not be written as a sum of two primes.

So, the answer is <u>No.</u>

3. (i) False
(ii) False

4)

Example 10 *In how many ways can one distribute eight identical balls into four distinct containers so that* (i) *no container is left empty?* (ii) *the fourth container gets an odd number of balls?*

▶ (i) First we distribute one ball into each container. Then we distribute the remaining 4 balls into 4 containers. The number of ways of doing this is the required number. This number is

$$C(4+4-1,4) = C(7,4) = \frac{7!}{4!\,3!} = 35.$$

(ii) If the fourth container has to get an odd number of balls, we have to put one or three or five or seven balls into it.

Suppose we put one ball into it (the fourth container). Then the remaining 7 balls can be distributed into the remaining three containers in

$$C(3+7-1,7) = C(9,7) \quad \text{ways.}$$

Similarly, putting 3 balls into the fourth container and the remaining 5 into the remaining 3 containers can be done in

$$C(3+5-1,5) = C(7,5) \quad \text{ways}$$

Next, putting 5 balls into the fourth container and the remaining 3 into the remaining 3 containers can be done in

$$C(3+3-1,3) = C(5,3) \quad \text{ways}$$

Lastly, putting 7 balls into the fourth container and the remaining 1 into the remaining 3 container can be done in

$$C(3+1-1,1) = C(3,1) = 3 \quad \text{ways.}$$

Thus, the total number ways of distributing the given balls so that the fourth container gets an odd number of balls is

$$C(9,7) + C(7,5) + C(5,3) + 3 = \frac{9!}{7!\,2!} + \frac{7!}{5!\,2!} + \frac{5!}{3!\,2!} + 3$$

$$= 36 + 21 + 10 + 3 = 70$$ ∎

5

Example 11 *How many positive integers n can we form using the digits* 3, 4, 4, 5, 5, 6, 7 *if we want n to exceed* 5,000,000*?*

▸ Here n must be of the form

$$n = x_1x_2x_3x_4x_5x_6x_7$$

where $x_1, x_2, \ldots, x_7$ are the given digits with $x_1 = 5, 6$ or 7. Suppose we take $x_1 = 5$. Then $x_2x_3x_4x_5x_6x_7$ is an arrangement of the remaining 6 digits which contains two 4's and one each of 3, 5, 6, 7. The number of such arrangements is

$$\frac{6!}{2!\,1!\,1!\,1!\,1!} = 360.$$

Next, suppose we take $x_1 = 6$. Then, $x_2x_3x_4x_5x_6x_7$ is an arrangement of 6 digits which contains two each of 4 and 5 and one each of 3 and 7. The number of such arrangements is

$$\frac{6!}{1!\,2!\,2!\,1!} = 180.$$

Similarly, if we take $x_1 = 7$, the number of arrangements is

$$\frac{6!}{1!\,2!\,2!\,1!} = 180.$$

Accordingly, by the Sum Rule, the number of n's of the desired type is

$$360 + 180 + 180 = 720.$$ ■

6)

(iii) The general term in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ is

$$\binom{6}{n_1,\ n_2,\ n_3}(2x^3)^{n_1}(-3xy^2)^{n_2}(z^2)^{n_3}$$

For $n_3 = 0, n_2 = 2, n_1 = 3$ this becomes

$$\binom{6}{3,\ 2,\ 0}(2^3x^9)(3^2x^2y^4) = 72 \times \frac{6!}{3!\,2!\,0!}x^{11}y^4.$$

Thus, the required coefficient is

$$72 \times \frac{6 \times 5 \times 4}{2} = 4320.$$

Example *A sequence $\{a_n\}$ is defined recursively by*

$$a_1 = 4, \; a_n = a_{n-1} + n \quad for \quad n \geq 2.$$

Find a_n in explicit form.

Using the given recursive formula repeatedly, we find that

$$\begin{aligned}
a_n &= a_{n-1} + n \\
&= [a_{n-2} + (n-1)] + n \\
&= a_{n-3} + (n-2) + (n-1) + n \\
&= a_{n-4} + (n-3) + (n-2) + (n-1) + n \\
&\quad \cdots\cdots\cdots \\
&\quad \cdots\cdots\cdots \\
&= a_1 + 2 + 3 + 4 + \cdots + n \\
&= (a_1 - 1) + (1 + 2 + 3 + \cdots + n)
\end{aligned}$$

