Modified

CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Find the Laplace transform of:
(i) $\left(\frac{4t+5}{e^{2t}}\right)^2$ (ii) $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$ (iii) $t\cos at$. **(10 Marks)**

b. The square wave function f(t) with period 2a defined by $f(t)=\begin{cases}1 & 0\le t<a\\ -1 & a\le t<2a\end{cases}$. Show that $\left(\frac{1}{s}\right)\tanh\left(\frac{as}{2}\right)$. **(05 Marks)**

c. Employ Laplace transform to solve $\frac{d^2y}{dt^2}-\frac{dy}{dt}=0$, $y(0)=y_1(0)=3$. **(05 Marks)**

OR

2 a. Find (i) $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\}$ (ii) $\cot^{-1}\left(\frac{s}{2}\right)$ (iii) $L^{-1}\left\{\frac{s}{(s+2)(s+3)}\right\}$ **(10 Marks)**

b. Find the inverse Laplace transform of, $\frac{1}{s(s^2+1)}$ using convolution theorem. **(05 Marks)**

c. Express $f(t)=\begin{cases}2 & \text{if } 0<t<1\\ \frac{t^2}{2} & \text{if } 1<t<\frac{\pi}{2}\\ \cos t & t>\frac{\pi}{2}\end{cases}$ in terms of unit step function and hence find its Laplace transformation. **(05 Marks)**

Module-2

3 a. Obtain the Fourier series of $f(x)=\begin{cases}2 & -2<x<0\\ x & 0<x<2\end{cases}$. **(08 Marks)**

b. Find the half range cosine series of, $f(x)=(x+1)$ in the interval $0\le x\le 1$. **(06 Marks)**

c. Express $f(x)=x^2$ as a Fourier series of period 2π in the interval $0<x<2\pi$. **(06 Marks)**

OR

4 a. Compute the first two harmonics of the Fourier Series of f(x) given the following table :

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

(08 Marks)

b. Find the half range size series of e^x in the interval $0 \le x \le 1$. **(06 Marks)**

c. Obtain the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ valid in the interval $(-\pi \quad \pi)$ **(06 Marks)**

Module-3

5 a. Find the Infinite Fourier transform of $e^{-|x|}$. **(07 Marks)**

b. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$. **(06 Marks)**

c. Solve $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$, given $u_0 = u_1 = 0$. **(07 Marks)**

OR

6 a. If $f(x) = \begin{cases} 1 & \text{for } |x| \le a \\ 0 & \text{for } |x| > a \end{cases}$, find the infinite transform of f(x) and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$.

(07 Marks)

b. Obtain the Z-transform of $\cosh n\theta$ and $\sinh n\theta$. **(06 Marks)**

c. Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$ **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4^{th} degree terms and find the value of $y(1.1)$. **(07 Marks)**

b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} + y = 2x$ at x = 1.1 given $y(1) = 3$ (Take h = 0.1) **(06 Marks)**

c. Apply Milne's predictor-corrector formulae to compute y(0.4) given $\frac{dy}{dx} = 2e^x y$, with

(07 Marks)

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

OR

8 a. Given $\frac{dy}{dx} = x + \sin y$; y(0) = 1. Compute y(0.4) with h = 0.2 using Euler's modified method. **(07 Marks)**

b. Apply Runge-Kutta fourth order method, to find y(0.1) with h = 0.1 given $\frac{dy}{dx} + y + xy^2 = 0$; y(0) = 1. **(06 Marks)**

c. Using Adams-Bashforth method, find y(4.4) given $5x\left(\frac{dy}{dx}\right) + y^2 = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

(07 Marks)

Module-5

9 a. Solve by Runge Kutta method $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ for x = 0.2 correct 4 decimal places, using initial conditions $y(0) = 1,\ y'(0) = 0,\ h = 0.2$. **(07 Marks)**

b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left[\frac{\partial f}{\partial y'}\right] = 0$. **(06 Marks)**

c. Find the extramal of the functional, $\int_{x_1}^{x_2} y^2 + (y')^2 + 2ye^x dx$. **(07 Marks)**

