CBCS SCHEME

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18EC55

Fifth Semester B.E. Degree Examination, Jan./Feb. 2021
Electromagnetic Waves

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. State and explain Coulomb's law in vector form. **(05 Marks)**

b. Derive the relationship between dot products between unit vectors of the three coordinate systems. Transform the following vectors to spherical system at the point given :
   i) $10a_x$ at P(3, 2, 4)
   ii) $10a_y$ at Q(5, 30°, 4) **(07 Marks)**

c. Four 10nc positive charges are located in z = 0 plane at the corners of a square 8cm on a side. A fifth 10nc charge is located at a point 8cm distant from other charges. Calculate the magnitude of total force on this fifth charge for $E = E_0$. **(08 Marks)**

OR

2 a. Using Coloumb's law, derive the expression for electric field Intensity 'E' due to an infinite sheet of charge of surface charge density ρ_s c/m². **(08 Marks)**

b. Four uniform sheets of charge are located as 20 Pc/m² at y = 7 ; −8 Pc/m² at y = 3 ; 6 P c/m² at y = −1 ; −18Pc/m² at y = −4. Find E at i) P_A (2, 6, −4) ii) P_B $(10^6, 10^6, 10^6)$. **(06 Marks)**

c. Find the net outward flux (ψ) through the surface of a cube 2m on an edge centered at origin if $D = 5x^2 ax + 10za_z$ c/m². (The edges of cube are parallel to coordinate axes). **(06 Marks)**

Module-2

3 a. State and prove Gauss law in Integral form. **(05 Marks)**

b. Find the volume charge density at the points indicated if

   i) $D = 4\rho z \sin\phi \, a_\rho + 2\rho z \cos\phi \, a_\phi + 2\rho^2 \sin\phi \, a_z$ c/m² at $P_A\left(1, \frac{\pi}{2}, 2\right)$

   ii) $D = \sin\theta \cos\phi \, a_r + \cos\theta \cos\phi \, a_\phi - \sin\phi \, a_\phi$ c/m² at $P_B\left(2, \frac{\pi}{3}, \frac{\pi}{6}\right)$ **(07 Marks)**

c. Evaluate both sides of Divergence Theorem if $D = \frac{5r^2}{4} a_r$ c/m² in spherical co-ordinate for the volume enclosed between r = 1m and r = 2m. **(08 Marks)**

OR

4 a. Find the work done in moving a 5μc charge from origin to P(2, −1, 4) through $E = 2xyza_x + x^2 2a_y + x^2 y \, a_z$ V/m via the path :
   i) Straight line segments (0, 0, 0) to (2, 0, 0) to (2, −1, 0) to (2, −1, 4)
   ii) Straight line x = −2y ; z = 2x. **(08 Marks)**

b. Find 'E' at P(3, 60°, 25°) in free space, given $V = \frac{60 \sin\theta}{r^2}$ V. **(06 Marks)**

c. Derive equation of continuity. Given $J = -10^6 z^{1.5} a_z$ A/m² in a region 0 ≤ ρ ≤ 20μm, find the total current crossing a surface z = 0.1m. **(06 Marks)**

Module-3

5 a. Derive the expression for capacitance of a cylindrical capacitor using Laplace equation.

b. Assume $V = V_0$ at $\rho = a$ and $V = 0$ at $\rho = b$, $b > a$. **(08 Marks)**

In spherical co-ordinate $V = 865$ V at $r = 50$cm and $E = 748.2\ a_r$ at $r = 85$cm. Determine the location of voltage reference if potential depends only on 'r'. **(08 Marks)**

c. Verify whether the potential function $V = 2x^2 - 3x^2 + z^2$ satisfies Laplace equation. **(04 Marks)**

OR

6 a. Derive the expression for magnetic field intensity 'H' at the centre of a square current carrying loop of I amps with side 'L' meters using Biot Savart's law. **(08 Marks)**

b. Given $H = \frac{x+2y}{z^2} a_y + \frac{2}{z} a_z$ A/m. find J. Use J to find total current passing through the surface $z = 4$, $1 \le x \le 2$, $3 \le y \le 5$. **(08 Marks)**

c. Explain the concept of scalar and vector magnetic potential. **(04 Marks)**

Module-4

7 a. The point charge $Q = 18$nc has a velocity of 5×10^6m/s in the direction $a_v = 0.6\ a_x + 0.75a_y + 0.3a_z$. Calculate the magnitude of the force exerted on the charge by the field.

i) $B = -3a_x + 4a_y + 6a_z$ mT

ii) $E = -3a_x + 4a_y + 6a_z$ kV/m **(08 Marks)**

b. The magnetization in a magnetic material for which $\chi_m = 8$ is $150z^2\ a_x$ A/m. At $z = 4$cm, find the magnitude of i) J ii) J_T iii) J_B. **(06 Marks)**

c. Derive the expression for the force between two differential current elements. **(06 Marks)**

OR

8 a. Derive the expression for the boundary conditions between two magnetic medias. **(06 Marks)**

b. Let the permitivity be 5μH/m in region A where $x < 0$ and 20 μH/m in region B where $x < 0$. and 20 μH/m in region B where $x > 0$. If $K = 150a_y - 200a_z$ A/m at $x = 0$ and $H_A = 300a_x - 400a_y + 500a_z$ A/m. Find : i) $|H_{tA}|$ ii) $|H_{NA}|$ iii) $|H_tB|$ iv) $|H_{NB}|$. **(08 Marks)**

c. A circular loop of radius 10cm radius is located in x – y plane in a magnetic field $B = 0.5 \cos(377t)(3a_y + 4a_z)$ T. Determine the voltage induced in the loop. **(06 Marks)**

Module-5

9 a. What is the inconsistency of Ampere's law with continuity equation? Derive the modified Ampere's law by Maxwell for time varying fields. **(06 Marks)**

b. Given $E = E_m \sin(\omega t - \beta z)\ a_y$ V/m, find i) D ii) B iii) H. sketch E and H at t = 0. **(08 Marks)**

c. Prove that the conduction current is equal to the displacement current between the two plates for $V = V_0 e^{j\omega t}$ in a parallel plate capacitor. **(06 Marks)**

OR

10 a. Show that the intrinsic impedance of the perfect dielectric $\eta = \frac{|E|}{|H|} = \sqrt{\frac{\mu}{E}}$ and show that its value in free space is 377Ω. **(08 Marks)**

b. A uniform plane wave of a frequency 300MHz travels in +x direction in a lossy medium with $E_r = 9$, $\mu_r = 1$ and $\sigma = 10$ mhos/m. Calculate γ, α, β and η. **(06 Marks)**

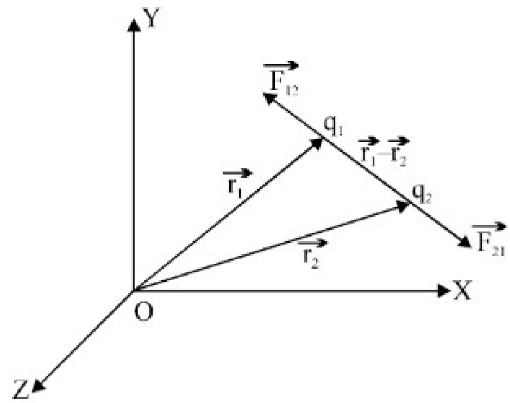
c. State and prove Poynting theorem. **(06 Marks)**

* * * * *

1(a).

Coulomb's law in vector form

Suppose the position vectors of two charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, then, electric force on charge q_1 due to charge q_2 is,



$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

Similarly, electric force on q_2 due to charge q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$$

Here q_1 and q_2 are to be substituted with sign.

Position vector of charges q_1 and q_2 are $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ respectively.

Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are
the co-ordinates of charges q_1 and q_2 respectively

1(b).

Dot products of unit vectors in cylindrical and rectangular coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x\cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y\cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z\cdot$	0	0	1

Dot products of unit vectors in spherical and rectangular coordinate systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

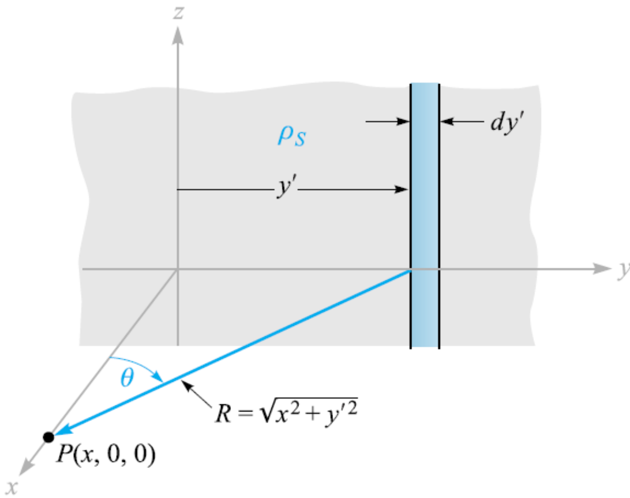
Ans. $-5.57\mathbf{a}_r - 6.18\mathbf{a}_\theta - 5.55\mathbf{a}_\phi;\ 3.90\mathbf{a}_r + 3.12\mathbf{a}_\theta + 8.66\mathbf{a}_\phi;$

1(c).

Arrange the charges in the xy plane at locations (4,4), (4,-4), (-4,4), and (-4,-4). Then the fifth charge will be on the z axis at location $z = 4\sqrt{2}$, which puts it at 8cm distance from the other four. By symmetry, the field on the fifth charge will be z-directed, and will be four times the z component of force produced by each of the four other charges.

$$E = \frac{4}{\sqrt{2}} \times \frac{q}{4\pi\epsilon_0 d^2} = \frac{4}{\sqrt{2}} \times \frac{(10^{-8})}{4\pi(8.85\times10^{-12})(0.08)^2} = 40000 \text{ V/m}$$

2(a).



charge per unit length, is $\rho_L = \rho_S\, dy'$, and the distance from this line charge to our general point P on the x axis is $R = \sqrt{x^2 + y'^2}$. The contribution to E_x at P from this differential-width strip is then

$$dE_x = \frac{\rho_S\, dy'}{2\pi\epsilon_0\sqrt{x^2 + y'^2}} \cos\theta = \frac{\rho_S}{2\pi\epsilon_0} \frac{x dy'}{x^2 + y'^2}$$

Adding the effects of all the strips,

$$E_x = \frac{\rho_S}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{x\, dy'}{x^2 + y'^2} = \frac{\rho_S}{2\pi\epsilon_0} \tan^{-1} \frac{y'}{x} \Bigg]_{-\infty}^{\infty} = \frac{\rho_S}{2\epsilon_0}$$

If the point P were chosen on the negative x axis, then

$$E_x = -\frac{\rho_S}{2\epsilon_0}$$

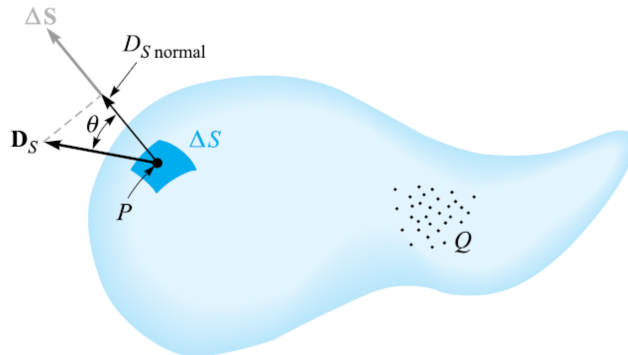
for the field is always directed away from the positive charge. This difficulty in sign is usually overcome by specifying a unit vector $\mathbf{a}_N$, which is normal to the sheet and directed outward, or away from it. Then

$$\boxed{\mathbf{E} = \frac{\rho_S}{2\epsilon_0}\mathbf{a}_N}$$

2(b). (i)(-20-8+6-18)/Ɛo, (ii) (20-8+6-18)/Ɛo,

2(c). 80C

3(a). Gauss's law: The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.



The electric flux density $\mathbf{D}_S$ at P arising from charge Q. The total flux passing through ΔS is $\mathbf{D}_S \cdot \Delta\mathbf{S}$.

At any point P, consider an incremental element of surface ΔS and let $\mathbf{D}_S$ make an angle θ with $\Delta\mathbf{S}$, as shown in Figure . The flux crossing ΔS is then the product of the normal component of $\mathbf{D}_S$ and $\Delta\mathbf{S}$,

$$\Delta\Psi = \text{flux crossing } \Delta S = D_{S,\text{norm}}\Delta S = D_S \cos\theta \Delta S = \mathbf{D}_S \cdot \Delta\mathbf{S}$$

where we are able to apply the definition of the dot product

The *total* flux passing through the closed surface is obtained by adding the differential contributions crossing each surface element $\Delta\mathbf{S}$,

$$\Psi = \int d\Psi = \oint_{\substack{\text{closed}\\\text{surface}}} \mathbf{D}_S \cdot d\mathbf{S}$$

The charge enclosed might be several point charges, in which case

$$Q = \Sigma Qn$$

or a line charge,

$$Q = \int \rho_L \, dL$$

or a surface charge,

$$Q = \int_S \rho_S dS \qquad \text{(not necessarily a closed surface)}$$

or a volume charge distribution,

$$Q = \int_{\text{vol}} \rho_v \, dv$$

The last form is usually used, and we should agree now that it represents any or all of the other forms. With this understanding, Gauss's law may be written in terms of the charge distribution as

$$\boxed{\oint_S \mathbf{D}_S \cdot d\mathbf{S} = \int_{\text{vol}} \rho_v \, dv}$$

3b.

(i) $D = 4\rho z \sin\phi \, a_\rho + 2\rho z \cos\phi \, a_\rho + 2\rho^2 \sin\phi \, a_z$

Volume charge density

$\rho_v = \nabla \cdot D$

$$\nabla \cdot D = \frac{1}{\rho}\frac{\partial(\rho D_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{\rho} \cdot 8\rho z \sin\phi - 2z \sin\phi + (2\rho^2 \sin\phi) \times 0$$

$$= 8z \sin\phi - 2z \sin\phi$$

$$= 6z \sin\phi \quad \text{at } \left(1, \frac{\pi}{2}, 2\right)$$

$$= 2 \ C/m^3$$

(ii) $$\nabla \cdot D = \left(\frac{1}{r^2}\left(\frac{\partial r^2 D_r}{\partial r}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(D_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi} \right)$$

$$= 0 + \frac{\cos\phi}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta \cdot \cos\theta) + \frac{1}{r\sin\theta}(\cos\phi)$$

$$= \frac{\cot\theta}{r}\left[-\sin^2\theta + \cos^2\theta\right] + \frac{\cos\phi}{r\sin\theta} \quad \text{at } \left(2, \frac{\pi}{3}, \frac{\pi}{6}\right)$$

$$= \frac{\cos\frac{\pi}{3}}{2}\left[-\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] + \frac{\cos \pi/6}{2\sin \pi/3}$$

