Q6 a) Derive characteristic impedance of microstrip line with diagram.

Characteristic impedance

Characteristic impedance Z0 of microstrip is also a function of the ratio of the height to the width W/H (and ratio of width to height H/W) of the transmission line, and also has separate solutions depending on the value of W/H. The characteristic impedance Z0 of microstrip is calculated by:

$$when \left(\frac{W}{H}\right) < 1$$

$$Z_0 = \frac{60}{\sqrt{\varepsilon_{eff}}} \ln\left(8\frac{H}{W} + 0.25\frac{W}{H}\right) \ (ohms)$$

$$when \left(\frac{W}{H}\right) \geq 1$$

$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{eff}} \times \left[\frac{W}{H} + 1.393 + \frac{2}{3}\ln\left(\frac{W}{H} + 1.444\right)\right]} \ (ohms)$$

6b)

5. Calculate the approximate directivity from the half power beam widths of a unidirectional antenna if the normalized power pattern is given by (a) $P_n = \cos\theta$, (b) $P_n = \cos^2\theta$, (c) $P_n = \cos^3\theta$, and $P_n = \cos^n\theta$. In all cases these patterns are unidirectional (+z direction) with P_n having a value only for zenith angles $0° \le \theta \le 90°$ and $P_n = 0$ for $90° \le \theta \le 180°$. The patterns are independent of the azimuth angle ϕ.

Solution: (a) $P_n = \cos\theta$

At half power, $P_n = 0.5$

$$\cos\theta = 0.5$$

$$\theta = \cos^{-1}(0.5)$$

$$= 60°$$

$$HPBW = \theta_{HP} = 2 \times 60° = 120°$$

$$D = \frac{40{,}000}{(120)^2} = 2.78$$

(b) $$\theta_{HP} = 2\cos^{-1}(\sqrt{0.5}) = 90°$$

$$D = \frac{40{,}000}{(90)^2} = 4.94$$

(c) $$\theta_{HP} = 2\cos^{-1}(\sqrt[3]{0.5}) \simeq 74.93°$$

$$D = \frac{40{,}000}{[illegible]} \simeq 7.12$$

(d) $$\theta_{HP} = 2\cos^{-1}(\sqrt[n]{0.5})$$

$$D = \frac{10000}{\cos^{-1}(\sqrt[n]{0.5})^2}.$$

6c)

Radiation intensity: defined as

The power radiated from antenna per unit solid angle U(θ,ϕ)

U ⇒ watt/solid angle = watt/m^2/r^2
U = $r^2 W_{rad}$

$$U(\theta,\phi) = \frac{r^2}{2\eta}|\mathbf{E}(r,\theta,\phi)|^2 \simeq \frac{r^2}{2\eta}\left[|E_\theta(r,\theta,\phi)|^2 + |E_\phi(r,\theta,\phi)|^2\right] \quad \text{(2-12a)}$$

where

$\mathbf{E}(r,\theta,\phi)$ = far-zone electric-field intensity of the antenna

E_θ, E_ϕ = far-zone electric-field components of the antenna

η = intrinsic impedance of the medium

U = radiation intensity (W/unit solid angle)

The total power is obtained by integrating the radiation intensity, as given by (2-12), over the entire solid angle of 4π. Thus

$$P_{rad} = \oiint_\Omega U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi \quad \text{(2-13)}$$

radiation intensity of an isotropic source $U_0 = \frac{P_{rad}}{4\pi}$

Directivity

is the ratio of radiation intensity in a given direction to isotropic radiation intensity

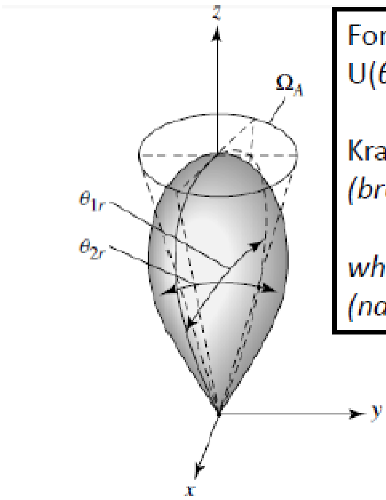
$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}}$$