OR

10 a. Apply Milne's predictor corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values:

x	0	0.1	0.2	0.3
y	1	1.1103	1.2427	1.3990
y′	1	1.2103	1.4427	1.6990

(07 Marks)

b. Find the extramal for the functional, $\int_0^{\frac{\pi}{2}} \left[y^2 - y'^2 - 2y\sin x\right] dx$; y(0) = 0; $y\left(\frac{\pi}{2}\right) = 1$. **(06 Marks)**

c. Prove that geodesics of a plane surface are straight lines. **(07 Marks)**

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Corrections / suggestions

Subject: Transform Calculus, Fourier Series and Numerical Techniques

Subject code: 18MAT31 (DEC. 2019 / Jan 2020)

1 (a) (ii) $f(t) = \frac{\sin^2 2t}{t} = \frac{1}{2}\left(\frac{1-\cos 4t}{t}\right)$ —— (1M)

$L(f(t)) = \frac{1}{2}\int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4^2}\right) ds = \frac{1}{2}\log\frac{\sqrt{s^2+4^2}}{s}$ —— (3M)

(2) (c) $f(t) = 2 + \left(\frac{t^2}{2} - 2\right) u(t-1) + \left(\cos t - \frac{t^2}{2}\right) u\left(t - \frac{\pi}{2}\right)$ —— 1M

$L(f(t)) = \frac{2}{s} + e^{-s}\left[\frac{1}{s^3} + \frac{1}{s^2} - \frac{3}{2s}\right] - e^{-\frac{\pi}{2}s}\left[\frac{1}{s^2+1} + \frac{1}{s^3} + \frac{\pi}{s^2} + \frac{\pi^2}{8s}\right]$

(3) (a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ —— (1M)

$a_0 = 3$ —— (2M) $a_n = \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]$ —— (2M)

$b_n = \frac{-2}{n\pi}$ —— (2M)

$f(x) = \frac{3}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} \cos\left(\frac{n\pi x}{2}\right) - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{2}\right)$ —— (1M

(8) (b)

x	$x_0 = 0$	$x_1 = 0.1$
y	$y_0 = 1$	$y_1 = ?$

—— (1M)

$k_1 = -0.1$, $k_2 = -0.0995$, $k_3 = -0.0996$, $k_4 = -0.0981$ —— (4M)

$y(0.1) = 0.9006$ —— (1M)

(10) (b) $y'' + y = \sin x$ —— 2M

$\left.\begin{array}{l} y_c = c_1 \cos x + c_2 \sin x \\ y_p = -\frac{x\cos x}{2} \end{array}\right\}$ —— (2M)

$\left.\begin{array}{l} c_1 = 0,\ c_2 = 1 \\ y = \sin x - \frac{x\cos x}{2} \end{array}\right\}$ —— (2M)

(4) (b) it's Half range Sine series

(6)(a) $F(u) = \frac{2\sin au}{u}$.