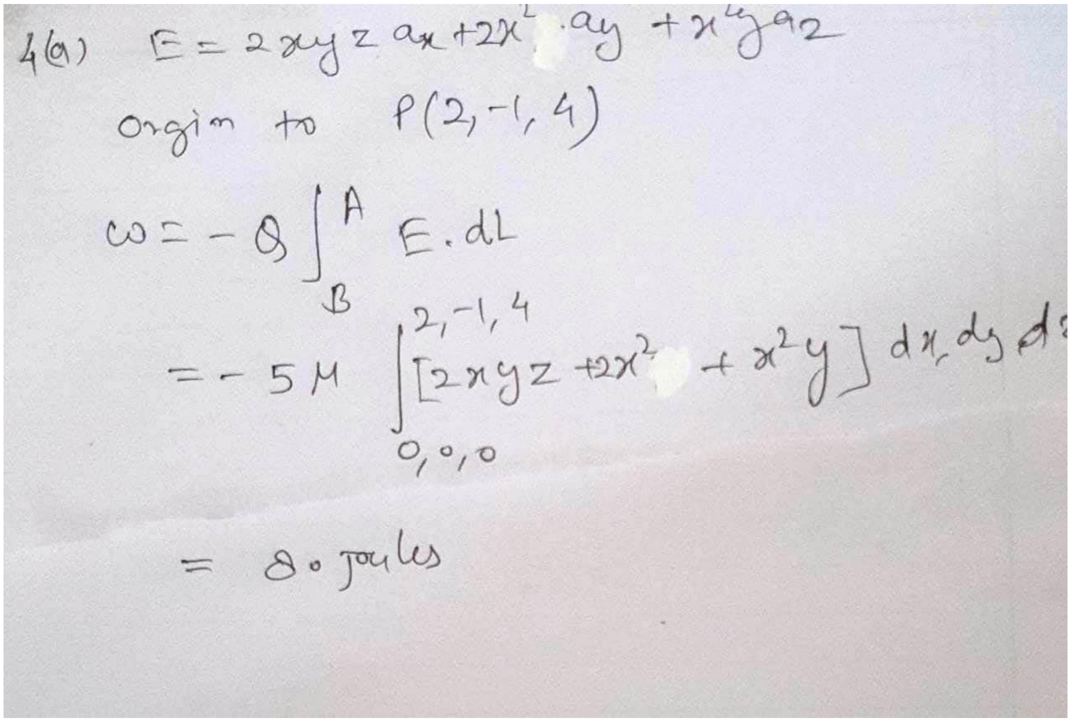
$$= \frac{1}{4}\left[-\frac{1}{2}\right] + \frac{1}{2}$$

$$= \frac{3}{8} \ C/m^3$$

3(b).

3(c). $\iint \left(5r^2/4\right) \cdot \left(r^2 \sin\theta \, d\theta \, d\phi\right)$, which is the integral to be evaluated. Since it is double integral, we need to keep only two variables and one constant compulsorily. Evaluate it as two integrals keeping r = 1 for the first integral and r = 2 for the second integral, with $\varphi = 0 \rightarrow 2\pi$ and $\theta = 0 \rightarrow \pi$. The first integral value is 80π, whereas the second integral gives -5π. On summing both integrals, we get 75π.

4(a).



4(a) $E = 2xyz\, a_x + 2x^2\, a_y + x^2 y\, a_z$

Orgin to $P(2,-1,4)$

$$W = -Q \int_B^A E \cdot dL$$

$$= -5\mu \int_{0,0,0}^{2,-1,4} [2xyz + 2x^2 + x^2 y]\, dx, dy, dz$$

$= 80$ Joules

80J

4(b).

4.(b)

$$V = \frac{60 \sin\theta}{r^2} \quad , \quad P(3, 60^\circ, 25^\circ)$$

$$\therefore E = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\hat{a}_\phi \right)$$

$$\frac{\partial V}{\partial r} = (60\sin\theta)(-2)r^{-3} = -\frac{120\sin\theta}{r^3}$$

$$\frac{\partial V}{\partial \theta} = \left(\frac{60}{r^2}\right)(\cos\theta)$$

$$\frac{\partial V}{\partial \phi} = 0$$

$$\therefore \vec{E} = -\left[-\frac{120\sin\theta}{r^3}\hat{a}_r + \frac{1}{r}\cdot\left(\frac{60}{r^2}\right)\cos\theta\,\hat{a}_\theta \right]$$

$\therefore$ at $P(3, 60^\circ, 25^\circ)$,

$$\vec{E} = -\left[-\frac{120\sin(60^\circ)}{3^3}\hat{a}_r + \frac{60}{3^3}\cos(60^\circ)\,\hat{a}_\theta \right]$$

$$= \left(3.84\,\hat{a}_r - 1.11\,\hat{a}_\theta\right) \text{ V/m}$$

4(c). Continuity equation of current and Problem:

Continuity of Current

principle of Conservation of charge and continuity equation — follows the principle

Charges can neither be created nor be destroyed, although equal amounts of positive and negative charge may be simultaneously created, obtained by separation, destroyed or lost by recombination.

(Q)

Consider any region bounded by a closed surface. Current through the closed surface,

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s}}$$

This outward flow of positive charge must be balanced by a decrease of positive charge within that closed surface.

Charge inside closed surface $\rightarrow Q_i \Rightarrow$ rate of decrease of $Q_i = -\frac{dQ_i}{dt}$.

Principle of conservation of charge

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s} = -\frac{dQ_i}{dt}}$$ Integral form of Continuity equation

Differential form:

Divergence theorem

$$\oint_S \vec{J}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{J})\,dv.$$

& $Q_i = \int_V \rho_v\,dv.$

$$\frac{dQ_i}{dt} = \frac{d}{dt}\int_V \rho_v\,dv.$$

closed Surface is constant $\Rightarrow \frac{dQ_i}{dt} = \int_V \frac{\partial \rho_v}{\partial t}\,dv.$

$$\therefore \boxed{I = \int_V (\nabla\cdot\vec{J})\,dv = -\int_V \frac{\partial \rho_v}{\partial t}\,dv.}$$

Since the expression is true for any volume, however small, it is true for an incremental volume Δv.

$$\therefore \boxed{(\nabla\cdot\vec{J})\,\Delta v = -\frac{\partial \rho_v}{\partial t}\,\Delta v.}$$

$$\boxed{\nabla\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ Point form of Continuity equation.

$\Downarrow$

Current or charge per second diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point.

Problem:

4) c) $\vec{J} = -10^6 z^{1.5}\vec{a}_z\ A/m^2$; $0<\rho<20\mu m$, $z=0.1m$

$$I = \iint \vec{J}\cdot\vec{ds} = \iint -10^6 z^{1.5}\vec{a}_z \cdot ds\,\vec{a}_z = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{20\times10^{-6}} -10^6 z^{1.5}\,\rho\, d\rho\, d\phi \Big|_{z=0.1m}$$

$$= -10^6\times(0.1)^{1.5}\int_0^{2\pi} d\phi \times \int_{\rho=0}^{20\times10^{-6}} \rho\, d\rho$$

$$= -10^6\times(0.1)^{1.5}\times 2\pi\times\left[\frac{\rho^2}{2}\right]_0^{20\times10^{-6}}$$

$$\boxed{I = -39.7\,\mu A}$$

5(a).Capacitance of a cylindrical capacitor:

Cylindrical co-ordinates:

Variations with respect to z are nothing new.

Variation with ρ only.

$$\nabla^2 V = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right)=0$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{dV}{d\rho}\right)=0.$$

multiply by ρ, $\frac{d}{d\rho}\left(\rho\frac{dV}{d\rho}\right)=0.$

Integrate $\rho\frac{dV}{d\rho} = A.$ (Constant).

$$\frac{dV}{d\rho} = \frac{A}{\rho}.$$

Integrate,

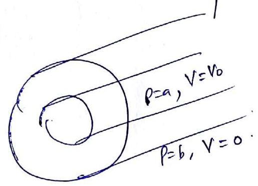
$$\boxed{V = A \ln \rho + B.}$$

Boundary conditions

Equipotential surfaces given by $\rho =$ constant are cylinders.

