CBCS SCHEME

USN

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Engineering Mathematics – III

Time: 3 hrs. Max. Marks: 100

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 a. Find the Fourier series to represent the periodic function $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. **(08 Marks)**

b. The following table gives the variations of periodic current over a period.

t sec	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
A amp.	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Expand A as a Fourier series upto first harmonic. Obtain the amplitude of the first harmonic. **(06 Marks)**

c. Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. **(06 Marks)**

OR

2 a. Find the Fourier series of $f(x) = 2x - x^2$ in $(0, 3)$. **(08 Marks)**

b. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier series expansion of y as given in the following table: **(06 Marks)**

x:	0	1	2	3	4	5	6
y:	9	18	24	28	26	20	9

c. Obtain the half-range sine series for the function,

$$f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$$ **(06 Marks)**

Module-2

3 a. Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| \le a \\ 0, & |x| > a \end{cases}$. Hence deduce that

$$\int_0^\infty \frac{(\sin x - x\cos x)}{x^3} \cos\frac{x}{2}\, dx = \frac{3\pi}{16}.$$ **(08 Marks)**

b. Find the Z-transform of,
(i) $\cos n\theta$ and (ii) $\cosh n\theta$ **(06 Marks)**

c. Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = 0 = y_1$, using z-transforms technique. **(06 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

OR

4 a. Find the Fourier cosine transform of e^{-ax}. Hence evaluate $\int_0^\infty \frac{\cos\lambda x}{x^2+a^2}dx$ **(08 Marks)**

b. Find the Z-transform of,
(i) $(n+1)^2$ (ii) $\sin(3n+5)$ **(06 Marks)**

c. Find the inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$. **(06 Marks)**

Module-3

5 a. Find the two regression lines and hence the correlation coefficient between x and y from the data. **(08 Marks)**

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	28	36	41	49	40	50

b. Fit a second degree parabola to the following data: **(06 Marks)**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

c. Using Newton-Raphson method find the root of $x\sin x + \cos x = 0$ near $x = \pi$ corrected to 4 decimal places. **(06 Marks)**

OR

6 a. Two variables x and y have the regression lines $3x + 2y = 26$ and $6x + y = 31$. Find the mean values of x and y and the correlation coefficient between them. **(08 Marks)**

b. Fit a curve of the form, $y = ae^{bx}$ to the following data: **(06 Marks)**

x:	5	15	20	30	35	40
y:	10	14	25	40	50	62

c. Using Regula-Falsi method find the root of $xe^x = \cos x$ in the interval (0, 1) carrying out four iterations. **(06 Marks)**

Module-4

7 a. Using Newton's forward and backward interpolation formulae, find f(1) and (10) from the following table: **(08 Marks)**

x	3	4	5	6	7	8	9
f(x)	4.8	8.4	14.5	23.6	36.2	52.8	73.9

b. Given that $f(5) = 150$, $f(7) = 392$, $f(11) = 1452$, $f(13) = 2366$, $f(17) = 5202$. Using Newton's divided difference formulae find f(9). **(06 Marks)**

c. Using Simpson's $\frac{1}{3}^{rd}$ rule evaluate $\int_0^{0.6} e^{-x^2}dx$ by taking seven ordinates. **(06 Marks)**

OR

8 a. Using Newton's Backward difference interpolation formula find f(105) from, **(08 Marks)**

x	80	85	90	95	100
f(x)	5026	5674	6362	7088	7854

b. If $f(1) = -3$, $f(3) = 9$, $f(4) = 30$, $f(6) = 132$ find Lagrange's interpolation polynomial that takes the same value as f(x) at the given point. **(06 Marks)**

c. Evaluate $\int_4^{5.2} \log_e x\,dx$ by Simpson's $\frac{3}{8}^{th}$ rule with $h = 0.1$. **(06 Marks)**

Module-5

9 a. Verify Green's theorem for $\oint_C (xy + y^2)dx + x^2dy$ where C is bounded by $y = x$ and $y = x^2$. **(08 Marks)**

b. Using Gauss divergence theorem evaluate $\iint_S \vec{F} \cdot \hat{n}\, ds$,

where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallel piped $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$. **(06 Marks)**

c. With usual notations derive Euler's equation, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(06 Marks)**

OR

10 a. If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{i}$, evaluate $\oint_C \vec{F}.d\bar{r}$ along the curve C in the xy-plane, $y = x^3$ from (1, 1) to (2, 8). **(08 Marks)**

b. Find the extremals of the functional with y(0) = 0 and y(1) = 1. **(06 Marks)**

c. Show that Geodesics on a plane arc straight lines. **(06 Marks)**

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Engineering Mathematics - III