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		Allocated
1 (a)	i) $e^{-4t}(16t^2+40t+25)$ (1M+4) $F(s)=\frac{32}{(s+4)^3}+\frac{40}{(s+4)^2}+\frac{25}{(s+4)}$ (2M)	3M
	ii) $f(t)=\frac{\sin^2 2t}{t}=\frac{1-\cos 2t}{t}$ (1M) $=\int_s^\infty \frac{1}{s}-\frac{s}{s^2+4}\,ds$ (1M) $=\left[\log s-\frac{1}{2}\log(s^2+4)\right]_{s}^{\infty}$ (1M) $f(s)=\log\left[\frac{\sqrt{s^2+4}}{s}\right]$ (1M) $(1+1+1+1)=4M$	4M
	iii) $F(s)=-\frac{d}{ds}\left[\frac{s}{s^2+a^2}\right]$ (1M) $=\frac{s^2-a^2}{(s^2+a^2)^2}$ (2M) $(1+2)=3M$	3M 3+4+3=10M
b	$F(s)=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt=\frac{1}{1-e^{-2as}}\int_0^{2a}e^{-st}f(t)\,dt$ (1M) $F(s)=\frac{1}{1-e^{-2as}}\int_0^a e^{-st}\,dt-\int_a^{2a}e^{-st}\,dt$ (1M). $F(s)=\frac{(1-e^{-as})^2}{s(1-e^{-2as})}$ (2M) $=\frac{\tanh\left(\frac{as}{2}\right)}{s}$ (1M) (1+1+2+1)	$\boxed{1+1+2+1}$ 5 Marks
c	$s^2L\{y(t)\}-sy(0)-y_1(0)-sL\{y(t)\}+y(0)=0$ (1M) $L\{y(t)\}(s^2-s)-3s-\cancel{3}+\cancel{3}=0$ (1M) $L\{y(t)\}=\frac{3s}{s^2-s}=\frac{3\cancel{s}}{\cancel{s}(s-1)}=\frac{3}{s-1}$ (2M) $y(t)=3e^{t}$ (1M)	$\boxed{1+1+2+1}$ 5 Marks
2 (a)	OR i) $f(t)=1-3t+2t^2$ (2M) ii) $F'(s)=\frac{-2}{s^2+4}$ (2M) $f(t)=\frac{\sin 2t}{t}$ (2M) (2+2) iii) $f(t)=Ae^{-2t}+Be^{-3t}$ (2M) Finding A & B (2M). $\frac{s}{(s+2)(s+3)}=\frac{A}{s+2}+\frac{B}{s+3}$: $\left.\begin{array}{l}A=-2\\B=3\end{array}\right\}$.	2+4+4 10 Marks

Number | Solution

b $L^{-1}\{F(s)G(s)\} = \int_0^t f(u)\, g(t-u)\, du$ ---(1M)

$f(t) = 1 \quad g(t) = \sin t$

$\left.\begin{array}{l} f(u) = 1 \quad g(t-u) = \sin(t-u) \end{array}\right\}$ 2m

$L^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = \int_0^t 1 \cdot \sin(t-u)\,du = \left[\cos(t-u)\right]_{u=0}^{t}$

$L^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = 1 - \cos t$ (2 m)

1+2+2 5 Marks

C $L\{f(t)\} = 2 + \left(\frac{t^2}{2} - 2\right)U(t-1) + \left(\cos t - \frac{t^2}{2}\right)U(t-\pi/2)$ --(1m)

Apply L.T

$L\{f(t)\} = \frac{2}{s} + \underbrace{\frac{e^{-s}}{2}\left[\frac{2}{s^3} + \frac{2}{s^2} - \frac{3}{s}\right]}_{2m} - e^{-\frac{\pi s}{2}}\left[\frac{1}{s^2+1} + \underbrace{\frac{1}{s^3}}_{2m} + \frac{\pi^2}{8s} + \pi/s^2\right]$ ---(1m)

(1+4) 5 Marks

$\boxed{\text{Module-2}}$

3(a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right)$ ---(1m)

$(-l, l) = (-2, 2) \quad \boxed{l=2}$

$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx = \frac{1}{2}\int_{-2}^{0} 2\,dx + \int_0^2 x\,dx = 3$ --(2m)

$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\left(\frac{n\pi}{2}x\right)dx = \frac{1}{2}\int_{-2}^{0} 2\cos\left(\frac{n\pi}{2}x\right)dx + \int_0^2 x\cos\left(\frac{n\pi}{2}x\right)dx$

$a_n = \frac{2\left[1-(-1)^n\right]}{n^2\pi^2}$ ----(2m)

$b_n = \frac{1}{2}\int_{-2}^{2} f(x)\sin\left(\frac{n\pi}{2}x\right)dx = \frac{1}{2}\int_{-2}^{0} 2\sin\left(\frac{n\pi}{2}x\right)dx + \int_0^2 x\sin\left(\frac{n\pi}{2}x\right)dx$

$b_n = -\frac{2}{n\pi}$ ----(2m)

1+2+2+2 +1 = 8m

$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)}{n^2\pi^2}\cos\left(\frac{n\pi}{2}x\right) + \sum_{n=1}^{\infty}\left(\frac{-2}{n\pi}\right)\sin\left(\frac{n\pi x}{2}\right)$ (1m)

b $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{l}x\right)$ ------(1m)