↓

Coaxial capacitor (or) Coaxial transmission line



$b > a.$

$$V_0 = A \ln a + B.$$

$$0 = A \ln b + B$$

$$B = -A \ln b.$$

$$V_0 = A \ln a + (-A \ln b)$$

$$V_0 = A\left[\ln (a/b)\right]$$

$$\boxed{A = V_0 / \ln (a/b).} \quad ; \quad \boxed{B = -\frac{V_0 \ln b}{\ln (a/b)}}$$

$$\therefore V = \frac{V_0}{\ln(a/b)} \ln \rho - \frac{V_0}{\ln(a/b)} \ln b$$

$$\boxed{V = V_0 \frac{\ln (b/\rho)}{\ln (b/a)}}$$

$$\vec{E} = -\nabla V$$

$$= -\frac{\partial V}{\partial \rho} \vec{a_\rho} = -\frac{\partial}{\partial \rho}\left[\frac{V_0 \ln (b/\rho)}{\ln (b/a)}\right] \vec{a_\rho}.$$

$$= -\frac{\partial}{\partial \rho}\left[\frac{V_0 \ln (b)}{\ln (b/a)} - \frac{V_0 \ln \rho}{\ln (b/a)}\right] \vec{a_\rho}.$$

$$\boxed{\vec{E} = \frac{V_0}{\rho} \cdot \frac{1}{\ln (b/a)} \vec{a_\rho}}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{D} = \frac{\varepsilon V_0}{\rho} \frac{1}{\ln(b/a)} \vec{a_\rho}$$

$$D_N = |\vec{D}|_{\rho=a} = \frac{\varepsilon V_0}{a} \frac{1}{\ln(b/a)}$$

$$\rho_s = \frac{\varepsilon V_0}{a} \frac{1}{\ln(b/a)}.$$

$$Q = \int_s \rho_s \, ds = \frac{\varepsilon V_0}{a} \frac{2\pi a L}{\ln(b/a)}$$

$$C = \frac{Q}{V_0} = \frac{\varepsilon V_0}{a V_0} \frac{2\pi a L}{\ln(b/a)}$$

$$\boxed{C = \frac{2\pi\varepsilon L}{\ln(b/a)}}$$

5(b).Problem:

5) b) Problem: Spherical co-ordinates

$V = V_0$ at $r = a$

$V = 0$ at $r = b$, $b > a$

$V = 865\,V$ at $r = 50\,cm$

$\vec{E} = 748.2\,\vec{a_r}$ at $r = 85\,cm$.

Determine location of voltage reference $V(r)$.

Laplace's equation: $\nabla^2 V = 0$

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\frac{d}{dr}\left(r^2 \frac{dv}{dr}\right) = 0$$

$$r^2 \frac{dv}{dr} = c_1$$

$$\boxed{\frac{dv}{dr} = \frac{c_1}{r^2}} \rightarrow (1)$$

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial r} \vec{a_r}\right]$$

$$\boxed{\vec{E} = -\frac{c_1}{r^2} \vec{a_r} \; V/m} \rightarrow (3)$$

$$\boxed{V = \frac{-c_1}{r} + c_2 \; V} \rightarrow (2)$$

Boundary conditions, At $r = 50\,cm$, $V = 865\,V \rightarrow (4)$

At $r = 85\,cm$, $\vec{E} = 748.2\,\vec{a_r}\; V/m \rightarrow (5)$

Applying (5) in (3)

$$\Rightarrow 748.2\,\vec{a_r} = \frac{-c_1}{(85 \times 10^{-2})^2} \vec{a_r}$$

$$\boxed{c_1 = -540.5745} \rightarrow (6)$$

Applying ④ & ⑥ in ②

$$865 = \frac{540.5745}{(50\times10^{-2})} + c_2$$

$$c_2 = 865 - \frac{540.5745}{(50\times10^{-2})}$$

$$c_2 = 865 - 1081.149$$

$$\boxed{c_2 = -216.149} \rightarrow ⑦$$

$$\therefore \quad V(r) = \frac{-c_1}{r} + c_2$$

$$\boxed{V(r) = \frac{540.5745}{r} - 216.149}$$

To find r at $V=0$:

$$0 = \frac{540.5745}{r} - 216.149$$

$$\frac{540.5745}{r} = 216.149$$

$$r = \frac{540.5745}{216.149}$$

$$\boxed{r = 2.5\ m}$$

5(c).Problem:

Problem:

5c) $V = 2x^2 - 3x^2 + z^2$

$V = -x^2 + z^2$

Laplace's equation: $\nabla^2 V = 0$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{\partial^2(-x^2+z^2)}{\partial x^2} + 0 + \frac{\partial^2}{\partial z^2}(-x^2+z^2)$$

$$= \frac{\partial}{\partial x}(-2x) + \frac{\partial}{\partial z}(2z)$$

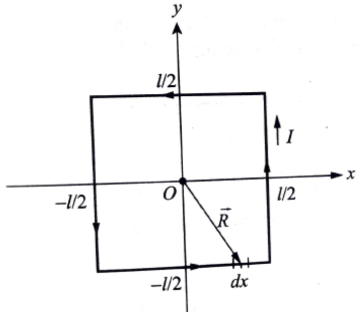
$$= -2+2$$

$$\boxed{\nabla^2 V = 0}$$ Given potential function satisfies Laplace equation.

6(a).Magnetic field intensity at the centre of a square loop:

Solution

Consider a square loop, located in xy-plane, carrying current I in anticlockwise direction as shown in Fig. E4.8.



Due to symmetry, each half side contributes same amount of magnetic field $\vec{H}$ at the center. Using Biot-Savart's law, the differential magnetic field at the origin for a half side $0 \le x \le l/2$ and $y = -l/2$ is

$$d\vec{H} = \frac{I d\vec{l} \times \vec{a}_R}{4\pi R^2} = \frac{I d\vec{l} \times \vec{R}}{4\pi R^3}$$

where the current element $Id\vec{l} = I\,dx\,\vec{a}_x$ and the distance vector $\vec{R} = -x\vec{a}_x + (l/2)\,\vec{a}_y$. Hence,

$$d\vec{H} = \frac{(I\,dx\,\vec{a}_x) \times \left(-x\vec{a}_x + (l/2)\vec{a}_y\right)}{4\pi\left[x^2 + (l/2)^2\right]^{3/2}}$$

$$= \frac{I\,dx\,(l/2)\,\vec{a}_z}{4\pi\left[x^2 + (l/2)^2\right]^{3/2}} \qquad (\text{since } \vec{a}_x \times \vec{a}_x = 0 \text{ and } \vec{a}_x \times \vec{a}_y = \vec{a}_z)$$

There are 8 half sides and all contribute to $\vec{H}$ in the same direction. Therefore, the total magnetic field intensity at the origin is

$$\vec{H} = 8\int_0^{l/2} \frac{I\,dx\,(l/2)\,\vec{a}_z}{4\pi\left[x^2 + (l/2)^2\right]^{3/2}} = \frac{Il\vec{a}_z}{\pi}\int_0^{l/2} \frac{dx}{\left[x^2 + (l/2)^2\right]^{3/2}}$$

$$= \frac{Il\vec{a}_z}{\pi}\left[\frac{4x}{l^2\sqrt{x^2 + (l/2)^2}}\right]_0^{l/2} = \frac{I\vec{a}_z}{\pi l}\left[\frac{2l}{\sqrt{(l/2)^2 + (l/2)^2}}\right]$$

$$= \frac{2\sqrt{2}I}{\pi l}\vec{a}_z = \frac{2\sqrt{2}I}{\pi l}\vec{a}_n \text{ A/m}$$

where $\vec{a}_n$ is the unit normal to the plane of the loop as given by the right hand rule. If the current flows in clockwise direction, the magnetic field intensity will be in $-\vec{a}_z$ direction i.e., in negative z-direction.

