CBCS SCHEME

USN | | | | | | | | | | |

18EC54

Fifth Semester B.E. Degree Examination, Jan./Feb. 2021
Information Theory and Coding

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Derive the expression for average information contents of symbols in long independent sequence. **(06 Marks)**

b. Find the relationship between Hartley's, nats and bits. **(06 Marks)**

c. A code is composed of dots and dashes. Assuming that a dash is 3 times as long as a dot and has one-third the probability of occurrence. Calculate:
(i) The information in a dot and dash
(ii) The entropy of dot-dash code
(iii) The entropy rate of information, if a dot losts for 10 ms and this time is allowed between symbols. **(08 Marks)**

OR

2 a. Consider a second order mark-off source as shown in Fig.Q2(a). Here s = {0, 1} and states are A{0, 0}, B = {0, 1}, C = {1, 0} and D = {1, 1}.
(i) Compute the probability of states (ii) Compute the entropy of the source

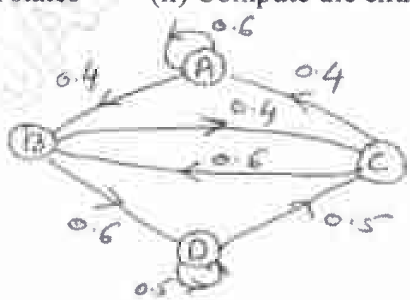

Fig.Q2(a) **(10 Marks)**

b. Prove that entropy of zero memory extension source is given by $H(s^n) = nH(s)$. **(10 Marks)**

Module-2

3 a. A Discrete Memory Source (DMS) has an alphabet X = $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ and source statistics. P = {0.3, 0.25, 0.20, 0.12, 0.08, 0.05}. Construct binary Huffman code. Also find the efficiency and redundancy of coding. **(10 Marks)**

b. Apply Shannon encoding algorithm to the following set of messages and obtain code efficiency and redundancy. **(10 Marks)**

m_1	m_2	m_3	m_4	m_5
1/8	1/16	3/16	1/4	3/8

OR

4 a. A source having alphabet s = $\{s_1, s_2, s_3, s_4, s_5\}$ produces a symbols with respective probabilities 1/2, 1/6, 1/6, 1/9, 1/18.
(i) When the symbols are coded as shown 0, 10, 110, 1110, 1111 respectively.
(ii) When the code is as 00, 01, 10, 110, 111
Find code efficiency and redundancy **(12 Marks)**

b. State and prove Kraft McMillan inequality. **(08 Marks)**

Module-3

5 a. Discuss the binary Erasure Channel (BEC) and also derive channel capacity equation for BEC. **(08 Marks)**

b. A channel has the following characteristics

$$P\left[\frac{Y}{X}\right] \quad \begin{array}{c c} & \begin{array}{cccc} Y_1 & Y_2 & Y_3 & Y_4 \end{array} \\ \begin{array}{c} X_1 \\ \\ X_2 \end{array} & \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ & & & \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \end{array}$$

Find H(X), H(Y), H(X, Y) and channel capacity if r = 1000 symbols/sec. **(12 Marks)**

OR

6 a. Determine the rate of transmission of information through a channel whose noise characteristics is as shown in Fig.Q6(a).

Given $P(X_1) = P(X_2) = \frac{1}{2}$. Assume $r_s = 10{,}000$ symbols/sec.

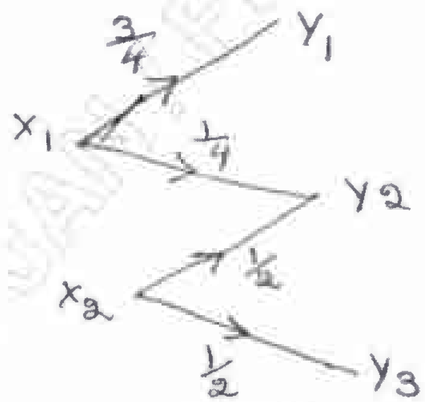

Fig.Q6(a) **(10 Marks)**

b. What is mutual information? Mention its properties and prove that $I(X:Y) = H(X) - H\left(\frac{X}{Y}\right)$; $I(X:Y) = H(Y) - H\left(\frac{Y}{X}\right)$. **(10 Marks)**