$(0, l) = (0, 1) \quad l = 1$

$$a_0 = \frac{2}{1}\int_0^1 (x+1)\,dx = \cancel{2}\left.\frac{(x+1)^2}{\cancel{2}}\right]_{x=0}^{1} = 4-1 = 3 \text{ --- (2m)}$$

$$a_n = \frac{2}{1}\int_0^1 (x+1)\cos(n\pi x)\,dx = \frac{2}{1}\left[(x+1)\left\{\frac{\sin n\pi x}{n\pi}\right\} - (1)\left\{-\frac{\cos(n\pi x)}{n^2\pi^2}\right\}\right]_{x=0}^{1}$$

$$a_n = \frac{2}{n^2\pi^2}\left(\cos n\pi - \cos 0\right) = \frac{2[(-1)^n - 1]}{n^2\pi^2} \text{ --- (2m)}$$ 1+2+2+1

$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty}\frac{2((-1)^n-1)}{n^2\pi^2}\cos n\pi x \text{ .... (1m)}$$ 6 Marks

C $f(x) = x^2$,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{\pi}\left[\frac{x^3}{3}\right]_{x=0}^{2\pi} = \frac{8\pi^2}{3} \text{ --- (1m)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2\cos nx\,dx = \frac{1}{\pi}\left[x^2\frac{\sin nx}{n} + (2x)\left(\frac{\cos nx}{n^2}\right) - \frac{2\sin nx}{n^3}\right]_{x=0}^{2\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{4\pi\cos 2n\pi}{n^2} - 0\right] = 4/n^2 \text{ --- (2m)}$$ 1+2+2+1

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\,dx = \frac{1}{\pi}\left[\frac{-x^2\cos nx}{n} + \frac{(2x)\sin nx}{n^2} + \frac{2\cos nx}{n^3}\right]_{x=0}^{2\pi}$$ 6 Marks

$$b_n = \frac{1}{\pi}\left[\frac{-4\pi^2\cos 2n\pi}{n} + \frac{2\cos 2n\pi}{n^3} - \frac{2}{n^3}\right] = \frac{-4\pi}{n} \text{ -- (2m)}$$

$$x^2 = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4}{n^2}\cos nx - \sum_{n=1}^{\infty}\frac{4\pi}{n}\sin nx \text{ ---- (1m)}$$

OR

4(a) $f(x) = \frac{a_0}{2} + [a_1\cos x + b_1 \sin x] + [a_2\cos 2x + b_2 \sin 2x]$ (1m)

x	y	$y\cos x$	$y\sin x$	$y\cos 2x$	$y\sin 2x$
0	7.9	7.9	0	7.9	0
60	7.2	3.6	6.2354	-3.6	6.2354
120	3.6	-1.8	3.1177	-1.8	-3.1177
180	0.5	-0.5	0	0.5	0
240	0.9	-0.45	-0.7794	-0.45	0.7794
300	6.8	3.4	-5.8890	-3.4	-5.8890
Total	26.9	12.15	2.6847	-0.85	-1.9918

(4m) 1+4+3 (8m)

$a_0 = 8.9667$; $a_1 = \frac{2}{N}\sum y\cos x = 4.05$; $a_2 = \frac{2}{N}\sum y\cos 2x = -0.2833$

$b_1 = \frac{2}{N}\sum y\sin x = 0.8949$; $b_2 = \frac{2}{N}\sum y\sin 2x = -0.6639$

(3m)

Q.5 (a) $F\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{isx} dx \longrightarrow$ 1M

$= \int_{-\infty}^{\infty} e^{(1+is)x} dx + \int_{-\infty}^{\infty} e^{-(1+is)} dx \longrightarrow$ 3

$= \frac{1}{1+is} + \frac{1}{1-is} \longrightarrow$ 2

$\therefore F\{f(x)\} = \frac{2}{s^2+1} \longrightarrow$ 1

$\longrightarrow$ 07 M.