Using $a_1 = 4$ and the standard result

$$1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n+1),$$

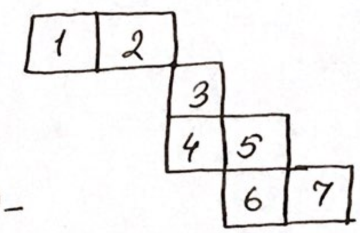
this becomes

$$a_n = 3 + \frac{1}{2}n(n+1).$$

This is the explicit formula for a_n.*

8. $r_1 = n = 7$

$r_2 = 16$



{Non-capturing positions when 2 rooks are placed on board-

(1,3), (1,4), (1,5), (1,6), (1,7), (2,3), (2,4), (2,5), (2,6), (2,7)
(3,5), (3,6), (3,7), (4,6), (4,7), (5,7)}

$r_3 = 13$ (Non-capturing positions when 3 rooks are placed on board)

(1, 3, 5), (1, 3, 6), (1, 3, 7), (2, 3, 5), (2, 3, 6) (2, 3, 7)

(3, 5, 7) (1, 4, 6), (1, 4, 7), (1, 5, 7) (2, 4, 6), (2, 4, 7), (2, 5, 7)

$r_4 = 2$ (when 4 rooks are placed -

(1, 3, 5, 7), (2, 3, 5, 7)

∴ The rook polynomial is -

$$r(C, x) = 1 + 7x + 16x^2 + 13x^3 + 2x^4$$

9) Replace n by 15 in the following solution

4)

Example *There are n pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to n children, and thereafter the left gloves are also distributed to them at random. Find the probability that* (i) *no child gets a matching pair,* (ii) *every child gets a matching pair,* (iii) *exactly one child gets a matching pair, and* (iv) *at least 2 children get matching pairs.*

▶ Any one distribution of n right gloves to n children determines a set of n places for the n pairs of gloves. Let us take these as the natural places for the pairs of gloves. The left gloves can be distributed to n children in $n!$ ways.

(i) The event of no child getting a matching pair occurs if the distribution of the left gloves is a derangement. The number of derangements is d_n. Therefore, the required probability, in this case, is

$$p_1 = \frac{d_n}{n!} = \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right)$$

(ii) The event of every child getting a matching pair occurs in only one distribution of the left gloves. Therefore, the required probability, in this case, is $p_2 = \dfrac{1}{n!}$.

(iii) The event of exactly one child getting a matching pair occurs when only one left glove is in the natural place, and all others are in wrong places. The number of such distributions is d_{n-1}. The required probability, in this case, is

$$p_3 = \frac{d_{n-1}}{n!} = \frac{1}{n}\left\{1 - \frac{1}{1!} + \frac{1}{2!} + \cdots + (-1)^{n-1}\frac{1}{(n-1)!}\right\}.$$

10) (i) Number of ways of selecting 7 questions out of 10 $= {}^{10}C_7$

$= 120$

(ii) Number of ways of selecting three from first five questions $=$

${}^{5}C_3$

Number of ways of selecting 4 questions from last five $=$

${}^{5}C_4$

$\therefore$ Answer in this particular case $= {}^{5}C_3 \times {}^{5}C_4 = 50$

11) WKT $(x+y)^n = \sum_{r=0}^{n} {}^{n}C_r \, x^r y^{n-r}$

$$(2x-3y)^7 = \sum_{r=0}^{7} {}^{7}C_r (2x)^r (-3y)^{7-r}$$

$$= \sum_{r=0}^{7} {}^{7}C_r \, 2^r (-3)^{7-r} x^r y^{7-r}$$

x^5y^2 corresponds to $r=5$ in the above term.

$\therefore$ the coefficient of x^5y^2 is ${}^{7}C_5 \, 2^5 (-3)^2 = 6048$

12)

(vi) Here, $a_1 = 3$, $a_2 = 1$, $a_3 = 3, \ldots$ $a_n = 2 - (-1)^n$ and $a_{n+1} = 2 - (-1)^{n+1}$. These give

$$a_{n+1} - a_n = -(-1)^{n+1} + (-1)^n = (-1)^n \left\{1 + (-1)^2\right\} = 2(-1)^n$$

or

$$a_{n+1} = a_n + 2(-1)^n \quad \text{for} \quad n \geq 1.$$

Thus, $a_1 = 3$ and $a_{n+1} = a_n + 2(-1)^n$ for $n \geq 1$

is a recursive definition of the given sequence. ■

(13.) No. of rows = 4

No. of columns = 5

The min no. is 4. $\therefore$ The degree of the rook poly. is 4.

(14.) $d_4 = 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right) = 9$

(15.) $x_1 + x_2 + x_3 + x_4 = 7$

Here $n = 4$, $r = 7$

The no. of non negative integers solution is

$$C(n+r-1, r) = C(10, 7) = 120$$