Module-4

7 a. For a (6, 3) linear block code the check bits are related to the message bits as per the equations given below:

$c_1 = d_1 \oplus d_2$

$c_2 = d_1 \oplus d_2 \oplus d_3$

$c_3 = d_2 \oplus d_3$

i) Find the generator matrix G
ii) Find all possible code words
iii) Find error detecting and error correcting capabilities of the code. **(12 Marks)**

b. The generator polynomial of a (7, 4) cyclic code is $g(x) = 1 + x + x^2$. Find the 16 code words of this code by forming the code polynomial v(x) using V(X) = D(X)G(X) where D(X) is the message polynomial. **(08 Marks)**

OR

8 a. Design a linear block code with a minimum distance of 3 and a message block size of 8 bits. **(08 Marks)**

b. For a (6, 3) cyclic code, find the following:
   (i) G(x)
   (ii) G in systematic form
   (iii) All possible code words
   (iv) Show that every code polynomial is multiple of g(x). **(12 Marks)**

Module-5

9 a. For the convolution encoder shown in Fig.Q9(a) the information sequence is d = 10011. Find the output sequence using the following two approaches.
   (i) Time domain approach
   (ii) Transfer domain approach

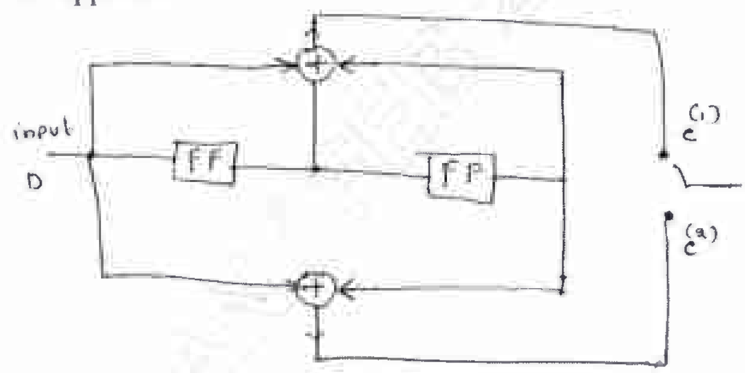

Fig.Q9(a) **(10 Marks)**

b. Consider a (3, 1, 2) convolution encoder with $g^{(1)} = 110$, $g^{(2)} = 101$ and $g^{(3)} = 111$.
   (i) Draw the encoder diagram
   (ii) Find the code word for message sequence (11101) using Generator matrix and Transfer domain approach. **(10 Marks)**

OR

10 a. Consider the rate $r = \frac{1}{2}$ and constraint length K = 2 convolution encoder shown in Fig.Q10(a).
   (i) Draw the state diagram.
   (ii) Draw the code tree
   (iii) Draw Trellis diagram,
   (iv) Trace the path through the tree that corresponds to the message sequence {1, 0, 1}.