b	$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}x\right)$; here $l = 1$ - - - - (1m) $b_n = \frac{2}{1}\int_0^1 f(x)\sin\left(\frac{n\pi}{1}x\right)dx = 2\int_0^1 e^x(\sin n\pi x)\,dx$ $b_n = 2\left[\frac{e^x\{(1\sin n\pi x) - n\pi\cos n\pi x\}}{1^2+n^2\pi^2}\right]_0^1 = \frac{2n\pi}{1+n^2\pi^2}\left[-e(-1)^n+1\right]$ - - (4m) $x=0$ $f(x) = \sum_{n=1}^{\infty}\frac{2n\pi}{1+n^2\pi^2}\left[1-e(-1)^n\right]\sin n\pi x$ - - (1m)	(1+5) 6m
c	$f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ is even function. So we find a_0 & a_n . . . . (1m) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ $a_0 = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi^2}{12} - \frac{x^2}{4}\right)dx = 0$ (2m) $a_n = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi^2}{12} - \frac{x^2}{4}\right)\cos nx\,dx$ $f(x) = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$ (1m) $a_n = \frac{(-1)^{n+1}}{n^2}$ (2m)	1+2+2+1 6 Marks
5(a)	$\boxed{\text{Module 3}}$ $F(u) = \cancel{\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} \cdot e^{iux}\,dx = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}+iux}\,dx = \int_{-\infty}^{\infty} e^{-\left(\frac{x^2-2iux}{2}\right)}\,dx}$ $\cancel{= \int_{-\infty}^{\infty} e^{-\frac{(x-iu)^2+u^2}{2}} = \int_{-\infty}^{\infty} e^{-u^2/2}\cdot e^{-\frac{(x-iu)^2}{2}}\,dx}$ (4m) on substitution $\frac{x-iu}{\sqrt{2}} = t$ $dx = \sqrt{2}\,dt$ $\cancel{F(u) = e^{-u^2/2}(\sqrt{2})(\sqrt{\pi})}$ (3m)	7 Marks (4+3)
b	$F_c[f(x)] = \int_0^{\infty} f(x)\cos ux\,dx = \int_0^{\infty} e^{-2x}(\cos ux)\,dx + 4\int_0^{\infty} e^{-3x}(\cos ux)\,dx$ - - (1m) $F_c[f(x)] = \left[\frac{e^{-2x}(-2\cos ux + u\sin ux)}{4+u^2}\right]_{x=0}^{\infty} + 4\left[\frac{e^{-3x}(-3\cos ux + u\sin ux)}{9+u^2}\right]_{x=0}^{\infty}$ - - (3m) $F_u = \frac{2}{4+u^2} + 4\left[\frac{3}{9+u^2}\right] = \frac{2}{4+u^2} + \frac{12}{9+u^2}$ - - (2m)	(1+3+2) 6 Marks
(c)	$z^2(\bar{u}(z) - u_0 - u_1 z^{-1}) - 3z(\bar{u}(z) - u_0) + 2\bar{u}(z) = \frac{z}{z-3}$ (2m) $u_0 = u_1 = 0$ $\bar{u}(z)[z^2 - 3z + 2] = \frac{z}{z-3}$; $\bar{u}(z) = \frac{z}{(z-1)(z-2)(z-3)}$ (3m) apply Z_T^{-1} $Z_T^{-1}\left(\frac{z}{z-1}\right) = 1^n$: $Z_T^{-1}\left\{\frac{z}{z-2}\right\} = 2^n$; $Z_T^{-1}\left\{\frac{z}{z-3}\right\} = 3^n$ (2m) $u_n = \frac{1}{2}(1)^n - 2^n + \frac{3^n}{2}$ (2m) Find A B C by Partial fra $A = \frac{1}{2}$; $B = -1$; $C = \frac{1}{2}$	2+3+2 7 Marks
6(a)	OR $F(u) = \int_{-a}^{a} f(x)e^{iux}\,dx = \int_{-a}^{a} e^{iux}\,dx = \left.\frac{e^{iux}}{iu}\right]_{x=-a}^{a}$ - - (2m) $F(u) = \frac{1}{iu}(e^{iua} - e^{-iua}) = \frac{1}{iu}(2i\sin ua) = \left(\frac{2\sin ua}{u}\right)$ - - (3m)	