6(b).Problem:

The magnetic field intensity is given in a certain region of space as

$$\mathbf{H} = \frac{x + 2y}{z^2}\mathbf{a}_y + \frac{2}{z}\mathbf{a}_z \text{ A/m}$$

a) Find $\nabla \times \mathbf{H}$: For this field, the general curl expression in rectangular coordinates simplifies to

$$\nabla \times \mathbf{H} = -\frac{\partial H_y}{\partial z}\mathbf{a}_x + \frac{\partial H_y}{\partial x}\mathbf{a}_z = \underline{\frac{2(x + 2y)}{z^3}\mathbf{a}_x + \frac{1}{z^2}\mathbf{a}_z \text{ A/m}}$$

b) Find $\mathbf{J}$: This will be the answer of part a, since $\nabla \times \mathbf{H} = \mathbf{J}$.

c) Use $\mathbf{J}$ to find the total current passing through the surface $z = 4$, $1 < x < 2$, $3 < y < 5$, in the $\mathbf{a}_z$ direction: This will be

$$I = \int\int \mathbf{J}\big|_{z=4} \cdot \mathbf{a}_z\,dx\,dy = \int_3^5 \int_1^2 \frac{1}{4^2}dx\,dy = \underline{1/8 \text{ A}}$$

6(c).Scalar and Vector Magnetic Potentials:

Scalar & Vector Magnetic Potentials:

electrostatic potential V $\rightarrow$ greatly simplified electrostatic field problems.

Can a scalar magnetic potential be defined?

Let us assume the existence of scalar magnetic potential, V_m.

whose negative gradient gives magnetic field intensity.

Let, $\vec{H} = -\nabla V_m$.

w.k.t.

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{H} = \vec{J} = \nabla \times (-\nabla V_m)$$

$$\nabla \times (-\nabla V_m) = 0 \Rightarrow \vec{J} = 0.$$

Vector Identity:

$\nabla \times \nabla V_m = 0.$

Curl of gradient of a scalar is zero.

$\therefore$ If a scalar magnetic potential is defined for a region, then current density must be zero through out the region

$$\therefore \boxed{\vec{H} = -\nabla V_m \quad (\vec{J} = 0).}$$

Scalar magnetic potential is useful in magnetic problems involving geometries in which current carrying conductors occupy a relatively small fraction of total region of interest & also in case of permanent magnets

V_m is in amperes.

This scalar potential also satisfies Laplace's equation.

In free space,

$$\vec{B} = \mu_0 \vec{H}$$

(from $\nabla \cdot \vec{B} = 0$).

$$\nabla \cdot \vec{B} = \nabla \cdot \mu_0 \vec{H} = 0.$$

$$\mu_0 (\nabla \cdot \vec{H}) = 0$$

$$\mu_0 (\nabla \cdot (-\nabla V_m)) = 0$$

$$\boxed{\nabla^2 V_m = 0 \quad \text{for } \vec{J} = 0}$$

(In homogeneous magnetic materials).

Electrostatic potential (V) is a conservative field.

Magnetostatic potential (V_m) is not a conservative field.

Vector Magnetic Potential:

extremely useful in studying radiation from antennas, transmission lines, & radiation leakage from waveguides & microwave ovens.

used in regions where current density is zero or nonzero. & extended to time varying case.

w.k.t.

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Divergence of curl of a vector field is zero.

Let $\boxed{\nabla \cdot (\nabla \times \vec{A}) = 0.}$

Therefore we get

Useful Definition of $\vec{A}$. $\therefore \boxed{\vec{B} = \nabla \times \vec{A}}$ where $\vec{A}$ signifies Vector magnetic potential

$$\therefore \vec{H} = \frac{1}{\mu_0}(\nabla \times \vec{A}).$$

$$\boxed{\nabla \times \vec{H} = \vec{J} = \frac{1}{\mu_0}(\nabla \times \nabla \times \vec{A})}$$

taking curl twice

Vector Identity

$$\boxed{\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}}$$

$\boxed{\text{(x) } \vec{A} \text{ is in } Wb/m.}$

7(a).Problem:

Problem:

The point charge $Q = 18\,nC$ has a velocity of 5×10^6 m/s in the direction $\vec{a_v} = 0.6\,\vec{a_x} + 0.75\,\vec{a_y} + 0.3\,\vec{a_z}$. Calculate the magnitude of force exerted on the charge by the fields $\vec{B} = -3\vec{a_x} + 4\vec{a_y} + 6\vec{a_z}$ mT & $\vec{E} = -3\vec{a_x} + 4\vec{a_y} + 6\vec{a_z}$ kV/m acting together.

$$\boxed{\vec{F} = Q(\vec{E} + \vec{v}\times\vec{B})}$$

$$\vec{v}\times\vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0.6 & 0.75 & 0.3 \\ -3 & 4 & 6 \end{vmatrix} \times 5\times10^6\times10^{-3} = 5\times10^6\left[\vec{a_x}(3.3) - \vec{a_y}(4.5) + \vec{a_z}(4.65)\right]\times10^{-3}$$

$$\vec{F} = 18\times10^{-9}\left(-3000\,\vec{a_x} + 4000\,\vec{a_y} + 6000\,\vec{a_z} + 5\times10^6\times10^{-3}(3.3\vec{a_x} - 4.5\vec{a_y} + 4.65\vec{a_z})\right)$$

$$= \cancel{0.297\,\vec{a_x} - 0.344\,\vec{a_y} + 0.236\,\vec{a_z}}$$

$$\boxed{\vec{F} = 2.43\times10^{-4}\,\vec{a_x} - 3.33\times10^{-4}\,\vec{a_x} + 5.265\times10^{-4}\,\vec{a_z}}\ N$$

$$\boxed{|\vec{F}| = 668.68\ \mu N}$$

7(b).Problem:

Problem:

7) b) $\chi_m = 8$

$\vec{M} = 150z^2\,\vec{a_x}$ A/m

at $z = 4$ cm, Find $|\vec{J}|$, $|\vec{J_T}|$ & $|\vec{J_B}|$

Solution:

$\mu_r = 1 + \chi_m = 9$

$\mu = \mu_0\mu_r = 4\pi\times10^{-7}\times9 = 36\pi\times10^{-7}$ H/m

$$\vec{H} = \frac{\vec{M}}{\chi_m} = \frac{150z^2\,\vec{a_x}}{8} \Rightarrow \boxed{18.75z^2\,\vec{a_x}\ A/m = \vec{H}}$$

$$\vec{B} = \mu\vec{H} = 36\pi\times10^{-7}\times18.75z^2\,\vec{a_x}$$

$$\boxed{\vec{B} = 675\pi\times10^{-7}z^2\,\vec{a_x}\ T}$$

$$\vec{J} = \vec{\nabla}\times\vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 18.75z^2 & 0 & 0 \end{vmatrix} = -\vec{a_y}\left(-\frac{\partial}{\partial z}(18.75z^2)\right) = 37.5z\,\vec{a_y}$$

$$\boxed{\vec{J} = 37.5z\,\vec{a_y}\ A/m^2}$$

At $z = 4$ cm,

$$\vec{J} = 37.5\times4\times10^{-2}\,\vec{a_y}$$

$$\boxed{\vec{J} = 1.5\,\vec{a_y}\ A/m^2} \quad \boxed{|\vec{J}| = 1.5\ A/m^2 \text{ at } z = 4\text{cm}}$$