1) a) Fourier Series of $f(x) = x - x^2$ in $(-\pi, \pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\,dx$$

$$= \frac{1}{\pi}\left(\frac{x^2}{2} - \frac{x^3}{3}\right)_{-\pi}^{\pi} = -\frac{2\pi^2}{3}$$

$$\boxed{\frac{a_0}{2} = -\frac{\pi^2}{3}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x - x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x - x^2)\left(\frac{\sin nx}{n}\right) - (1 - 2x)\left(-\frac{\cos nx}{n^2}\right) + (-2)\left(-\frac{\sin nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= -\frac{4}{n^2}\cos n\pi \qquad \left(\text{since } \sin n\pi = 0,\ \cos n\pi = (-1)^n\right)$$

$$= -\frac{4}{n^2}(-1)^n = \frac{4(-1)^{n+1}}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(-\frac{\cos nx}{n}\right) - (1-2x)\left(-\frac{\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$= \frac{1}{-n\pi}\left((\pi-\pi^2)\cos n\pi - (-\pi-\pi^2)\cos n\pi\right)$$

$$= -\frac{1}{n\pi}(2\pi\cos n\pi)$$

$$= -\frac{2}{n}(-1)^n$$

$$= \frac{2}{n}(-1)^{n+1}$$

Thus $(x-x^2) = -\pi^2/3 + 4\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n^2}\cos nx$

$$+ 2\sum_{1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx.$$

1) b) Given

t sec	0	T/6	T/3	T/2	2T/3	5T/6	T
	9	18.2	24.4	27.8	27.5	22	9

$\theta = 2\pi (t/T)$ here when $\theta = 0$, $t = 0$

when $\theta = 2\pi$ $t = T$.

y	θ	$y \cos\theta$	$y \sin\theta$
9.0	0	9	0
18.2	60	9.1	15.762
24.4	120	−12.2	21.1304
27.8	180	−27.8	0
27.5	240	−13.75	−23.815
22	300	11	−19.052
		−24.65	−5.9754

$a_0 = \frac{2}{N} \Sigma y = \frac{2}{6}(128.9)$

$\Rightarrow \frac{a_0}{2} = 21.4835$

It is the direct current part of the variable current.

CMR

First harmonic

$\therefore A = a_0/2 + (a_1 \cos\theta + b_1 \sin\theta)$

$a_1 = 2/N \sum y \cos\theta = 2/6 (-24.65)$

$\boxed{a_1 = -8.217}$

$b_1 = 2/N \sum y \sin\theta = 2/6 (-5.9754)$

$\boxed{b_1 = -1.9918}$

Amplitude of first harmonic $= \sqrt{a_1^2 +}$

$= 8.455$

CMR

Cosine Series of $f(x) = (x-1)^2$ in $0 \le x \le 1$.

Solution: $l = 1$ (half Range)

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx \qquad a_n = \frac{2}{l}\int_0^l f(x)\cos n\pi x\,dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = 2\int_0^1 (x-1)^2\,dx = 2\left(\frac{(x-1)^3}{3}\right)_0^1$$

$$= \frac{2}{3}\left(0-(-1)^3\right) = 2/3$$

$$\boxed{\frac{a_0}{2} = 1/3}$$

$$a_n = 2\int_0^1 (x-1)^2 \cos n\pi x\,dx$$

$$= 2\left[(x-1)^2 \frac{\sin n\pi x}{n\pi} - 2(x-1)\frac{-\cos n\pi x}{(n\pi)^2} + 2\left(-\frac{\sin \pi x}{n^3\pi^3}\right)\right]_0^1$$

$$= \frac{4}{n^2\pi^2}\left((x-1)\cos n\pi x\right)_0^1 = 4/n^2\pi^2$$

$$\therefore \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$$

$$= \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x$$

—x—

2) a) $f(x) = 2x - x^2$ in $(0, 3)$

The period $f(x) = 3 - 0 = 3$ $\quad \therefore 2l = 3$

$l = 3/2$

$$f(x) = \frac{a_0}{2} + \sum_{1}^{\infty} a_n \cos \frac{2n\pi x}{3} + \sum_{1}^{\infty} b_n \sin \frac{2n\pi x}{3}$$

$$a_0 = 2/3 \int_0^3 f(x)\,dx = 2/3 \int_0^3 (2x - x^2)\,dx.$$

$$= 2/3 \left(\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3} \right)_0^3 = 0$$

$$\boxed{\frac{a_0}{2} = 0}$$

$$a_n = \frac{2}{3}\int_0^3 (2x-x^2)\cos\frac{2n\pi x}{3}\,dx$$

Bernoulli rule

$$a_n = \frac{2}{3}\left[(2x-x^2)\cdot\frac{\sin\frac{2n\pi x}{3}}{2n\pi/3} - (2-2x)\frac{-\cos\frac{2n\pi x}{3}}{\left(\frac{2n\pi}{3}\right)^2} + (-2)\frac{-\sin\frac{2n\pi x}{3}}{(2n\pi/3)^3}\right]_0^3$$

$$= \frac{2}{3}\cdot\frac{9}{4n^2\pi^2}\left(\left[(2-2x)\cos\frac{2n\pi x}{3}\right]_0^3\right)$$

$$a_n = -\frac{9}{n^2\pi^2}$$

$$b_n = \frac{2}{3}\int_0^3 (2x-x^2)\sin\frac{2n\pi x}{3}\,dx.$$

$$b_n = \frac{2}{3}\left[(2x-x^2)\cdot\left(\frac{-\cos\frac{2n\pi x}{3}}{\frac{2n\pi}{3}} - (2-2x)\frac{-\sin\frac{2n\pi x}{3}}{(2n\pi/3)^2} + (-2)\frac{\cos\frac{2n\pi x}{3}}{(2n\pi/3)^3}\right)\right.$$

$$= \frac{2}{3}\left[\frac{-3}{2n\pi}\left\{(2x-x^2)\cos\frac{2n\pi x}{3}\right\} - \frac{54}{8n^3\pi^3}\left\{\cos\right.\right.$$

$$b_n = \frac{3}{n\pi}$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{-9}{n^2\pi^2}\cos\frac{2n\pi x}{3} + \sum_{n=1}^{\infty}\frac{3}{n\pi}\sin 2$$

b)

x	0	1	2	3	4	5	6
y	9	18	24.	28	26	20	9.