If the direction is not specified →(maximum directivity)

$$D_{max} = D_0 = \frac{U|_{max}}{U_0} = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}}$$

Beam solid Angle:

For antennas with one narrow major lobe and very negligible minor lobes, the beam solid angle is approximately equal to the product of the half-power beam widths in two perpendicular planes



For radiation intensity

$U(\theta,\phi) = B_0 \cos^n(\theta)$ at $0<\theta<\pi/2$, $0<\phi<2\pi$ and 0 elsewhere

Kraus' formula is more accurate for small values of *n* *(broader patterns)* *(let us take it as n* <10 according to table 2.1)

while Tai & Pereira's is more accurate for large values of *n* *(narrower patterns).*

Beam solid angles $\Omega_A = \theta_{1r} \cdot \theta_{2r}$

$$D_0 = \frac{4\pi}{\Omega_A} \simeq \frac{4\pi}{\theta_{1r} \cdot \theta_{2r}} \text{ (Kraus)} \quad \text{or} \quad D_0 \simeq \frac{32 \ln 2}{\theta_{1r}^2 + \theta_{2r}^2} \text{ (Tai-Pereira)}$$

θ_{1r} = half-power beamwidth in one plane (rad)

θ_{2r} = half-power beamwidth in a plane at a right angle to the other (rad)

Antenna Beam-Efficiency, Stray-Factor

The distribution of radiation over the sphere is not uniform for any antenna. At certain points there seems to be no radiation at all. The shape of the antenna beam can give a rough estimation of what fraction of the power is radiated in required direction.

Beam efficiency: Ratio of solid angle of the main beam to the sum of solid angles subtended by all lobes (including main lobe).

$$\varepsilon_M = \frac{\text{solid angle subtended by the main beam}}{\text{sum of solid angles subtended by all the lobes}} = \frac{\Omega_M}{\Omega_A}$$

Stray factor: Ratio of sum of solid angles subtended only by minor lobes to the sum of solid angles subtended by all lobes (including main lobe).

$$\varepsilon_m = \frac{\text{sum of solid angles subtended by the minor lobes}}{\text{sum of solid angles subtended by all the lobes}} = \frac{\Omega_m}{\Omega_A}$$

Thus the sum of these two factors is unity

$$\varepsilon_M + \varepsilon_m = 1$$

7a

Power Theorem

Statement : If the Poynting vector is known at all points on a sphere of radius r from a point source in a lossless medium, the total power radiated by the source is the integral over the surface of the sphere of the radial component S_r of the average Poynting vector.

$$P = \oiint S_r \, ds$$

where

P = power radiated, W

S_r = radial component of average Poynting vector, $W m^{-2}$

ds = infinitesimal element of area of sphere
$= r^2 \sin\theta \, d\theta \, d\phi$, m^2

For an isotropic source, S_r is independent of θ and ϕ so

$$P = S_r \oiint ds = S_r \times 4\pi r^2$$

and

$$S_r = \frac{P}{4\pi r^2} \quad (W m^{-2})$$

Radiation Resistance of a Short Dipole

- The Poynting vector of the far field is integrated over a large sphere to obtain the total power radiated.
- This power is then equated to I^2R where I is the rms current on the dipole and R is a resistance, called the radiation resistance of the dipole.
- The average Poynting vector is given by
- $S = \frac{1}{2}Re(\mathrm{E} \text{ x } H^*)$ (1)
- The far-field components are E_θ and H_φ so that the radial component of the Poynting vector is
- $S_r = \frac{1}{2}Re(E_\theta \text{ x } H_\varphi{}^*)$ (2)
- Where E_θ and H_φ are complex.
- The far-field components are related by the intrinsic impedance of the medium.
- $E_\theta = H_\varphi Z = H_\varphi\sqrt{\frac{\mu}{\varepsilon}}$ (3)
- Therefore (2) now becomes
- $S_r = \frac{1}{2}Re\ \mathrm{Z}H_\varphi\ H_\varphi{}^* = \frac{1}{2}\left|H_\varphi\right|^2\sqrt{\frac{\mu}{\varepsilon}}$ (4)