$$J_b = \vec{\nabla}\times\vec{M} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 150z^2 & 0 & 0 \end{vmatrix} = -\vec{a_y}\left(-\frac{\partial}{\partial z}(150z^2)\right)$$

$$\boxed{\vec{J_b} = 300z\,\vec{a_y}\ A/m^2}$$

$\vec{J}_b$ at 4cm $\Rightarrow$ $\vec{J}_b = 300 \times 4 \times 10^{-2}\ \vec{a}_y\ A/m^2$

$$\boxed{\vec{J}_b = 12\ \vec{a}_y\ A/m^2}$$

$$\boxed{|\vec{J}_b| = 12\ A/m^2 \quad \text{at} \quad z = 4\text{cm}}$$

$$\vec{J}_T = \frac{\vec{\nabla} \times \vec{B}}{\mu_0} = \vec{\nabla} \times (18.75 z^2\ \vec{a}_x \times 9) = \vec{\nabla} \times (168.75 z^2\ \vec{a}_x)$$

$$= \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 168.75 z^2 & 0 & 0 \end{vmatrix}$$

$$= \vec{a}_y (168.75 \times 2z)$$

$$\boxed{\vec{J}_T = 337.5 z\ \vec{a}_y\ A/m^2}$$

At $z = 4$cm $\Rightarrow$ $\boxed{\begin{array}{l} \vec{J}_T = 13.5\ \vec{a}_y\ A/m^2 \\ |\vec{J}_T| = 13.5\ A/m^2 \end{array}}$

7(c).Force between two differential current elements:

8.3 FORCE BETWEEN DIFFERENTIAL CURRENT ELEMENTS

The concept of the magnetic field was introduced to break into two parts the problem of finding the interaction of one current distribution on a second current distribution. It is possible to express the force on one current element directly in terms of a second current element without finding the magnetic field. Because we claimed that the magnetic-field concept simplifies our work, it then behooves us to show that avoidance of this intermediate step leads to more complicated expressions.

The magnetic field at point 2 due to a current element at point 1 was found to be

$$d\mathbf{H}_2 = \frac{I_1 d\mathbf{L}_1 \times \mathbf{a}_{R12}}{4\pi R_{12}^2}$$

Now, the differential force on a differential current element is

$$d\mathbf{F} = I\, d\mathbf{L} \times \mathbf{B}$$

and we apply this to our problem by letting $\mathbf{B}$ be $d\mathbf{B}_2$ (the differential flux density at point 2 caused by current element 1), by identifying $I\, d\mathbf{L}$ as $I_2 d\mathbf{L}_2$, and by symbolizing the differential amount of our differential force on element 2 as $d(d\mathbf{F}_2)$:

$$d(d\mathbf{F}_2) = I_2 d\mathbf{L}_2 \times d\mathbf{B}_2$$

Because $d\mathbf{B}_2 = \mu_0 d\mathbf{H}_2$, we obtain the force between two differential current elements,

$$d(d\mathbf{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\mathbf{L}_2 \times (d\mathbf{L}_1 \times \mathbf{a}_{R12}) \tag{13}$$

The total force between two filamentary circuits is obtained by integrating twice:

$$\begin{aligned} \mathbf{F}_2 &= \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[d\mathbf{L}_2 \times \oint \frac{d\mathbf{L}_1 \times \mathbf{a}_{R12}}{R_{12}^2} \right] \\ &= \mu_0 \frac{I_1 I_2}{4\pi} \oint \left[\oint \frac{\mathbf{a}_{R12} \times d\mathbf{L}_1}{R_{12}^2} \right] \times d\mathbf{L}_2 \end{aligned} \tag{14}$$

8(a).

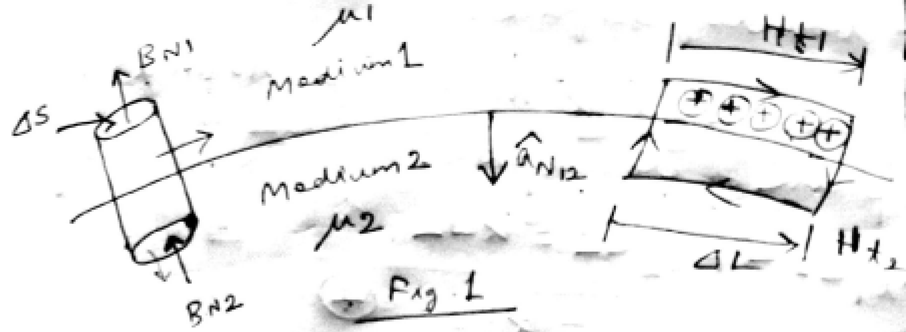


Fig. 1

$y = a$

$(-\hat{a}_x)$

Gauss's law of magnetostatics, $\oiint \vec{B} \cdot \vec{ds} = 0$

$$B_{N1}\Delta S - B_{N2}\Delta S = 0$$

or $B_{N1}\Delta S = B_{N2}\Delta S$

or $\boxed{B_{N1} = B_{N2}}$ .. (1)

Normal comp. of B is continuous

Note
B_{N1}, H_{t1}, M_{N1}
B_{N2}, H_{t2}, M_{N2}
B_N, H_N, M_N
B_t, H_t, M_t

The fig. 1 shows a boundary b/w two isotropic homogeneous linear materials with permeabilities μ_1 and μ_2.

Now, $\vec{B} = \mu \vec{H}$

$\therefore B_{N1} = \mu_1 H_{N1}$ and $B_{N2} = \mu_2 H_{N2}$

From (1), $\mu_1 H_{N1} = \mu_2 H_{N2}$

or $\boxed{H_{N2} = \frac{\mu_1}{\mu_2} H_{N1}}$ .. (2)

$\vec{M} = \chi_m \vec{H}$

$M_{N1} = \chi_{m1} H_{N1}$ and $M_{N2} = \chi_{m2} H_{N2}$

$$\frac{M_{N2}}{\chi_{m2}} = \frac{\mu_1}{\mu_2} \frac{M_{N1}}{\chi_{m1}}$$

or $\boxed{M_{N2} = \frac{\mu_1}{\mu_2} \cdot \frac{\chi_{m2}}{\chi_{m1}} \cdot M_{N1}}$ .. (3)

Small closed path in a plane normal to the boundary surface.

Surface current $\vec{k}$ with comp. normal to the plane of closed path.

$$H_{t1}\Delta L - H_{t2}\Delta L = k\Delta L$$

or $\boxed{H_{t1} - H_{t2} = k}$ ... (4)

or $\boxed{(\vec{H_1} - \vec{H_2}) \times \vec{a}_{N12} = \vec{k}}$

$\vec{B} = \mu\vec{H}$ $\therefore$ From (4),

$\left[\because B_{t1} = \mu_1 H_{t1}\right.$

$\frac{B_{t1}}{\mu_1} = H_{t1}$

$$\boxed{\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = k} \quad \text{... (5)}$$

From (4),

$$H_{t1} - H_{t2} = k$$

$\left[\because \vec{M} = \chi_m \vec{H}\right]$

$$\frac{M_{t1}}{\chi_{m1}} - \frac{M_{t2}}{\chi_{m2}} = k$$

or $\left(\frac{M_{t1}}{\chi_{m1}} - k\right) = \left(\frac{M_{t2}}{\chi_{m2}}\right)$

$$\boxed{M_{t2} = \frac{\chi_{m2}}{\chi_{m1}} M_{t1} - \chi_{m2} k} \quad \text{... (6)}$$

8(b).