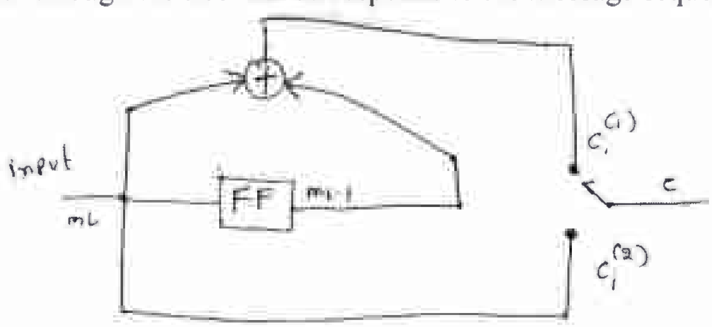

Fig.Q10(a) **(14 Marks)**

b. Explain Viterbi decoding. **(06 Marks)**

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Sl no.	Sub code	Subject Name	Remarks
1	18EC54	Information Theory and coding	The following questions answers are modified. Refer the attachment. Q1a, 1c, 2a, 3a. Q4a II code solutions are not given. Refer updated solutions in the attachment. Q5b.data missing. In the scheme p1(x)=p2(x)=1/2 or any other assumption consider for awarding the marks. Q8a. n values not shown. Refer the attachment. Q9a,b detailed solution given in the attachment. Students may do the problem using matrix method or formula approach.Valuers can consider any approach. Q9b. Refer attachment for marks split-up. Q10 a. state table and trellis diagram is added in the attachment.

Corrections for Q 1a, 1c, 2a, 3a, 4a 5b 8a, 9a, 9b, 10a.

Scheme & Solution

Subject: Information Theory and Coding

Code: 18EC54

Exam: Jan-Feb-2021

Semester - 5th sem CBCS / BE

1a. In a long message containing N symbols emitted by a source alphabet of M-symbols, the information content of i^{th} symbol is,

$$I(s_i) = \log_2 \frac{1}{P_i} \text{ bits.} \quad — ②$$

To derive
↳ Total information content $= I_{total} = \sum_{i=1}^{M} N P_i \log_2 \frac{1}{P_i}$ — ②

$$H = \frac{I_{total}}{N} = \sum_{i=1}^{M} P_i \log_2 \frac{1}{P_i} \text{ bits/symbol.} \quad — ②$$

1c. $P_{dot} = 3/4$; $P_{dash} = 1/4$

$I_{dot} = 0.415$ bits.

$I_{dash} = 2$ bits. — ③

$H(S) = 0.813$ bits/symbol — ②

Symbol rate
↳ $r_s = 4$ symbols/100 ms $= 40$ symbols/sec. — ②

Information rate $= R = r_s H = 40 \times 0.8113 = 32.45$ bits/sec — ①

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2 a) (i) $P(A) = 0.6\,P(A) + 0.4\,P(C) \Rightarrow \boxed{P(A) = P(C)}$

$P(B) = 0.4\,P(A) + 0.6\,P(C) \Rightarrow \boxed{P(B) = P(C)}$ (using above)