- The total power P radiated is then
- $P = \iint S_r \, ds = \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}} \int_0^{2\pi} \int_0^{\pi} \left|H_\varphi\right|^2 r^2 sin\theta \, d\theta d\varphi$ (5)
- $\left|H_\varphi\right| = \frac{\omega I_0 L \, sin\theta}{4\pi c r}$ (6)
- Substituting this into (5), we have
- $P = \frac{1}{32}\sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{\pi^2} \int_0^{2\pi} \int_0^{\pi} sin^3\theta \, d\theta d\varphi$ (7)

- The double integral equals $\frac{8\pi}{3}$ and (7) becomes
- $P = \sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi}$ (8)
- This is the average power or rate at which energy is streaming out of a sphere surrounding the dipole. Hence it is equal to the power radiated.
- Assuming no losses this is also equal to the power delivered to the dipole.
- Therefore, P must be equal to the square of the rms current I flowing on the dipole times a resistance R_r called the radiation resistance of the dipole.
- $\sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi} = (\frac{I_0}{\sqrt{2}})^2 R_r$ (9)
- Solving for R_r,
- $R_r = \sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 L^2}{6\pi}$ (10)
- For air or vacuum $\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 = 120\pi\Omega$, so that (10) becomes
- Dipole with uniform current :
- $R_r = 80\pi^2(\frac{L}{\lambda})^2$ (11)

7c

To find the total power radiated by the cosine source we use power theorem and integrate only over the upper hemisphere.

$$\therefore P = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} U_m \cos\theta \sin\theta \, d\theta \, d\phi$$

$$= 2\pi U_m \frac{1}{2} \int_{\theta=0}^{\pi/2} \sin 2\theta \, d\theta$$

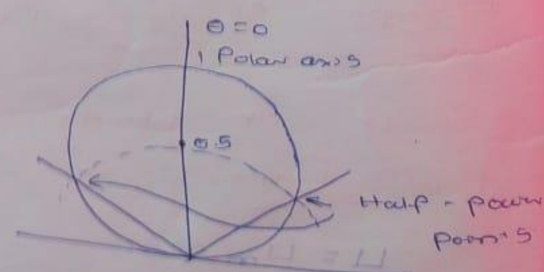
$$= \pi U_m \left[\frac{-\cos 2\theta}{2} \right]_0^{\pi/2}$$

$$= \frac{\pi U_m}{2} (-) (-1-1) = \pi U_m$$

If the power radiated by the unidirectional cosine source is the same as for an isotropic source, ($P = 4\pi U_0$) we can equate them.

i.e., $\pi U_m = 4\pi U_0$

$$\text{Directivity } (D) = \frac{U_m}{U_0} = 4$$



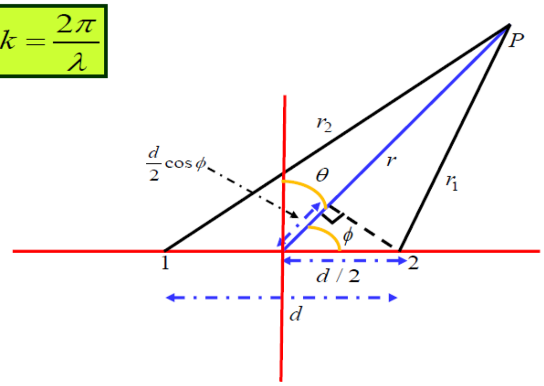
8a

Array of Two Isotropic Point Sources

$$E = E_o e^{-j\beta r_1} + E_o e^{-j\beta r_2}$$

$$\beta = k = \frac{2\pi}{\lambda}$$

$$\left.\begin{aligned} r_1 &\cong r + \frac{d}{2}\cos\phi \\ r_2 &\cong r + \frac{d}{2}\cos\phi \end{aligned}\right) r >> d, \phi = 90 - \theta$$