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To evaluate By IFT $f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(u)\,e^{-iux}\,dx$

$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{2\sin ua}{u}e^{-iux}\,dx$; $f(0)=\frac{2}{2\pi}\int_{-\infty}^{\infty}\frac{\sin ua}{u}\,dx$

$1=\frac{2}{\pi}\int_{0}^{\infty}\frac{\sin ua}{u}\,dx$; $\int_{0}^{\infty}\frac{\sin x}{x}\,dx=\pi/2$ — (3m)

$a=1$ & $u=x$

(2+2+3) 7 Marks

6(b) $Z_T\{\cosh n\theta\}=\frac{1}{2}Z_T\{e^{n\theta}+e^{-n\theta}\}=\frac{1}{2}\left[\frac{z}{z-e^{\theta}}+\frac{z}{z-e^{-\theta}}\right]$ on simp

$Z_T\{\cosh n\theta\}=\frac{z(z-\cosh\theta)}{z^2-2z\cosh\theta+1}$ ---------- 3m

$Z_T\{\sinh n\theta\}=\frac{1}{2}Z_T\{e^{n\theta}-e^{-n\theta}\}=\frac{1}{2}\left[\frac{z}{z-e^{\theta}}-\frac{z}{z-e^{-\theta}}\right]$ Simplify

$Z_T\{\sinh n\theta\}=\frac{z\sinh\theta}{z^2-2z\cosh\theta+1}$ ------- (3m)

$\therefore$ 3+3 = 6m

(c) $\bar{u}(z)=\frac{4z^2-2z}{(z-1)(z-2)^2}$ (1m) $Z_T^{-1}\left\{\frac{z}{z-1}\right\}=1^n$; $Z_T^{-1}\left\{\frac{z}{z-2}\right\}=2^n$; $Z_T^{-1}\left\{\frac{2z}{(z-2)^2}\right\}=2^n n$ --(3m)

$Z_T^{-1}\left\{\frac{4z^2-2z}{(z-1)(z-2)^2}\right\}=A\left[\frac{z}{z-1}\right]+B\left[\frac{z}{z-2}\right]+C\left[\frac{2z}{(z-2)^2}\right]$; $A=2$, $B=-2$, $C=3$ (2m)

$u_n=2-2(2^n)+3(2^n)(n)$ -(1m)

1+3+2+1

Module-4

7(a) $y_1=e^x-y$; $y_1=e^0-2=-1$ when $x_0=0$, $y_0=2$

$y_2=e^x-y_1$; $y_2=1-(-1)=2$

$y_3=e^x-y_2=y_3=1-2=-1$

$y_4=e^x-y_3=y_4=1-(-1)=2$ 4m

4+1+1+1 7 Marks

Taylor Series: about x_0 upto 4th degree

$f(x)=y_0+\frac{x}{1!}y_1+\frac{x^2}{2!}y_2+\frac{x^3}{3!}y_3+\frac{x^4}{4!}y_4$ ...(1m)

$f(x)=2-x+x^2-\frac{x^3}{6}+\frac{x^4}{12}$ ---- (1m)

$f(1.1)=2.0101$ ------ (1m)

(b) $f(x,y)=2x-y$

x	$x_0=1$	$x_1=1.1$
y	$y_0=3$	$y_1=?$

$h=0.1$ ....(1m)

$y_1=y_0+K$; $K_1=-0.1$ $K_2=-0.0850$ $K_3=-0.0858$ (4m)

$K_4=-0.0714$ $y_1=3+\frac{1}{6}\left[K_1+2(K_2+K_3)+K_4\right]=2.9145$ (1m)

1+4+1 6 Marks

C. $f(x,y) = 2ye^{x}$

x	y	$f=2ye^x$
$x_0 = 0$	$2.4 = y_0$	$f_0 = 4.8$
$x_1 = 0.1$	$2.473 = y_1$	$f_1 = 5.466$
$x_2 = 0.2$	$3.127 = y_2$	$f_2 = 7.6435$
$x_3 = 0.3$	$4.059 = y_3$	$f_3 = 10.9581$