$P(C) = 0.4\,P(B) + 0.5\,P(D)$

$P(D) = 0.5\,P(D) + 0.6\,P(B)$

$\therefore\ 0.5\,P(D) = 0.6\,P(B)$

$$\boxed{P(D) = 1.2\,P(A)}$$

$\therefore\ P(A) + P(B) + P(C) + P(D) = 1$

$$\boxed{P(A) = P(B) = P(C) = \frac{5}{21}}$$

then $\boxed{P(D) = \frac{2}{7}}$

(ii) $H_A = 0.6 \log \frac{1}{0.6} + 0.4 \log \frac{1}{0.4} = 0.9708$ bits/sy

$H_B = 0.4 \log \frac{1}{0.6} + 0.6 \log \frac{1}{0.6} = 0.9708$ bits/sy

$H_C = 0.6 \log \frac{1}{0.6} + 0.4 \log \frac{1}{0.4} = 0.9708$ —"—

$H_D = 0.5 \log \frac{1}{0.5} + 0.5 \log \frac{1}{0.5} = 1$ bits/sy

$H = \sum_{i=1}^{4} P_i H_i = P_A H_A + P_B H_B + P_C H_C + P_D H_D$

$H = 0.9791$ bits/sy

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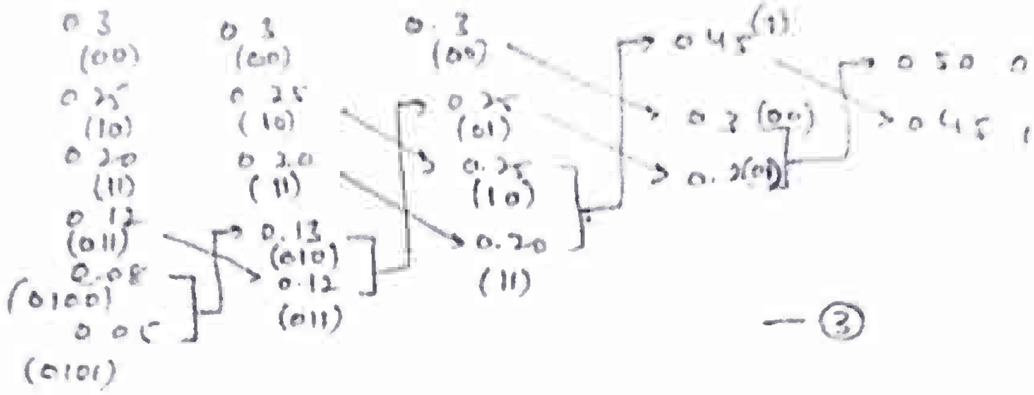
3a) Binary Huffman coding:

$H(x) = 2.36$ bits/symbol — (2)

Codewords



— (3)

codeword lengths — (2)

$L = \sum_i p_i n_i = 2.38$ bits/symbol — (2)

$\eta = \frac{H(x)}{L} = 0.99$ — (1)

Redundancy $= 1 - \eta = 0.01$ — (1)

1(a)

	I-code		II-code	
1/2	0	1	00	2
1/6	10	2	01	2
1/6	110	3	10	2
1/9	1110	4	110	3
1/18	1111	4	111	3

Two set of codewords are specified solution is to find, codeword length, Entropy, η, R.

$\Rightarrow L = 2.16$ bit/sy

$\eta = 90\%$

$R = 0.1$ — (6)

$H(c) = \sum_i p_i \log_2 \frac{1}{p_i} = 1.945$ bit/symb

$L = \sum_i p_i n_i = 2$ bit/symbol

$\eta = 97.27\%$ & $R = 2.75$ — (6)

5b. To determine i/p entropy, we need input probabilities, which are not provided in the example.

For the assumed input probabilities, marks can be provided.

$H(X) = 2$ Marks
$H(X,Y) = 2$ Marks

~~Grace marks can~~ be ~~given~~.

8a) Single error correcting Hamming code.

$$n \leq 2^{n-k} - 1$$

for $k = 8$, by iteratively solving, $n = 12$. — 2

H^T = Transpose of parity check polynomial matrix — 2

H - matrix — 2

G - Matrix — 2

9a) Time-domain approach:

$$c_i^{(1)} = \sum_{l=0}^{M} g_i^{(1)} m_{i-l}$$ $\boxed{\text{or}}$ Matrix Method

$$c_i^{(2)} = \sum_{l=0}^{M} g_i^{(2)} m_{i-l}$$

The outputs are $c_i^{(1)} = \{1111001\}$

$c_i^{(2)} = \{1011111\}$

$C = \{11, 10, 11, 11, 01, 01, 11\}$. — 5

Transform domain approach:

$$C(x) = 1 + x + x^2 + x^4 + x^5 + x^6 + x^7 + x^9 + x^{11} + x^{12} + x^{13}$$

$C = \{11, 10, 11, 11, 01, 01, 11\}$ — 5

9b) Encoder diagram — 2

$g_i^{(1)} = \{110\}$; $g_i^{(2)} = \{101\}$; $g_i^{(3)} = \{111\}$.

$$G = \begin{bmatrix} 111 & 101 & 011 & 000 & 000 & 000 & 000 \\ 000 & 111 & 101 & 011 & 000 & 000 & 000 \\ 000 & 000 & 111 & 101 & 011 & 000 & 000 \\ 000 & 000 & 000 & 111 & 101 & 011 & 000 \\ 000 & 000 & 000 & 000 & 111 & 101 & 011 \end{bmatrix}$$