Sln:

$2l = 6 \Rightarrow l = 3.$

$$y = f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin$$

$\theta = \frac{\pi x}{3}$, $y = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta.$

$N = 6$

x	$\theta = \pi x/3$	y	$y\cos\theta$	$y\sin\theta$
0	0	9	9	0
1	60	18	9	15.588
2	120	24	−12	20.784
3	180	28	−28	0
4	240	26	−13	−22.516
5	300	20	10	−17.32
			−25	−3.464

$$a_0 = \frac{2}{N}\Sigma y = \frac{125}{3} = 41.67 \quad \& \quad \frac{a_0}{2} = 20.835$$

$$a_1 = \frac{2}{N}\Sigma y\cos\theta = \frac{-25}{3} = -8.333$$

$$b_1 = 2/N \ \Sigma y\sin\theta = \frac{-3.464}{3} = -1.155$$

2c) Half Range $f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx.$$

$$= \frac{2}{\pi} \left\{ \int_0^{\pi/2} x \sin nx \, dx + \int_{\pi/2}^{\pi} (\pi - x) \sin nx \right.$$

$$= \frac{2}{\pi} \left\{ \left[\left(x - \frac{\cos nx}{1} - 1 \cdot \frac{(-\sin nx)}{n^2} \right]_0^{\pi/2} + \left((\pi - x) - \frac{\cos nx}{n} - (-1)\left(-\frac{\sin nx}{n^2}\right) \right)_{\pi/2}^{\pi} \right\}$$

$$= \frac{2}{\pi} \left[-\frac{\pi}{2n} \cos \frac{n\pi}{2} + \frac{1}{n^2} \sin \frac{n\pi}{2} + \frac{\pi}{2n} \cos \frac{n\pi}{2} + \frac{1}{n^2} \sin \right.$$

$$b_n = \frac{4}{\pi n^2} \sin \frac{n\pi}{2}$$

$$\therefore f(x) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{2} \sin nx$$

3) a) $f(x) = \left\{ \begin{array}{ll} a^2 - x^2 , & |x| \leq a \\ 0 , & |x| > a \end{array} \right\}$

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{iux} dx = \int_{-a}^{a} (a^2 - x^2) e^{iux} dx.$$

$$F(u) = \left[(a^2 - x^2) \frac{e^{iux}}{iu} - (-2x) \frac{e^{iux}}{(iu)^2} + (-2) \frac{e^{iux}}{(iu)^3} \right]_{-a}^{a}$$

$$= \frac{-i}{u} \left((a^2 - x^2) e^{iux} \right)_{-a}^{a} - \frac{2}{u^2} \left(x e^{iux} \right)_{-a}^{a} - \frac{2i}{u^3} (e^{iux})$$

$$= 0 - 2/u^2 \left(a e^{iua} - (-a) e^{-iua} \right) - \frac{2i}{u^3} (e^{iua} - e$$

$$= \frac{-2a}{u^2} \left(e^{iua} + e^{-iua} \right) - \frac{2i}{u^3} (e^{iua} - e^{-iua})$$

$$\left\{ \begin{array}{l} e^{iua} = \cos ua + i \sin ua \\ e^{-iua} = \cos ua - i \sin ua \end{array} \right\}$$

$$-\frac{4a\cos ua}{u^2} - (2i)\frac{2i\sin ua}{u^3}$$

$$= \frac{-4ua\cos ua + 4\sin ua}{u^3}$$

$$= \frac{4(\sin ua - au\cos ua)}{u^3}.$$

To deduce $\int_0^\infty \frac{\sin x - x\cos x}{x^3.} \cos x/2 \, dx$

Using Inverse Fourier transform

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)\, e^{-iux}\, du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} 4\frac{(\sin ua - au\cos ua)}{u^3} e^{-iux}\, du$$

Put $x = 1/2$

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin au - au\cos ua}{u^3}\right)\left(\cos u/_2 - i\sin u/_2\right) du = f(1/_2)$$

$$= 1-(1/_2)^2$$

$$2\int_0^{\infty}\frac{\sin u - u\cos u}{u^3}\cdot\cos u/_2\, du = 3/_4 \times \pi/_2$$ (Comparing Real part) & put $a=1$

$$\therefore \int_0^{\infty}\frac{\sin u - u\cos u}{u^3}\cos u/_2\, du = \frac{3\pi}{16} \quad //\text{Ans.}$$

—×—

Find ZT of $\cos n\theta$ & $\cosh n\theta$.