$\mu_A = 5\,\mu H/m$, $\mu_B = 20\,\mu H/m$

$x < 0$ $\quad$ $x > 0$

$\vec{K} = 150\,\hat{a}_y - 200\,\hat{a}_z$.

$\vec{H}_A = 300\,\hat{a}_x - 400\,\hat{a}_y + 500\,\hat{a}_z$ A/m

$\therefore \vec{H}_{NA} = 300\,\hat{a}_x$ A/m

$\therefore \boxed{\text{(ii)}\ |H_{NA}| = 300 \text{ A/m}}$

(i) $\vec{H}_{tA} = \vec{H}_A - \vec{H}_{NA} = -400\,\hat{a}_y + 500\,\hat{a}_z$

$\therefore |\vec{H}_{tA}| = \sqrt{(400)^2 + (500)^2} = 640{\cdot}31$ A/m.

$\boxed{\text{(i)}\ |\vec{H}_{tA}| = 640{\cdot}31 \text{ A/m}}$

$\vec{H}_{tB} = \vec{H}_{tA} - \hat{a}_{NAB} \times \hat{k}$

$= -400\,\hat{a}_y + 500\,\hat{a}_z - \hat{a}_x \times (150\,\hat{a}_y - 200\,\hat{a}_z)$

$= -400\,\hat{a}_y + 500\,\hat{a}_z - 150\,\hat{a}_z - 200\,\hat{a}_y$

$= -600\,\hat{a}_y + 350\,\hat{a}_z$

$\therefore \boxed{|\vec{H}_{tB}| = \sqrt{(-600)^2 + (350)^2} = 694{\cdot}62 \text{ A/m}}$

$\therefore B_{tB} = \mu_B H_{tB} = (20 \times 10^{-6}) \times (-600\,\hat{a}_y + 350\,\hat{a}_z)$

$= (-12\,\hat{a}_y + 7\,\hat{a}_z)$ mT

$B_{NB} = B_{NA} = (\mu_A \cdot H_A) \cdot \hat{a}_x$

$= (5 \times 10^{-6}) \times 300\ \hat{a}_x$

$= {\cdot}0015\,\hat{a}_x$

$\therefore H_{NB} = \dfrac{B_{NB}}{\mu_B} = \dfrac{{\cdot}0015\,\hat{a}_x}{20 \times 10^{-6}} = 75$ A/m.

$\therefore \boxed{\text{(iv)}\ |H_{NB}| = 75 \text{ A/m}}$

8(c).

Solution:

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\oint_s \vec{B}\cdot\vec{ds}$$

$$e = -\frac{d}{dt}\left[\oint_s 0.5\cos(377t)(3\hat{a}_y+4\hat{a}_z)\cdot\vec{ds}\right]$$

$$= -\frac{d}{dt}\left[\oint_s 0.5\cos(377t)(3\hat{a}_y+4\hat{a}_z)\cdot ds\,\hat{a}_z\right]$$

$$= -\frac{d}{dt}\left[\oint_s 0.5\cos(377t)\cdot 4\,ds\right]$$

$$= -\frac{d}{dt}\,0.5\cos(377t)\cdot 4\cdot\pi(0.1)^2 \quad \left[\because r = 0.1\,m = 10\,cm\right]$$

$$= 0.5\sin(377t)\cdot 377\cdot 4\cdot\pi\cdot(0.1)^2$$

$$= 23.69\sin(377t)\ V$$

$\therefore$ Induced voltage $\boxed{e = 23.69\sin(377t)\ V}$

9(a).

According to Amperes's law,

$$\vec{\nabla} \times \vec{H} = \vec{J} \quad \ldots (1)$$

$$\therefore \quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) \equiv \vec{\nabla} \cdot \vec{J} = 0$$

$$[\because \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0 \text{ identically}]$$

But According to continuity of current eqn,

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad \cdot (2)$$

For time varying fields we add an unknown term G to eqn. (1).

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{G}$$

$$\text{or} \quad \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{G}$$

$$\text{or} \quad 0 = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{G}$$

$$\text{or} \quad \vec{\nabla} \cdot \vec{J} = -\vec{\nabla} \cdot \vec{G} = \frac{-\partial \rho_v}{\partial t}$$

$$\therefore \quad \vec{\nabla} \cdot \vec{G} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{D})$$

$$[\because \vec{\nabla} \cdot \vec{D} = \rho_v]$$

$$\text{or} \quad \vec{\nabla} \cdot \vec{G} = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\text{or} \quad \boxed{\vec{G} = \frac{\partial \vec{D}}{\partial t}}$$

$$\therefore \quad \boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

$$\frac{\partial \vec{D}}{\partial t} = \vec{J}_D = \text{Displacement current density}$$

$\therefore$ Point form of Amperes's law,

$$\oint \vec{H} \cdot \vec{dl} = I + \iint \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

$\longrightarrow$ Integral form of Amperes's law

9(b).

$$\vec{E} = E_m \sin(\omega t - \beta z)\, \hat{a}_y \text{ V/m}$$

$$\therefore \vec{D} = \epsilon_0 \vec{E} = 8.854 \times 10^{-12} E_m \sin(\omega t - \beta z)\, \hat{a}_y \text{ C/m}^2$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}.$$

$$\therefore \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_m \sin(\omega t - \beta z) & 0 \end{vmatrix} = -\frac{\partial \vec{B}}{\partial t}$$

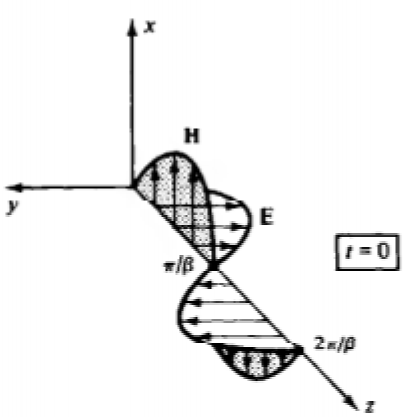
$$\text{or } -\frac{\partial \vec{B}}{\partial t} = \hat{a}_x \beta E_m \cos(\omega t - \beta z)$$

$$\text{or } \vec{B} = -\frac{\beta E_m}{\omega} \sin(\omega t - \beta z)\, \hat{a}_x \text{ T}$$

$$\vec{H} = -\frac{\beta E_m}{\omega \mu_0} \sin(\omega t - \beta z)\, \hat{a}_x \text{ A/m}$$

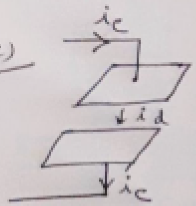
$$\text{at } t = 0, \quad E = -E_m \sin \beta z\, \hat{a}_y \text{ V/m}$$

$$H = \frac{\beta E_m}{\omega \mu_0} \sin(\beta z)\, \hat{a}_x \text{ A/m}$$



9(c).

9.(c)



$$i_c = C\frac{dv}{dt} = \frac{\epsilon A}{d}\frac{d}{dt}\left(V_0 e^{j\omega t}\right)$$

$$\text{or } \boxed{i_c = \left(\frac{\epsilon A}{d}\right)(j\omega)\, V_0 e^{j\omega t}} \quad - (1)$$

$$i_d = J_D \cdot A = \frac{\partial \vec{D}}{\partial t}\cdot A = \epsilon A \cdot \frac{d\vec{E}}{dt}$$

$$= \epsilon A \cdot \frac{d}{dt}\left(\frac{v}{d}\right) = \frac{\epsilon A}{d}\frac{d}{dt}\left(V_0 e^{j\omega t}\right)$$

$$\therefore \boxed{i_c = i_d} \qquad \boxed{i_d = \left(\frac{\epsilon A}{d}\right)(j\omega)\, V_0 e^{j\omega t}} \quad (2)$$

10(a).