$$E = E_o e^{-j\beta r}\left[e^{-j\beta\frac{d}{2}\cos\phi} + e^{j\beta\frac{d}{2}\cos\phi}\right]$$

$$= E_o e^{-j\beta r}\left[e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}}\right]$$

$$\psi = \beta d\cos\phi = \frac{2\pi d}{\lambda}\cos\phi$$

$$= \beta d \sin\theta = \frac{2\pi d}{\lambda}\sin\theta$$

$$E = 2E_o \cos\left(\frac{\psi}{2}\right) = 2E_o \cos\left(\frac{\pi d}{\lambda}\cos\phi\right)$$

Two Isotropic Point Sources of Same Amplitude and Phase

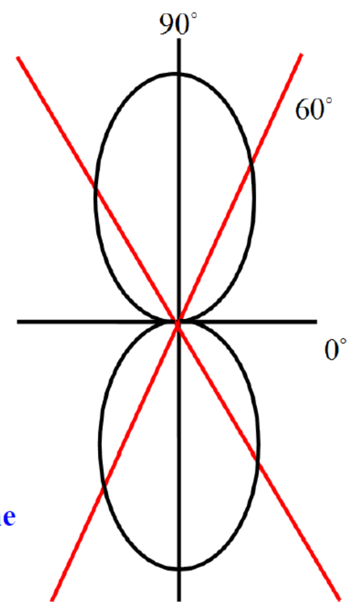
$$E = \cos\left(\frac{d_r}{2}\cos\phi\right)$$

$$d_r = \frac{2\pi d}{\lambda} = \beta d$$

For $d = \frac{\lambda}{2}$ $\qquad E = \cos\left(\frac{\pi}{2}\cos\phi\right)$

ϕ	**0°**	**90°**	**60°**
E	**0**	**1**	$1/\sqrt{2}$

HPBWs = 60° in one plane and 360° in another plane



8b

- The retarded value of the current at any point z on the antenna referred to a point at a distance s is
- $I = I_0 \sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \pm z\right)\right] e^{j\omega(t-(r/c))}$ (1)
- In (1), the function $\sin\left[\frac{2\pi}{\lambda}\left(\frac{L}{2} \pm z\right)\right]$ is the form factor for the current on the antenna.
- The expression $\left(\frac{L}{2} + z\right)$ is used when z < 0 and $\left(\frac{L}{2} - z\right)$ is used when z > 0.
- By regarding the antenna as made up of a series of infinitesimal dipoles of length dz, the field of the entire antenna may then be obtained by integrating the fields from all of the dipoles making up the antenna with the result
- $H_\varphi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos\left[\frac{\beta L cos\theta}{2}\right] - \cos(\frac{\beta L}{2})}{sin\theta}\right]$
- $E_\theta = \frac{j60[I_0]}{r}\left[\frac{\cos\left[\frac{\beta L cos\theta}{2}\right] - \cos(\frac{\beta L}{2})}{sin\theta}\right]$
- Where $[I_0] = I_0 e^{j\omega[t-(r/c)]}$ and E_θ=$120\pi H_\varphi$.

9a

Radiation Resistance of Loops

- $P = \frac{I_m^2}{2} R_r$
- $P = \iint S_r \, ds,$
-
- $S_r = \frac{1}{2}|H|^2\eta,$
-
- $ds = r^2 sin\theta d\theta d\phi$
- $R_r = 31{,}171(\frac{A}{\lambda^2})^2 = 197C_\lambda^4$

Where

- R_r is the radiation resistance of the loop antenna
- P is power radiated,
- I_m is peak value of current from loop,
- S_r is the radial component of the Poynting vector,
- ds is the area of small region in the sphere,
- η is the intrinsic impedance of free space equal to 120π ?,
- A is the area of the loop,
- C_λ is the circumference of the loop = $\frac{2\pi a}{\lambda} = \beta a$