$C = DG = [11101][G]$

$C = \{111, 010, 001, 110, 100, 101, 011\}$

— 4

Transform domain approach — 4

10a) For the given convolution encoder,

let $S_0 = 0$; $S_1 = 1$

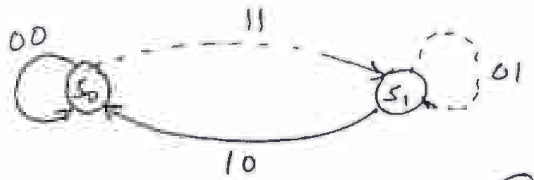
State table

$c^{(1)} = m_l + m_{l-1}$: $c^{(2)} = m_{l-1}$

Present state m_{l-1}	i/p (m_l)	Next state	outputs $c_i^{(1)}$	$c_i^{(2)}$
(S_0) 0	0	0 (S_0)	0	0
(S_0) 0	1	1 (S_1)	1	1
(S_1) 1	0	0 (S_0)	1	0
(S_1) 1	1	1 (S_1)	0	1

— 3

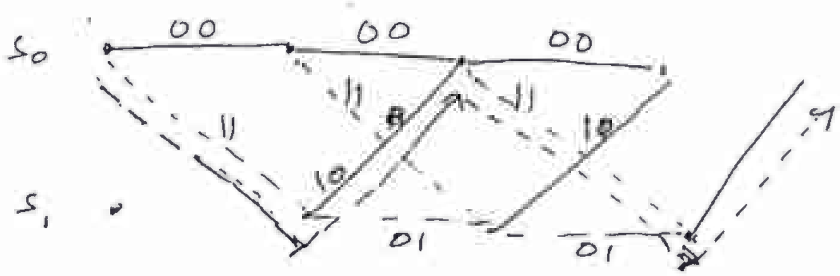
$S_0 \to S_0$ (00)
$S_0 \to S_1$ (11)
$S_1 \to S_0$ (10)
$S_1 \to S_1$ (01)



— 2

Tree diagram — 5

Trellis diagram:



trace of i/p {101} ⇒ {11, 10, 11, 10}

6.3.2021

— 4

Scheme & Solutions

Subject Title : Information Theory and coding **Subject Code :** 18EC54

Question Number	Solution	Marks Allocated
1 (a)	$H(S) = \sum_{i=1}^{q} p_i \log \frac{1}{p_i}$ bits/message symbol	6 m.
(b)	$I = \log_e \frac{1}{p}$ nats 1 Hartely $= \log_e P$ nats or 2.303 nats 1 Hartely $= \frac{1}{\log_{10} 2} = 3.32$ bits 1 nat $= \frac{1}{\log_e 2} = 1.443$ bits	3+3m
(c)	$P_{dot} = \frac{3}{4}$ $P_{dash} = \frac{1}{4}$ (i) information in dash $I_{dash} = 2$ bits $I_{dot} = \log \frac{1}{P_{dot}} = 0.415$ bits $I_{dash} = \log \frac{1}{P_{dash}} = 2$ bits ii) $H(s) = P_{dot} \log \frac{1}{P_{dot}} + P_{dash} \log \frac{1}{P_{dash}}$ $= 0.8113$ bits/msg-symbol	3+3+2

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Question Number	Solution	Marks Allocated
	$R_s = r_s H(S)$ $= 32.452$ bits/sec	
2 (a)	(i) $S^2 = \{00, 01, 10, 11\}$ $P_A = \frac{1}{4}$, $P_B = \frac{1}{4}$, $P_C = \frac{1}{4}$, $P_D = 1/4$ ii) $P_{AA} = 0.6$, $P_{BC} = 0.4$, $P_{DD} = 0.5$, $P_{CA} = 0.4$ $P_{AB} = 0.4$, $P_{BD} = 0.6$, $P_{DC} = 0.5$ $P_{CB} = 0.6$ remaining 0 iii) $H_A = \sum_{j=ABCD} P_{Aj} \log_2 \frac{1}{P_{Aj}} = 0.97$ bits/msg $H_B = \sum_{j=ABCD} P_{Bj} \log_2 \frac{1}{P_{Bj}} = 0.97$ bits/msg $H_C = \sum_{j=ABCD} P_{Cj} \log_2 \frac{1}{P_{Cj}} = 0.97$ bits/msg $H_D = \sum_{j=ABCD} P_{Dj} \log \frac{1}{P_{Dj}} = 1$ bits/message $\boxed{H = \sum_{i=ABCD} P_i H_i = 0.977 \text{ bits/message}}$	2+2+6m.
b)	Proof: $H(S^n) = n\,H(S)$	10 m.
3 a)	$H(x) = \sum_{i=1}^{6} P_i \log \frac{1}{P_i}$ $= 2.36$ bits/symbol	