Intrinsic Impedance for free space :-

(Find the relation b/w $\vec{E}$ and $\vec{H}$ in free space)

$$\nabla^2 \vec{E}_s = -k_0^2 \vec{E}_s \qquad \left[\vec{E}_s = E_{xs}\hat{a}_x + E_{ys}\hat{a}_y + E_{zs}\hat{a}_z\right]$$

considering only x-component of the above eqn.

$$\nabla^2 E_{xs} = -k_0^2 E_{xs}$$

$$\text{or } \frac{\partial^2 E_{xs}}{\partial x^2} + \frac{\partial^2 E_{xs}}{\partial y^2} + \boxed{\frac{\partial^2 E_{xs}}{\partial z^2}} = -k_0^2 E_{xs}$$

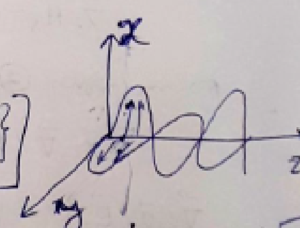
For a uniform plane wave, E_{xs} varies only with z.

$$\therefore \frac{d^2 E_{xs}}{\partial z^2} = -k_0^2 E_{xs}$$

Soln. of the eqn, $E_{xs}(z) = E_{x0} e^{-jk_0 z} + E_{x0}' e^{jk_0 z}$.

$$\therefore \frac{dE_{xs}}{dz} = -j\omega\mu_0 H_{ys} \qquad \left[\text{Note: } \vec{\nabla}\times\vec{E}_s = -j\omega\mu_0 \vec{H}_s \;\; \text{— (2)}\right]$$

$$H_{ys} = -\frac{1}{j\omega\mu_0}\frac{dE_{xs}}{dz}$$



$$\text{or } H_{ys} = -\frac{1}{j\omega\mu_0}\left[\frac{d}{dz}\left\{E_{x0}e^{-jk_0 z} + E_{x0}' e^{jk_0 z}\right\}\right]$$

$$\text{or } H_{ys} = -\frac{1}{j\omega\mu_0}\left[\left\{(-jk_0)E_{x0}e^{-jk_0 z} + (jk_0)E_{x0}' e^{jk_0 z}\right\}\right]$$

$$\frac{jk_0}{j\omega\mu_0} = \frac{\omega\sqrt{\mu_0\epsilon_0}}{\omega\mu_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}$$

$$H_{ys} = -\frac{1}{j\omega\mu_0}\left[(-jk_0)E_{x0}e^{-jk_0 z} + (jk_0)E'_{x0}e^{jk_0 z}\right]$$

$$H_{ys} = \left\{\sqrt{\frac{\epsilon_0}{\mu_0}}E_{x0}e^{-jk_0 z} - \sqrt{\frac{\epsilon_0}{\mu_0}}E'_{x0}e^{jk_0 z}\right\} \quad (1)$$

$$\boxed{\frac{k_0}{\omega\mu_0} = \frac{\omega\sqrt{\mu_0\epsilon_0}}{\omega\mu_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}}$$

Now, $H_{ys} = H_{y0}e^{-jk_0 z} + H'_{y0}e^{jk_0 z} \quad (2)$

Comparing (1) and (2),

$$H_{y0} = \sqrt{\frac{\epsilon_0}{\mu_0}}E_{x0}$$

or $$E_{x0} = \left(\sqrt{\frac{\mu_0}{\epsilon_0}}\right)H_{y0} = \eta_0 H_{y0}$$

where, $\eta_0 \rightarrow$ Intrinsic impedance of free space.

and $$\boxed{\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega = 120\pi\,\Omega}$$

10(b).

10.(b) $\gamma = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}$

or $\gamma = \sqrt{j(1884) \times 4\pi - (1884) \times 4\pi \times 8.854 \times 9 \times 10^{-7}}$

$= \sqrt{j(23{,}663.04) - 0.188}$

$= \sqrt{(559937{,}569 + .0353)^{1/2}} \cdot \frac{1}{2} \cdot \angle 89.99$

$= 153.82 \angle 45^\circ$

$= 108.76 + j108.76$

$= \alpha + j\beta.$

$\therefore \boxed{\begin{matrix} \alpha = 108.76 \\ \beta = 108.76 \end{matrix}}$

$\eta = \sqrt{\dfrac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\dfrac{j(2\pi \times 300 \times 10^6) \times 4\pi \times 10^{-7}}{10 + j(2\pi \times 300 \times 10^6) \times 9 \times 8.854 \times 10^{-12}}}$

$= \sqrt{23.66(10j + 0.15)}$

$= \sqrt{236.6j + 3.549}$

$= 15.38 \angle 44.57$

10(c).

Poynting's Theorem :-

For a conductive medium,

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Taking dot product of $\vec{E}$ on both sides,

$$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

or $$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\left(\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}\right) \quad \text{--- (1)}$$

According to vector identity,

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{E}\cdot(\vec{\nabla}\times\vec{H}) + \vec{H}\cdot(\vec{\nabla}\times\vec{E})$$

or $$\vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{H}) \quad \text{--- (2)}$$

Using (2) into eqn. (1),

$$\vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\left(\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}\right)$$

or $$\vec{H}\cdot\left(-\frac{\partial \vec{B}}{\partial t}\right) - \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\left(\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}\right)$$

$$\left[\because (\vec{\nabla}\times\vec{E}) = -\left(\frac{\partial \vec{B}}{\partial t}\right)\right]$$

Faraday's law,

$$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}$$

or $$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \mu\,\vec{H}\cdot\frac{\partial\vec{H}}{\partial t},$$

$$\mu\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)$$

and $$\epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)$$

Note

$$\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H})$$
$$= \vec{H}\cdot\frac{\partial\vec{H}}{\partial t} + \frac{\partial\vec{H}}{\partial t}\cdot\vec{H}$$
$$= 2\left(\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}\right)$$

or $$\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H})$$

$$\therefore -\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \epsilon\,\vec{E}\cdot\frac{\partial\vec{E}}{\partial t} + \mu\vec{H}\cdot\frac{\partial\vec{H}}{\partial t}.$$

i.e. $$-\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{E}\cdot\vec{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right) \quad \cdots (3)$$

Integrating over a volume,

$$-\iiint \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv = \iiint(\vec{E}\cdot\vec{J})\,dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)dv$$

or $$-\oiint(\vec{E}\times\vec{H})\cdot\vec{ds} = \iiint(\vec{E}\cdot\vec{J})\,dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{D}\cdot\vec{E}\right)dv + \iiint\frac{\partial}{\partial t}\left(\frac{1}{2}\vec{B}\cdot\vec{H}\right)dv. \quad \cdots (4)$$

The total power flowing out of the volume is,

$$\oiint (\vec{E} \times \vec{H}) \cdot \vec{ds} \quad \text{Watt}$$

The cross product, $(\vec{E} \times \vec{H}) \equiv \vec{S}$ W/m^2

Poynting's vector

In case of uniform plane wave,

$$E_x \hat{a}_x \times H_y \hat{a}_y = S_z \hat{a}_z$$

$$E_x = E_{x0} \cos(\omega t - \beta z)$$

$$H_y = \frac{E_{x0}}{\eta} \cos(\omega t - \beta z)$$

$$S_z = \frac{E_{x0}^2}{\eta} \cos^2(\omega t - \beta z)$$

The time-average power density, $\langle S_z \rangle$

For lossy dielectric, $\langle S_z \rangle = \frac{1}{2} \frac{E_{x0}^2}{\eta} e^{-2\alpha z} \cos\theta_n$.

where, $\eta = |\eta| \angle \theta_n$.