9c

Yagi-Uda Configuration

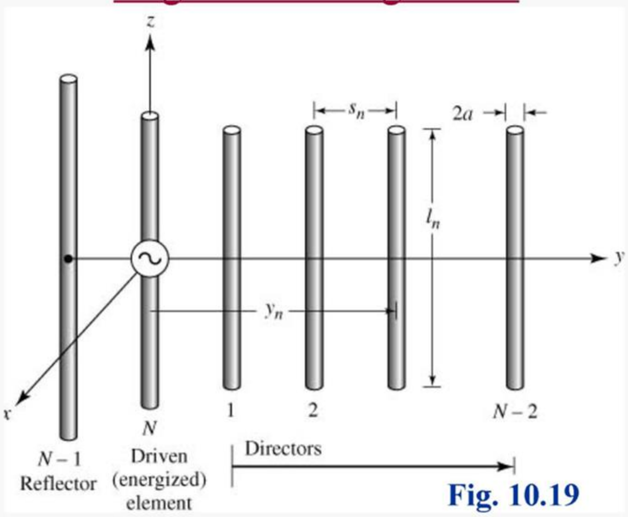


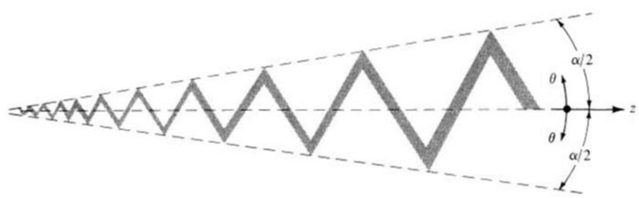
Fig. 10.19

Yagi-Uda Antenna

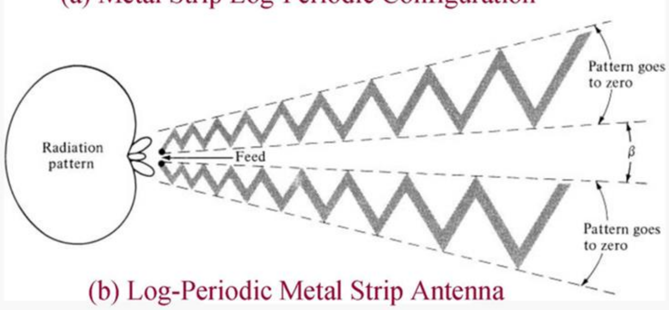
1. Applications
   a. Amateur radio
   b. TV antenna (usually single or few channels)
2. Frequency range
   a. HF (3-30 MHz)
   b. VHF (30-300 MHz)
   c. UHF (300-3,000 MHz)

Typically

A. Director lengths: (0.4 - 0.45)λ
B. Feeder length: (0.47 - 0.49)λ
(usually Folded Dipole)(resonant)
C. Reflector length: (0.5 - 0.525)λ
D. Reflector-feeder spacing : (0.2 - 0.25)λ
E. Director spacing: (0.3 - 0.4)λ



(a) Metal Strip Log-Periodic Configuration



(b) Log-Periodic Metal Strip Antenna

Fig. 11.5

Geometric Ratio τ (defines period):

$$\tau = \frac{R_n}{R_{n+1}} < 1 \qquad (11\text{-}23)$$

Width of Slot:

$$\chi = \frac{r_n}{R_{n+1}} < 1 \qquad (11\text{-}24)$$

$$\tau = \frac{f_1}{f_2} < 1, \qquad f_2 > f_1 \qquad (11\text{-}25)$$

f_1 and f_2 are one period apart.

Log-periodic structures provide frequency-independent operation above a certain low-frequency cutoff. This occurs when the longest tooth is approximately $\lambda/4$. Currents on the structure decay quite rapidly past the region where a $\lambda/4$ tooth exists.
This means that a smaller and smaller portion of the structure is used as the frequency is increased.

