CBCS SCHEME

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18EC44

Fourth Semester B.E. Degree Examination, Jan./Feb. 2021
Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive mean, variance and characteristic function for uniformly distributed random variable. **(10 Marks)**

b. The cdf for random variable z is $F_z(z) = \begin{cases} 1 - \exp(-2z^{3/2}) & z \geq 0 \\ 0 & \text{otherwise} \end{cases}$.

Evaluate $P(0.5 < z \leq 0.9)$ **(04 Marks)**

c. It is given that $E[X] = 2$ and $E[X^2] = 6$.
(i) Find standard deviation of X.
(ii) If $Y = 6X^2 + 2X - 13$. Find mean of Y. **(06 Marks)**

OR

2 a. Given the data in the following table:

K	1	2	3	4	5
x_K	2.1	3.2	4.8	5.4	6.9
$p(x_K)$	0.21	0.18	0.20	0.22	0.19

(i) Plot pdf and cdf of discrete random variable X.
(ii) Write expression for $f_x(x)$ and $F_x(x)$ using unit delta functions and unit step functions. **(08 Marks)**

b. The random variable X is uniformly distributed between 0 and 2. $Y = 3x^3$. What is the pdf of X? **(06 Marks)**

c. The ransom variable X is uniformly distributed between 0 and 5. The event B is $B = \{X > 3.7\}$. What are $f_{X/B}(x)$, $\mu_{X/B}$ and $\sigma^2_{X/B}$? **(06 Marks)**

Module-2

3 a. The joint pdf $f_{XY}(x, y) = c$, a constant when $(0 < x < 2)$ and $(0 < y < 3)$ and is 0 otherwise.
(i) What is the value of the constant c?
(ii) What are the pdfs for X and Y?
(iii) What is $F_{XY}(x, y)$ when $(0 < x < 2)$ and $(0 < y < 3)$?
(iv) What are $F_{XY}(x, \infty)$ and $F_{XY}(\infty, y)$?
(v) Are X and Y independent? **(06 Marks)**

b. Write a short note on Chi-Square random variables. **(08 Marks)**

c. Prove that correlation coefficient $\rho_{XY} = \pm 1$. **(06 Marks)**

OR

4 a. Mean and variance of random variable X are –2 and 3. Mean and variance of Y are 3 and 5. The correlation $E[X\,Y] = -8.7$. What are the covariance $COV[X\,Y]$ and correlation coefficient ρ_{XY}? **(04 Marks)**

b. The random variable X is uniformly distributed between ±1. Two independent realizations of X are added $Y = X_1 + X_2$, what is the pdf of Y? **(10 Marks)**

c. The random variable X = 3 + V where V is a Gaussian RV with mean of 0 and variance of 30. 72 independent realizations of X are averaged $Y = \frac{1}{72}\sum_{i=1}^{72} X_i$. What are the mean and variance of Y? **(06 Marks)**

Module-3

5 a. Define Autocorrelation function. List and prove various properties of autocorrelation function. **(08 Marks)**

b. PSD is shown in Fig.Q5(b) where a = 55, b = 5, w_0 = 1000, w_1 = 100. Calculate values for $E[X^2(t)]$, σ_X^2, $|\mu_x|$.

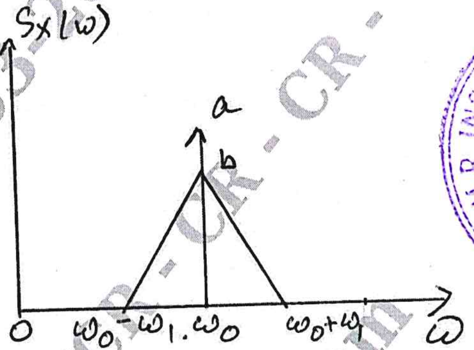

Fig.Q5(b) **(04 Marks)**

c. Random Processes X(t) and Y(t) are independent and have autocorrelation functions $R_X(\tau) = e^{-10|\tau|}$ and $R_X(\tau) = 6\cos(400\tau)$. If Z(t) = X(t)Y(t), what is the psd of Z(t)? **(08 Marks)**

OR

6 a. Explain Power Spectral Density and Wiener-Khinchin relation. **(06 Marks)**

b. A random process is described by $X(t) = A\sin(w_c t + \theta)$ where A and w_c are constants and θ is a random variable uniformly distributed between ±π. Is X(t) wide sense stationary? If not, then why not? If so, then what are mean and autocorrelation function for the random process. **(06 Marks)**

c. The PSD of random process X(t) and transfer function of a network are $S_x(w) = \frac{1}{w^2 + (100)^2}$ and $H(s) = \frac{s}{(s+10)(s+9000)}$. Y(s) = H(s) X(s). Find μ_Y, $S_{XY}(jw)$ and $S_Y(w)$. **(08 Marks)**

Module-4

7 a. Write the complete solution as x_p + multiplies of s in the null space.

$$x + 3y + 3z = 1$$
$$2x + 6y + 9z = 5$$
$$-x - 3y + 3z = 5$$

(06 Marks)

b. Find bases for the four subspaces associated with $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$. **(04 Marks)**

c. Find orthogonal vector A, B and orthonormal vector q_1 q_2 from a, b using Gram Schmidt process. Factorize into A = QR. $a = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. **(10 Marks)**

OR

8 a. Reduce A to echlon form. Which combination of rows of A produce zero row? What is the left Null space?

$$A = \begin{bmatrix} 1 & 2 & b_1 \\ 3 & 4 & b_2 \\ 4 & 6 & b_3 \end{bmatrix}$$ **(04 Marks)**

b. Project the vector b onto the line through a. Check that e is perpendicular to a.

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ **(08 Marks)**

c. In order to fit best straight line through four points passing through b = 0, 8, 8, 20 at t = 0, 1, 3, 4. Set up and solve normal equations $A^T A\hat{x} = A^T b$. **(08 Marks)**

Module-5

9 a. Find the eigen values and eigen vectors of A, A^2, A^{-1} and A + 4I. Find trace and determinant of A using eigen vectors.

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$ **(10 Marks)**

b. Reduce A to U and find determinant by product of pivots.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}$$ **(04 Marks)**

c. Test the matrix for positive definiteness.

$$S = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$ **(06 Marks)**

OR

10 a. Factorize matrix A into $X \wedge X^{-1}$.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$ **(08 Marks)**

b. Find the matrices U, Σ, V for $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$ and factorize $A = U \Sigma V^T$. **(12 Marks)**

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