3M

$y_4^p = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3) = 5.7606$ ...... (2m)

$f_4^p = 2y_4^p e^{x_4} = 17.1876$

$y_4^C = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4^p\right] = \boxed{y_4^C = 5.4177}$

$y_4^{C_1} = 5.3836$
$y_4^{C_2} = 5.3836$ —— 2m

(3+2+2) 7 Marks

OR

8(a) Given $f(x,y) = x + \sin y$

x	$x_0 = 0$	$x_1 = 0.2$	$x_2 = 0.4$
y	$y_0 = 1$	$y_1 =$	$y_2 = ?$

(1M)

Stage 1: $y(0.2)$ $y_1^p = y_0 + hf(x_0, y_0) = y_0 + 0.2[x_0 + \sin y_0] = 1.1683$

$y_1^{C_1} = y_0 + \frac{h}{2}[f(x_0,y_0) + f(x_1, y_1^p)] = 1.1962$

$y_1^{C_2} = 1.1972$ $y_1^{C_3} = 1.1973$ .... (3m)

Stage 2: $y(0.4)$ $y_2^p = 1.4235$; $y_2^{C_1} = 1.4473$

$y_2^{C_2} = 1.4497$; $y_2^{C_3} = 1.4497$ .. (3m)

1+3+3 7 Marks

(b) $f(x,y) = -y - xy^2$

x	$x_0 = 0$	$x_1 = 0.1$
y	$y_0 = 1$	$y_1 = ?$

--- (1m)

$K_1 = -0.1$, $K_2 = -0.0995$, $K_3 = -0.0996$, $K_4 = -0.0981$ .. (4m)

$y(0.1) = 0.9333$ -- (1m)

(1+4+1) 6m

(c)

x	y	$f = \frac{2-y^2}{5x}$
$x_0 = 4$	$y_0 = 1$	$f_0 = 0.05$
$x_1 = 4.1$	$y_1 = 1.0049$	$f_1 = 0.0483$
$x_2 = 4.2$	$y_2 = 1.0097$	$f_2 = 0.0467$
$x_3 = 4.3$	$y_3 = 1.0142$	$f_3 = 0.0452$
$x_4 = 4.4$	$y_4 = 2$	

(3m)

$y_4^p = y_3 + \frac{h}{24}\left[55f_3 - 59f_2 + 37f_1 - 9f_0\right]$

$y_4^p = 1.0187$ ---- (2m)

$f_4^p = 0.0437$

$y_4^C = 1.0186$; $f_4^p = 0.0437$

$y_4^{C} = 1.0186$ -- (2m)

3+2+2 (7m)

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[Module - 5]