This implies that the effective electrical aperture is essentially independent of frequency. Because the antenna cannot be extended to the origin, due to the presence of the transmission line, a high-frequency cutoff occurs when the shortest tooth is less than $\lambda/4$ long.

10a)

Geometry of Helical Antenna

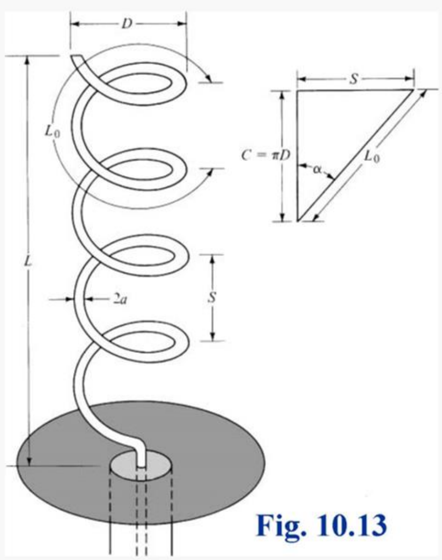


Fig. 10.13

End-Fire Mode

1. $12^\circ < \alpha < 14^\circ$
2. $\frac{3}{4}\lambda_o < C < \frac{4}{3}\lambda_o \quad (C \simeq \lambda_o \text{ near optimum})$
3. $N > 3$

$$R \simeq 140\left(\frac{C}{\lambda_o}\right) \quad \text{Accuracy}\,(\pm 20\%) \qquad \text{(10-30)}$$

$$\text{HPBW(degrees)} \simeq \frac{52\lambda_o^{3/2}}{C\sqrt{NS}} \qquad \text{(10-31)}$$

$$FNBW\text{(degrees)} \simeq \frac{115\lambda_o^{3/2}}{C\sqrt{NS}} \qquad \text{(10-32)}$$

$$D_o\text{(dimensionless)} \simeq 15N\frac{C^2S}{\lambda_o^3} \qquad \text{(10-33)}$$

$$AR = \frac{2N+1}{2N} \qquad \text{(10-34)}$$

Element Pattern (Axial Mode)

$$E(element) \cong \cos\theta$$

Total Field (Axial Mode)

$$E(total) = E(element) \cdot (AF)_n$$

$$E(total) = \cos\theta \frac{1}{N} \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)}$$

Chapter 10
Traveling Wave and Broadband Antennas

10b)

Small loop

- The field pattern of a small circular loop of radius "***a***" may be determined by considering a square loop of the same area, that is,
- $d^2 = \pi a^2$ (1)
- Where d is side length of square loop as shown in Fig 1
-

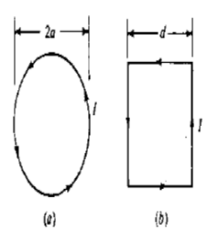


Fig 1: Circular loop (a) and square loop (b) of equal area

- It is assumed that the loop dimensions are small compared to the wavelength.
- It will be shown that the far-field patterns of circular and square loops of the same area are the same when the loops are small but different when they are large in terms of the wavelength.

- Let us consider the orientation of the loop as in Fig2 and the far-field is found to have only the E_φ component.
- To find the far-field pattern in the yz plane, it is only necessary to consider two of the four small linear dipoles (2 and 4).