$$e^{in\theta} = \cos n\theta + i\sin n\theta.$$

$$Z_T(e^{in\theta}) = \frac{z}{z-e^{i\theta}} = \frac{z(z-e^{-i\theta})}{(z-e^{-i\theta})(z-e^{-i\theta})}$$

$$= \frac{z\,(z - \cos\theta - i\sin\theta)]}{z^2 - z\,(e^{i\theta} + e^{-i\theta}) + 1}$$

$$= \frac{z\,[(z - \cos\theta) + i\sin\theta]}{z^2 - 2z\cos\theta + 1}$$

$Z_T(\cos n\theta) + i\,Z_T(\sin n\theta) = R.H.S$

Compari Real part

$$Z_T(\cos n\theta) = \frac{z\,(z - \cos\theta)}{z^2 - 2z\cos\theta + 1}$$

$$\cosh n\theta = \frac{1}{2}\left(e^{n\theta} + e^{-n\theta}\right)$$

$$Z_T(\cosh n\theta) = \frac{1}{2}\left(Z_T(e^{n\theta}) + Z_T(e^{-n\theta})\right)$$

$$= \frac{1}{2}\left(\frac{z}{z - e^{\theta}} + \frac{z}{z - e^{-\theta}}\right)$$

$$= \frac{z}{2}\left(\frac{1}{z-e^{\theta}}+\frac{1}{z-e^{-\theta}}\right)$$

$$= \frac{z}{2}\left\{\frac{z-e^{-\theta}+z-e^{\theta}}{z^2-z(e^{\theta}+e^{-\theta})+1}\right\}$$

$$= \frac{z}{2}\left\{\frac{2z-2\cosh\theta}{z^2-2z\cosh\theta+1}\right\}$$

$$= \frac{z\,(z-\cosh\theta)}{z^2-2z\cosh\theta+1}$$

Solve $y_{n+2}+6y_{n+1}+9y_n = 2^n$

with $y_0 = 0 = y_1$.

$$Z_T(y_{n+2})+6Z_T(y_{n+1})+9Z_T(y_n) = Z_T(2^n)$$

$$z^2\left(\bar{y}(z)-y_0-y_1 z^{-1}\right)+6z\left(\bar{y}(z)-y_0\right)+9\bar{y}(z) = \frac{z}{z-2}$$

$$(z^2+6z+9)\,\bar{y}(z) = \frac{z}{z-2}$$

$$\bar{y}(z) = \frac{z}{(z-2)(z+3)^2}$$

Now $$\frac{z}{(z-2)(z+3)^2} = \frac{Az}{(z-2)} + \frac{Bz}{(z+3)} + \frac{Cz}{(z+3)^2}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2)$$

Put $z=2 \Rightarrow 1 = A(25) \Rightarrow A = 1/25$

Put $z=-3 \Rightarrow 1 = C(-5) \Rightarrow C = -1/5$

equating coeff of z^2 $\quad 0 = A+B \Rightarrow \boxed{B = -1/25}$

$$\therefore \bar{y}(z) = \frac{1}{25}\frac{z}{z-2} - \frac{1}{25}\frac{z}{z+3} - \frac{1}{5}\frac{z}{(z+3)^2}$$

Applying Inverse.

For the last term

Multiply Nr & Dr by -3 $\quad Z_T^{-1}\left(\frac{kz}{(z-k)^2}\right) = k^n n$

$$y(z) = \frac{1}{25} Z_T^{-1}\left(\frac{z}{z-2}\right) - \frac{1}{25} Z_T^{-1}\left(\frac{z}{z-(-3)}\right)$$

$$- \frac{1}{5\times -3} \times \frac{-3z}{(z-(-3))^2}$$

$$\bar{y}(z) = \frac{1}{25}(2^n) - \frac{1}{25}(-3)^n + \frac{1}{15}(-3)^n \cdot n \text{ is the soln.}$$

—— x ——

4) a) FCT of e^{-ax} & hence evaluate $\int_0^\infty \frac{\cos \lambda x}{x^2+a^2}\,dx$.

$$F_c(u) = \int_0^\infty f(x)\cos ux\,dx = \int_0^\infty e^{-ax}\cos ux\,dx.$$

$$= \left[\frac{e^{-ax}}{(-a)^2+u^2}\left(-a\cos ux + u\sin ux\right)\right]_{x=0}^{\infty}$$

$$F_c(u) = \frac{a}{a^2+u^2}$$

Applying Inverse F.C.T

$$f(x) = \frac{2}{\pi}\int_0^\infty \frac{a}{a^2+u^2}\cos ux\,du$$

$$e^{-ax} = \frac{2a}{\pi}\int_0^\infty \frac{\cos ux}{a^2+u^2}\,du$$

change x to λ & u to x $\int_0^\infty \frac{\cos\lambda x}{a^2+x^2}\,dx = \frac{\pi}{2a}e^{-a\lambda}$

CMR

b) Find Z-T of (1) $(n+1)^2$ (2) $\sin(3n+5)$

$$Z_T(n+1)^2 = Z_T(n^2+2n+1)$$

$$= Z_T(n^2) + 2Z_T(n) + Z_T(1)$$

$$= \frac{z^2+z}{(z-1)^3} + 2\frac{z}{(z-1)^2} + \frac{z}{z-1}$$

$$Z_T(\sin 3n) = \frac{z\sin 3}{z^2-2z\cos 3+1}$$

Using $Z_T(\sin n\theta) = \frac{z\sin\theta}{z^2-2z\cos\theta+1}$

here $\theta = 3$

$$\left[\ Z_T(e^{in\theta}) = \frac{z}{z-e^{i\theta}} = \frac{z}{z-e^{i\theta}}\frac{(z-e^{-i\theta})}{(z-e^{-i\theta})}\right.$$

$$Z_T(\cos n\theta + i\sin n\theta) = \frac{z((z-\cos\theta)+i\sin\theta)}{z^2-2z\cos\theta+1}$$