9(a)	Assume $\frac{dy}{dx} = z$; given Eqn becomes $\frac{dy}{dx} = z = f(x,y,z)$ $\frac{dz}{dx} = xz^2 - y^2 = \phi(x,y,z)$. $x_0 = 0$; $y_0 = 1$; $Z_0 = 0$ and $h = 0.2$ } 2m $k_1 = 0$ $\quad k_2 = -0.02$ $\quad k_3 = -0.01998$ $k_4 = -0.03916$ $l_1 = -0.2$ $\quad l_2 = -0.1998$ $\quad l_3 = -0.1958$ $l_4 = -0.1985$ } 4m y at $x = 0.2$; 0.9801 } 1m		2+4+1 7 Marks
b	Book work		6 Marks
c	$f = y^2 + (y')^2 + 2ye^x$: $\frac{\partial f}{\partial y} = 2y + 2e^x$ (1m) $\frac{\partial f}{\partial y'} = 2y'$ (1m); Using E.E $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$ (1m) $\frac{d^2y}{dx^2} - y = e^x$; $(D^2 - 1)y = e^x$ (1m) } 4m $y_c = C_1e^x + C_2e^{-x}$ (1m) $+ \frac{xe^x}{2}$ (1m) } 3m		(4+3) 7 Marks
	(OR)		
10a	$\frac{dy}{dx} = z$; $z' = \frac{dz}{dx} = 1 + z$ $z_0' = 2$: $z_1' = 2.2103$, $z_2' = 2.4427$, $z_3' = 2.699$ } 3m $y_4^p = y_0 + \frac{4h}{3}[2z_1 - z_2 + 2z_3] = 1.5835$ $z_4^p = z_0 + \frac{4h}{3}[2z_1' - z_2' + 2z_3'] = 1.9835$ } 2m $\rightarrow z_4' = 2.9835$ $y_4^c = y_2 + \frac{h}{3}[z_2 + 4z_3 + z_4] = 1.58344$ $z_4^c = z_2 + \frac{h}{3}[z_2' + 4z_3' + z_4'] = 1.98344$ } 2m		3+2+2 7 Marks
b	$f = y^2 - y'^2 - 2y\sin x$: $\frac{\partial f}{\partial y} = 2y - 2\sin x$; $\frac{\partial f}{\partial y'} = -2y'$ EE $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$; $y'' + y = \sin x$ (2m) $\quad C_1 = C_2 = 0$ (1m) $y = C_1\cos x + C_2\sin x - \frac{x\cos x}{2}$ (2m) : $y = -\frac{x\cos x}{2}$ (2m)	2m	2+2+2 6 Marks
(c)	$S = I = \int_{x_1}^{x_2}\sqrt{1+y'^2}\,dx$ (1m) : getting $\frac{d^2y}{dx^2} = 0$ using EE (3m) Integrating twice $y = C_1x + C_2$ (3m)		1+3+3 6 Marks

Note: Any alternative methods, marks may be awarded.

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corrections / suggestions

Subject: Transform Calculus, Fourier Series and Numerical Techniques

Subject code : 18MAT31 (Dec 2019 / Jan 2020)

1 (a) (ii) $f(t) = \frac{\sin^2 2t}{t} = \frac{1}{2}\left(\frac{1-\cos 2t}{t}\right)$ —— (1M)

$L(f(t)) = \frac{1}{2}\int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4^2}\right) ds = \frac{1}{2}\log\frac{\sqrt{s^2+4^2}}{s}$ —— (3M)

(2) (c) $f(t) = 2 + \left(\frac{t^2}{2} - 2\right)u(t-1) + \left(\cos t - \frac{t^2}{2}\right)u\left(t-\frac{\pi}{2}\right)$ —— 1M

$L(f(t)) = \frac{2}{s} + e^{-s}\left[\frac{1}{s^3} + \frac{1}{s^2} - \frac{3}{2s}\right] - e^{-\frac{\pi}{2}s}\left[\frac{1}{s^2+1} + \frac{1}{s^3} + \frac{\pi}{s^2} - \frac{\pi^2}{8s}\right]$

(3) (a) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ —— (1M)

$a_0 = 3$ —— (2M) $a_n = \frac{2}{n^2\pi^2}\left[(-1)^n - 1\right]$ —— (2M)

$b_n = \frac{-2}{n\pi}$ —— (2M)

$f(x) = \frac{3}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cos\left(\frac{n\pi x}{2}\right) - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{2}\right)$ —— (1M

(8) (b)

x	$x_0 = 0$	$x_1 = 0.1$
y	$y_0 = 1$	$y_1 = ?$

—— (1M)

$K_1 = -0.1$, $K_2 = -0.0995$, $K_3 = -0.0996$, $K_4 = -0.0981$ —— (4M)

$y(0.1) = 0.9006$ —— (1M)

(10) (b) $y'' + y = \sin x$ —— 2M

$$\left.\begin{aligned} y_c &= c_1\cos x + c_2 \sin x \\ y_p &= -\frac{x\cos x}{2}\end{aligned}\right\} \text{—— (2M)}$$

$$\left.\begin{aligned} &c_1 = 0,\ c_2 = 1 \\ &y = \sin x - \frac{x\cos x}{2}\end{aligned}\right\} \text{—— (2M)}$$

(4) (b) it's Half range Sine series