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Question Number	Solution	Marks Allocated

symbol	P_i	code word	n_i
x_1	0.3	00	2
x_2	0.25	10	2
x_3	0.20	11	2
x_4	0.12	011	3
x_5	0.08	0100	4
x_6	0.05	0101	4

5+5

$$L = \sum_{i=1}^{6} n_i P_i = 2.38 \text{ binary digits/symbol}$$

$$\eta_c = \frac{H(x)}{L \log r} = 0.99 \text{ or } 99\%$$

$$\gamma = 1 - \eta_c = 0.01$$

b)

$\alpha_1 = 0$, $\alpha_2 = 0.375$, $\alpha_3 = 0.625$, $\alpha_4 = 0.8125$

$\alpha_5 = 0.9375$ $\alpha_6 = 1$

$L_1 = 2$, $L_2 = 2$, $L_3 = 3$, $L_4 = 3$, $L_5 = 4$ 5m

$S_1 = 00$, $S_2 = 01$, $S_3 = 101$, $S_4 = 110$, $S_5 = 1111$

$$L = \sum_{i=1}^{5} P_i l_i = 2.437 \text{ binits/msg symbol}$$

$$H(S) = \sum_{i=1}^{5} P_i \log \frac{1}{P_i} = 2.1085 \text{ bits/msg-sym}$$

$$\eta = \frac{H(S)}{L} = 0.865$$

$\% \eta_c = 86.5\%$ 5m

% code Redundancy = 13.5%

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Question Number	Solution	Marks Allocated
4 a)	$L = \sum_{i=1}^{5} P_i l_i = 2.0$ binits/msg-symbol $H(S) = \sum_{i=1}^{5} P_i \log \frac{1}{P_i} = 1.945$ bits/msg-sym $\eta = \frac{H(S)}{L} \times 100 = 97.25\%$ $R_{\eta c} = 1 - \eta = 2.75\%$	5+5m +2m
b)	state + proof	2+6m
5 a)	Explaination derivation $C = \bar{p}$	3+5m
b)	$H(X) = \sum_{i=1}^{2} P(x_i) \log \frac{1}{P(x_i)}$ $= 1$ bits/message-symbol $H(Y) = \sum_{j=1}^{4} P(y_j) \log \frac{1}{P(y_j)} = 2$ bits/message symbol $H(X,Y) = \sum_{i=1}^{2} \sum_{j=1}^{4} P(x_i, y_j) \log \frac{1}{P(x_i y_j)}$ $= 2.918$ bits/message-symbol $I(X,Y) = H(X) + H(Y) - H(X,Y)$ $C = \max[I(X,Y)] = 0.0817$ bits/message symbol	2×6 =12m

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Question Number	Solution	Marks Allocated

$\gamma = 1000$ symbols/sec

$C = \gamma \times 0.0817$

$C = 81.7$ bits/sec

6 a)

$H(Y/X) = 0.90564$ bits/message symbol

$H(Y) = 1.5612$

$I(X,Y) = H(Y) - H(Y/X)$ — 10 m.