Comparing Real part $Z_T(\sin n\theta)$

A c) Find Inverse transform of $\frac{2z^2+3z}{(z+2)(z-4)} = \bar{U}(z)$

$$\bar{U}(z) = \frac{z(2z+3)}{(z+2)(z-4)}$$

$$\frac{\bar{U}(z)}{z} = \frac{2z+3}{(z+2)(z-4)}$$

$$\frac{(2z+3)}{(z+2)(z-4)} = \frac{A}{(z+2)} + \frac{B}{(z-4)}$$

$A = 1/6$ put $z = -2$

& $B = 11/6$ put $z = 4$

$$\frac{\bar{U}(z)}{z} = \frac{1}{6} \cdot \frac{1}{z+2} + \frac{11}{6} \cdot \frac{z}{z-4}$$

$$Z_T^{-1}\left(\bar{U}(z)\right) = \frac{1}{6} Z_T^{-1}\left(\frac{z}{z+2}\right) + \frac{11}{6} \cdot Z_T^{-1}\left(\frac{z}{z-4}\right)$$

$$= \frac{1}{6} \cdot \left\{ (-2)^n + 11(4^n) \right\}$$

Module-3

5) a)

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	28	36	41	49	40	50

x	y	z = x−y	x^2	y^2	z^2
1	10	−9	1	100	81
2	12	−10	4	144	100
3	16	−13	9	256	169
4	28	−24	16	784	576
5	25	−20	25	625	400
6	36	−30	36	1296	900
7	41	−34	49	1681	1156
8	49	−41	64	2401	1681
9	40	−31	81	1600	961
10	50	−40	100	2500	1600
$\Sigma x = 55$	= 307	−252	385	11387	7624

$\Sigma z =$

$$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = 8.25 \Rightarrow \boxed{\sigma_x = 2.87}$$

CMR

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = 196.21$$

$$\boxed{\sigma_y = 14.01}$$

$$\sigma_z^2 = \sigma_{x-y}^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = 127.36$$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x \sigma_y} = 0.96$$

$$y - \bar{y} = r\frac{\sigma_y}{\sigma_x}(x - \bar{x}) \quad ; \quad (x - \bar{x}) = r\frac{\sigma_x}{\sigma_y}(y - \bar{y})$$

$$\underline{y = 4.687x + 4.927} \quad ; \quad \underline{x = 0.197y - 0.548}$$

b)

x:	5	15	20	30	35	40
y:	10	14	25	40	50	62

Find $y = ae^{bx}$

CMR

Fit a parabola for the data

Let $y = a + bx + cx^2$

$$\Sigma y = na + b\Sigma x + c\Sigma x^2$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$$

x	y	xy	x^2y	x^2	x^3	x^4
0	1	0	0	0	0	0
1	1.8	1.8	1.8	1	1	1
2	1.3	2.6	5.2	4	8	16
3	2.5	7.5	22.5	9	27	81
4	6.3	9.2	36.8	16	64	256
10	8.9	21.1	66.3	30	100	354

$5a + 10b + 30c = 8.9$

$10a + 30b + 100c = 21.1$

$30a + 100b + 354c = 66.3$

$a = 1.0771$ $b = 0.4157$ $c = -0.0214$

CMR

5 c) $x\sin x + \cos x = 0$ Near $x = \pi$

$f(x) = x\sin x + \cos x$

$f'(x) = x\cos x + \sin x - \sin x = x\cos x$

Also $x_0 = \pi$; $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$$x_1 = \pi - \frac{f(\pi)}{f'(\pi)} = \pi - \frac{1}{\pi} = 2.8233$$

$$x_2 = 2.8233 - \frac{(2.8233\sin(2.8233) + \cos(2.8233))}{2.8233\cos(2.8233)}$$

$x_2 = 2.7986$

$$x_3 = 2.7986 - \frac{(2.7986\sin(2.7986) + \cos(2.7986))}{2.7986\cos(2.7986)}$$

$x_3 = 2.7984$

Hence Real Root is 2.7984.

6 a) $3x+2y=26$

$6x+y=31$

To find $\bar{x}$ & $\bar{y}$

$$\begin{array}{r} 3\bar{x}+\cancel{2\bar{y}}=26 \\ -\ 12\bar{x}+\cancel{2\bar{y}}=62 \\ \hline -9\bar{x}=-36 \end{array}$$

$\boxed{\bar{x}=+4}$

$\bar{y}=31-6\bar{x}=\cancel{58}\ 31-24=7$

$\boxed{\bar{y}=7}$

$r=\sqrt{(\text{coeff of } x)(\text{coeff of } y)}$

$3x+2y=26.$

$6x+y=31 \Rightarrow x=\dfrac{31-y}{6}$ coeff of $\bar{y}=-0.166$

$3x+2y=26 \Rightarrow 2y=26-3x$

coeff of $\bar{x}=-1.5$

$r=\pm\sqrt{(-0.166)(-1.5)}=\pm 0.249$

b) fit $y = ae^{bx}$

x	5	15	20	30	35	40
y	10	14	25	40	50	62

$$\log_e y = \log_e a + bx$$

$$Y = Ax + B$$

$A = b$, $B = \log_e a$, $Y = \log_e y$

$\therefore$ The normal equations are

$$\Sigma xY = A\Sigma x^2 + B\Sigma x$$

$$\Sigma Y = A\Sigma x + Bn$$

x	y	$Y = \log_e y$	x_i^2	$x_i y_i$
5	10	2.3026	25	11.513
15	14	2.6391	225	39.5865
20	25	3.2189	400	64.378
30	40	3.6889	900	110.667
35	50	3.9120	1225	136.92
40	62	4.1271	1600	165.084
		19.8886	4375	528.1485