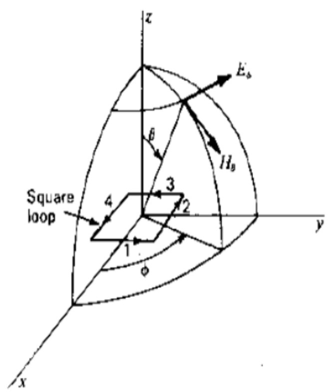


Fig 2: Relation of square loop to coordinates

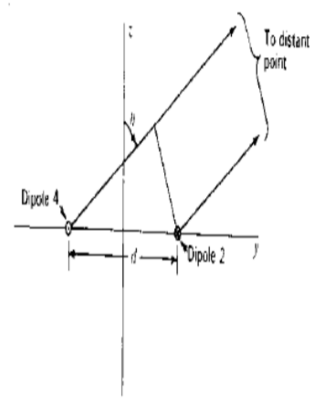


Fig 3: Construction for finding far field of dipoles 2 and 4 of square loop

- A cross section through the loop in the yz plane is presented in Fig 3.
- Since the individual small dipoles 2 and 4 are non-directional in the yz plane, the field pattern of the loop in this plane is the same as that for two isotropic point sources as treated earlier.
- $E_\varphi = -E_{\varphi 0}e^{j\psi/2} + E_{\varphi 0}e^{-j\psi/2}$ (2)
- Where $E_{\varphi 0}$ is electric field from individual dipole and
- $\psi = d_r sin\theta = \frac{2\pi d}{\lambda} sin\theta$ (3)
- It follows that
- $E_\phi = -2jE_{\phi 0}\sin(\frac{d_r}{2}sin\theta)$ (4)
- The factor j in (4) indicates that the total field E_ϕ is in phase quadrature with the field $E_{\phi 0}$ of the individual dipole.
- Now if $d \ll \lambda$, (4) can be written
- $E_\phi = -jE_{\phi 0}d_r sin\theta$ (5)
- In developing the fields of dipole, the z direction was considered where as in the present case it is in the x-direction (Fig 2 and 3).
- The angle θ in the dipole formula is measured from the dipole axis and is 90^0 in the present case.
- The angle θ in (5) is a different angle with respect to the dipole, being as shown in Figs 2 and 3.

- Therefore, we have for far field $E_{\phi 0}$ of the dipole
- $E_{\phi 0} = \frac{j60\pi[I]L}{r\lambda}$ (6)
- Where [I] is the retarded current on the dipole and r is the distance from the dipole.
- Substituting (6) in (5) then gives
- $E_{\phi} = \frac{60\pi[I]Ld_r sin\theta}{r\lambda}$ (7)
- However, the length L of the short dipole is the same as d, that is, L=d.
- Noting also that $d_r = \frac{2\pi d}{\lambda}$ and that the area A of the loop is d^2, (7) becomes
- $E_{\phi} = \frac{120\pi^2[I]sin\theta}{r}\frac{A}{\lambda^2}$ (8)
- This is the instantaneous value of the E_{ϕ} component of the field of a small loop of area A.
- The peak value of the field is obtained by replacing [I] by I_0, where I_0 is the peak current in time on the loop.
- The other component of the far field of the loop is H_{θ}, which is obtained by the intrinsic impedance of the medium, in this case, free space.
- $H_{\theta} = \frac{E_{\phi}}{120\pi} = \frac{\pi[I]\ sin\theta}{r}\ \frac{A}{\lambda^2}$ (9)

8C $\quad l = \frac{\lambda}{15}$, $R_{Loss} = 1\,\Omega$

$$R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2$$

$$= 80\pi^2 \left(\frac{\lambda/15}{\lambda}\right)^2$$

$$= 3.51\,\Omega$$

$$\eta = \frac{R_r}{R_r + R_L} = \frac{3.51}{3.51 + 1}$$

$$\cong 0.78$$

$$A_e = \frac{3\lambda^2}{8\pi} = 0.119\,\lambda^2\,m^2$$

9 b $\quad r = 1m$, $f = 0.9\,MHz$

$$\lambda = c/f = 3\times10^8/0.9\times10^6$$

$$= 333.33\,m$$

$$D = 0.68\,C_\lambda = 0.68\,\frac{2\pi a}{\lambda}$$

$$= \frac{2\pi \times 1}{333.33}$$

$$= 1.08$$

10 C $\quad D = 0.5m$, $f = 1MHz$ $\quad \lambda = c/f = 300\,m$

$$R_r = 197\,C_\lambda^4 = 197\left(\frac{2\pi a}{\lambda}\right)^2$$

$$= 197\left(\frac{2\pi\,0.25}{300}\right)^2$$

$$= 1.59\,\Omega$$