$= 0.655$ bits/message-symbol

$R_t = I(X,Y)\, \gamma_s$

$= 6556.4$ bit/sec

b) Defination — 2+4+4

mention properties

proof

7 a)

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(a)

$$G = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

(b)

D	C = DG	Hamming weight
000	000000	
001	011001	3
010	111010	4
011	100011	3
100	110100	3
101	101101	4
110	001110	3
111	010111	4

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Question Number	Solution	Marks Allocated
	(c) $d_{min} = 3$ maximum no of errors it can detect $= d_{min} - 1 = \underline{2}$ maximum no of errors it can correct $= \frac{1}{2}(d_{min} - 1)$ $= \underline{1}$ ..	5+5+ 2m
7. b)	$g(x) = 1 + x + x^2$	

message (D)	code vector (V)	message (D)	code-vector (V)
0000	0000000	1000	1101000
0001	0001101	1001	1100101
0010	0011010	1010	1110010
0011	0010111	1011	1111111
0100	0110100	1100	1011100
0101	0111001	1101	1010001
0110	0101110	1110	1000110
0111	0100011	1111	1001011

8 m

$$V(x) = D(x)\, g(x)$$

8 a)

$$n \leq 2^{n-k} - 1$$

given $k = 8$

$$n \leq 2^{n-8} - 1$$

$$H^T = \begin{bmatrix} P_{8\times 4} \\ \hline I_4 \end{bmatrix}$$

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Question Number	Solution	Marks Allocated
	see below	4+4m

$$H^T = \begin{bmatrix} 0&0&1&1 \\ 0&1&0&1 \\ 0&1&1&0 \\ 0&1&1&1 \\ 1&0&0&1 \\ 1&0&1&0 \\ 1&0&1&1 \\ 1&1&0&0 \\ \hline 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$$

$$H = \left[\begin{array}{cccccccc|cccc} 0&0&0&0&1&1&1&1&1&0&0&0 \\ 0&1&1&1&0&0&0&1&0&1&0&0 \\ 1&0&1&1&0&1&1&0&0&0&1&0 \\ 1&1&0&1&1&0&1&0&0&0&0&1 \end{array}\right]$$

$$G = [I_k \,\vdots\, P_{k\times(n-k)}] = [I_8 \,\vdots\, P_{8\times4}]$$

$$G = \left[\begin{array}{cccccccc|cccc} 1&0&0&0&0&0&0&0&0&0&1&1 \\ 0&1&0&0&0&0&0&0&0&1&0&1 \\ 0&0&1&0&0&0&0&0&0&1&1&0 \\ 0&0&0&1&0&0&0&0&0&1&1&1 \\ 0&0&0&0&1&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&1&0&1&0 \\ 0&0&0&0&0&0&1&0&1&0&1&1 \\ 0&0&0&0&0&0&0&1&1&1&0&0 \end{array}\right]$$