CMR

c) $xe^x = \cos x$ in $(0,1)$

$f(x) = xe^x - \cos x.$

$f(0.5) = -0.0532 < 0$

$f(0.6) = 0.2679 > 0$

$\therefore$ Root lies in $(0.5, 0.6)$

I iteration :

Let $a = 0.5$; $b = 0.6$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)} = 0.5166$$

$\therefore f(x_1) = f(0.5166) = -0.0035 < 0$

$\therefore$ Root lies btw $(0.5166, 0.6)$

$a = 0.5166$, $f(a) = -0.0035$

$b = 0.6$ $f(b) = 0.2679$

$\therefore$

$x_2 = 0.5177$

$\therefore f(0.5177) = -0.00017 < 0$

$\therefore$ Root lies in $(0.5177, 0.6)$

$f(a) = -0.00017$ $\quad$ $f(b) = 0.2679$

$\therefore x_3 = 0.5178$

is the required root.

— x —

Module-4.

7 a)

x					
3	4.8				
		3.6			
4	8.4		2.5		
		6.1		0.5	
5	14.5		3.0		0
		9.1		0.5	
6	23.6		3.5		0
		12.6		0.5	
7	36.2		4.0		0
		16.6		0.5	
8	52.8		4.5		
		21.1			
9	73.9				

To find $y(1)$

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0$$

$r = \frac{x - x_0}{h}$; $r = \frac{1-3}{1} = -2.$

$$y(1) = 4.8 + (-2)(3.6) + \frac{(-2)(-3)}{2}(2.5) + \frac{(-2)(-3)(-4)}{6}(0.5)$$

$y(1) = 3.1$

Now To find $y(10)$ $\quad r = \frac{x - x_n}{h} = \frac{10-9}{1} = 1$

$$y_r = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n$$

$$y(10) = 73.9 + 1(21.1) + \frac{(1)(2)}{2!}(4.5) + \frac{(1)(2)(3)}{3!}(0.5)$$

$y(10) = 100.$

7b) Given

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

To find $f(9)$

5	150			
		$\frac{392-150}{7-5}=121$		
7	392		$\frac{265-121}{11-5}=24$	
		$\frac{1452-392}{11-7}=265$		$\frac{32-24}{13-5}=$
11	1452		$\frac{457-265}{13-7}=32$	
		$\frac{2366-1452}{13-11}=457$		$\frac{42-32}{17-7}=$
13	2366		$\frac{709-457}{17-11}=42$	
		$\frac{5202-2366}{17-13}=709$		
17	5202			

$$y=f(x)=y_0+(x-x_0)f(x_0,x_1)+(x-x_0)(x-x_1)\,f(x_0,x_1,x_2)$$

$y_0=150$, $f(x_0,x_1)=121$, $f(x_0,x_1,x_2)=24$

$f(x_0,x_1,x_2,x_3)=1$

$$y=f(9)=150+(9-5)(121)+(9-5)(9-7)24+(9-5)(9-7)(9-11)$$

$$\boxed{f(9)=810 \; //}$$

CMR

7) c) $\int_0^{0.6} e^{-x^2} dx$, $h = \frac{0.6 - 0}{6} = 0.1$; $n = 6$.

Simpsons 1/3 rule

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y = e^{-x^2}$	1	0.99..	0.9608	0.9139	0.8521	0.77[illegible]	0.6977

$$\int_a^b y\,dx = \frac{h}{3}\left((y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right)$$

$$= \frac{0.1}{3}\left[(1 + 0.6977) + 4(0.99 + 0.9139 + 0.7788) + 2(0.9608 + 0.8521)\right]$$

$$= 0.5351$$

Thus $\int_0^{0.6} e^{-x^2} dx = 0.5351$.

8) a) Newton backward difference find f(105)

x	y				
80	5026				
		648			
85	5674		40		
		688		-2	
90	6362		38		4
		726		2	
95	7088		40		
		766			
100	7854				

$$y_r = y_n + r\nabla y_n + \frac{r(r+1)}{2!}\nabla^2 y_n + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_n + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_n + \cdots$$