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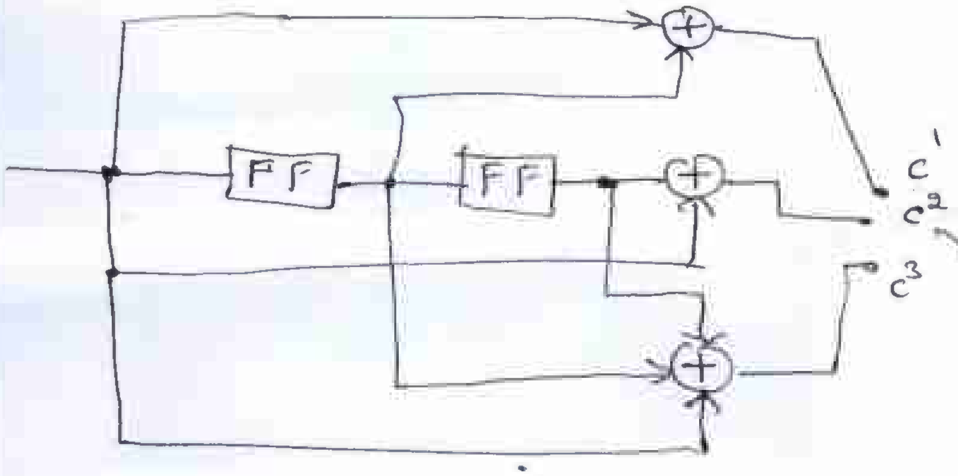
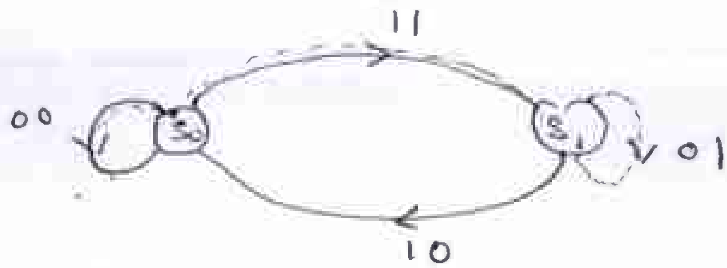
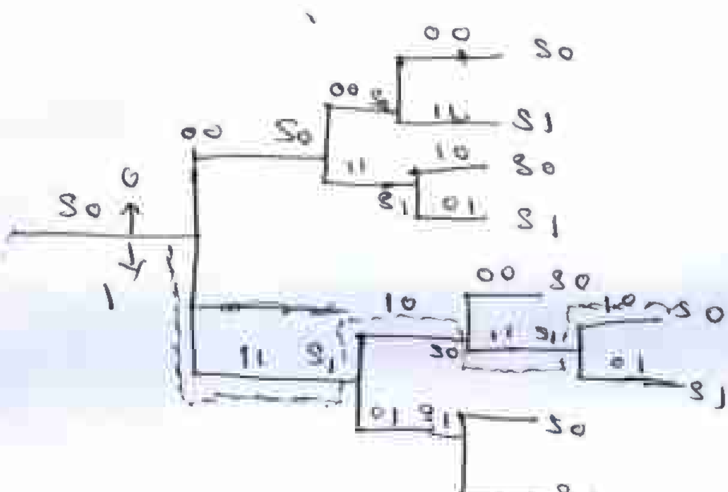
Question Number	Solution	Marks Allocated
8. b)	(i) $g(x) = 1+x^3$	2 m
	(ii) $G = \left[\begin{array}{ccc\|ccc} 1&0&0&1&0&0 \\ 0&1&0&0&1&0 \\ 0&0&1&0&0&1 \end{array}\right]$	2 m
	(iii) $P = I_3$ $C = \{000000, 001001, 010010, 011011, 100100, 101101, 110110, 111111\}$	3 m
	(iv) $x^2 \oplus x^5 = x^2 g(x)$ $x\,g(x)$, $(x^2 \oplus x)\,g(x)$, $1\,g(x)$, $(x^2\oplus 1)\,g(x)$ $(x\oplus 1)\,g(x)$, $(x^2 \oplus x \oplus 1)\,g(x)$	5 m
9 a)	$g^{(1)} = [111]$ $g^{(2)} = [101]$ i) Time domain: $G = \begin{bmatrix} 11 & 10 & 11 & 00 & 00 & 00 & 00 \\ 00 & 11 & 10 & 11 & 00 & 00 & 00 \\ 00 & 00 & 11 & 10 & 11 & 00 & 00 \\ 00 & 00 & 00 & 11 & 10 & 11 & 00 \\ 00 & 00 & 00 & 00 & 11 & 10 & 11 \end{bmatrix}$ $c = [11, 10, 11, 11, 01, 01, 11]$	5 m
	ii) Transform domain $d = [10011]$ $d(x) = 1+x^3+x^4$ $c^1(x) = 1+x+x^2+x^3+x^6$ $c^2(x) = 1+x^2+x^3+x^4+x^5+x^6$ $c(x) = 1+x+x^2+x^3+x^5+x^6+x^7+x^9+x^{11}$ $+x^{12}+x^{13}$ $c = [11, 10, 11, 11, 01, 01, 11]$	5 m

Subject Title : Information theory & coding **Subject Code :** 18EC54

Question Number	Solution	Marks Allocated
9 (b)	 C = 111, 010, 001, 110, 100, 101, 011	5+5m
10 a)	State diagram 	5m
	<u>Code tree</u>, 	5m
	Trellis diagram	4m

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Question Number	Solution	Marks Allocated
b)	Explanation of viterbi decoding	6m