$r = \frac{x - x_n}{h}$; $r = \frac{105-100}{5} = 1$

$\nabla y_n = 766$ $\quad \nabla^2 y_n = 40$ $\quad \nabla^3 y_n = 2$ $\quad \nabla^4 y_n = 4$

$$f(105) = 7854 + 1(766) + \frac{1\times 2}{2}\times 40 + \frac{1\times 2\times 3}{6}(2) + \frac{1\times 2\times 3\times 4}{24}(4)$$

$$\boxed{f(105) = 8666.}$$

$f(1)=-3$; $f(3)=9$ $f(4)=30$ $f(6)=132$

Lagrange Interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)y_0}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$+\frac{(x-x_0)(x-x_2)(x-x_3)y_1}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)y_2}{(x_2-x_0(x_2-x_1)(x_2-x_3)}.$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)y_3}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= \frac{(x-3)(x-4)(x-6)\,3}{(-2)(-3)(-5)} + \frac{(x-1)(x-4)(x-6)9}{2(-1)(-3)}$$

$$+\frac{(x-1)(x-3)(x-6)\,30}{3\times 1\times(-2)} + \frac{(x-1)(x-3)(x-4)132}{5\times 3\times 2}$$

$$f(x) = \frac{1}{10}\Big(-(x^3 - 13x^2 + 54x - 72) + 15(x^3 - 11x^2 + 34x - 2$$

$$-50(x^3 - 10x^2 + 27x - 18)$$

$$+44(x^3 - 8x^2 + 19x - 12)$$

$$= \frac{1}{5}(4x^3 - 2x^2 - 29x + 42)$$

c) $\int_4^{5.2} \log_e x \, dx$ simpsons $3/8$ rule with $h = 0.1$

Formula $\int_a^b y\,dx = \frac{3h}{8}\Big((y_0 + y_n) + 3(y_1 + y_2 + y_4 + \cdots$

$$+ 2(y_3 + y_6 + \cdots) \Big]$$

x	4	4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9
$\log_e x$	1.38629	1.41099	1.43508	1.4586	1.4816	1.5041	1.52606	1.5476	1.56862	1.58

	5	5.1	5.2
$\log_e x$	1.60944	1.6292	1.64866

$$\int_{4}^{5.2} \log_e x \, dx = \frac{3h}{8}\Big((y_0+y_{12}) + 3(y_1+y_2+y_4+y_5+y_7+y_8+y_{10}+y_{11})$$

$$+ 2(y_3+y_6+y_9)\Big)$$

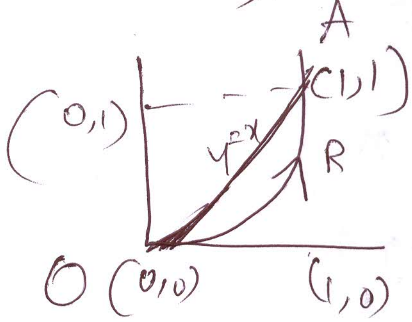
$$= 1.8279.$$

— x —

9) a) $\int_c (xy+y^2)\,dx + x^2\,dy$ bounded by $y=x$ & $y=x^2$

Point of intersection are $(0,0)$ & $(1,1)$

Sine $x=x^2$ (or) $x(1-x)=0 \Rightarrow x=0,1$



TP $\int_c M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$

$$\int_C (xy+y^2)\,dx + x^2 dy = \int_{OA} + \int_{AO} = I_1 + I_2$$

$$I_1 = \int_{x=0}^{1} (x \cdot x^2 + x^4)\,dx + x^2 \cdot 2x \; dx$$

$$= \int_0^1 (3x^3 + x^4)\,dx = 3\left(x^4/4\right)_0^1 + \left(x^5/5\right)_0^1 = \frac{19}{20}$$

Along AO $\quad y = x \quad \Rightarrow \quad dy = dx \quad$ & x varies from 1 to 0

$$I_2 = \int_{x=1}^{0} (x \cdot x + x^2)\,dx + x^2 dx = \int_{x=1}^{0} 3x^2\,dx = \left(x^3\right)_1^0 = -1$$

$$LHS = I_1 + I_2 = \frac{19}{20} - 1 = -\frac{1}{20}$$

RHS $\quad M = xy + y^2 \quad ; \quad N = x^2$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = 2x - (x + 2y) = x - 2y$$

R is the Region bounded by $y = x^2$ & $y = x$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy = \int_{x=0}^{1} \int_{y=x^2}^{x} (x - 2y)\,dy\,dx.$$

$$\int_{x=0}^{1} (x^4 - x^3)\,dx = \left(\frac{x^5}{5} - \frac{x^4}{4}\right)_0^1 = -\frac{1}{20}$$

— x —

9) b) $\iint_S \vec{F}\cdot\hat{n}\,ds = ?$

$\vec{F} = (x^2 - yz)\,i + (y^2 - zx)\,j + (z^2 - xy)\,\hat{k}$

over $0 \le x \le a$, $0 \le y \le b$ & $0 \le z \le c$.

Soln :

$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = \frac{\partial}{\partial x}(x^2 - yz) + \frac{\partial}{\partial y}(y^2 - zx) + \frac{\partial}{\partial z}(z^2 - xy)$$

$$= 2(x + y + z)$$

$$\text{LHS} = \iiint_V \operatorname{div}\vec{F}\,dv$$

$$= \int_{x=0}^{a}\int_{y=0}^{b}\int_{z=0}^{c} 2(x+y+z)\,dz\,dy\,dx$$

$$= \int_{x=0}^{a}\int_{y=0}^{b}\left(2xz + 2yz + z^2\right)_{z=0}^{c} dy\,dx$$

CMR

$$\int_{x=0}^{a}\int_{y=0}^{b}\left(2cx+2cy+c^2\right) dy\,dx$$

$$= \int_{x=0}^{a}\left(2cxy+cy^2+c^2y\right)_{y=0}^{b} dx.$$

$$= \int_{x=0}^{a}\left(2bcx+cb^2+c^2b\right) dx$$

$$= \left(bcx^2+b^2cx+bc^2x\right)_{x=0}^{a}$$

$$= a^2bc+ab^2c+abc^2$$

$$\text{LHS.} = abc\,(a+b+c)$$

9) c) Derive $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

Let I be an extremum along $y = y(x)$

Passing through $p(x_1, y_1)$ and $Q(x_2, y_2)$

Let $Y = y(x) + h\alpha(x)$ be the neighbouring curve

$\alpha(x_1) = 0$ at P and $\alpha(x_2) = 0$ at Q

when $h=0$ these curves coincide thus Making

I an extremum.

(ii) $I = \int_{x_1}^{x} f(x, y(x)+h\alpha(x), y'(x)+h\alpha'(x))\, dx$

is an extremum when $h=0$

This requires $\frac{dI}{dh} = 0$ when $h=0$ treating

I to be a function of h.

Using Leibnitz rule

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial}{\partial h} f(x, y(x)+h\alpha(x), y'(x)+h\alpha'(x))$$

Using chain rule

$$= \int_{x_1}^{x_2} \left\{ \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial h} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial h} + \frac{\partial f}{\partial y'} \cdot \frac{\partial y'}{\partial h} \right.$$

$\frac{\partial x}{\partial h} = 0$, h is independent of x & $\frac{\partial y}{\partial h} = \alpha(x)$

& $\frac{\partial y'}{\partial h} = \alpha'(x)$

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \left(\frac{\partial f}{\partial y} \alpha(x) + \frac{\partial f}{\partial y'} \alpha'(x) \right) dx.$$

$$\frac{dI}{dh} = \int_{x_1}^{x_2} \frac{\partial f}{\partial y} \alpha(x)\, dx + \left(\left(\frac{\partial f}{\partial y'} \alpha(x) \right)_{x_1}^{x_2} - \int_{x_1}^{x_2} \alpha(x) \frac{d}{dx}\left(\frac{\partial f}{\partial y'} \right.\right.$$

$$\int_{x_1}^{x_2} \frac{\partial f}{\partial y} \alpha(x)\, dx + \left[\frac{\partial f}{\partial y'} \alpha(x_2) - \frac{\partial f}{\partial y'} \alpha(x_1)\right] - \int_{x_1}^{x_2} \alpha(x) \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) dx$$

$\alpha(x_1) = 0$ & $\alpha(x_2) = 0$

$$\frac{dI}{dx} = \int_{x_1}^{x_2} \left(\left(\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)\right)\right) \alpha(x)\, dx$$

$\frac{dI}{dh} = 0$ hence integrand on RHS is zero

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \quad \text{proved.}$$

——— x ———

10) a) $\vec{F} = (5xy - 6x^2)\,\hat{i} + (2y - 4x)\,\hat{j}$

$\int_C \vec{F} \cdot d\vec{r} = ?$ along $y = x^3$ from $(1,1)$ to $(2,8)$

$$\frac{d\vec{r}}{dt} = \hat{i} + 3t^2\hat{j} + 0\hat{k}$$

$$\vec{f} = (5t^4 - 6t^2)\,\hat{i} + (2t^3 - 4t)\,\hat{j}$$

$$\vec{f} \cdot \frac{d\vec{r}}{dt} = \left[(5t^4 - 6t^2)\,\hat{i} + (2t^3 - 4t)\,\hat{j} + 0k\right] \cdot \left[\hat{i} + 3t^2\hat{j} + 0k\right]$$

$= 6t^5 + 5t^4 - 12t^3 - 6t^2$

$$\int_c \vec{f} \cdot d\vec{r} = \int_1^2 \left(\vec{f} \cdot \frac{d\vec{r}}{dt}\right) dt$$

$$= \int_1^2 (6t^5 + 5t^4 - 12t^3 - 6t^2)\, dt$$

$$= \left(6\frac{t^6}{6} + 5\frac{t^5}{5} - 12\frac{t^4}{4} - 6\frac{t^3}{3}\right)_1^2$$

$$= \left(t^6 + t^5 - 3t^4 - 2t^3\right)_1^2$$

$$\boxed{\int_c \vec{f} \cdot d\vec{r} = 35}$$

— x —

10) b) Find the extremals of the functional with $y(0)=0$ & $y(1)=1$.

Solution : In correct question, Since functional is not given.

10 c) Arc length btw P & Q is

$$S = \int_{x_1}^{x_2} \frac{ds}{dx}\, dx = \int_{x_1}^{x_2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$S = I = \int_{x_1}^{x_2} \sqrt{1+y'^2}\, dx.$$

To find the curve $y(x) \ni I$ is minimum

$$f(x, y, y') = \sqrt{1+y'^2}\, dx$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$$

$$\frac{d}{dx}\left(\frac{y'}{\sqrt{1+y'^2}}\right)=0$$

$$y''\sqrt{1+y'^2}-y'\frac{2y'y''}{2\sqrt{1+y'^2}}=0$$

$$y''(1+y'^2)-y''y'^2=0 \quad (\text{or}) \quad y''=0$$

$\frac{d^2y}{dx^2}=0$ Integrate twice w.r.to x

$y=c_1x+c_2$ which is a straight line