CBCS SCHEME

18CS36

USN 1CR19CS108

Third Semester B.E. Degree Examination, Jan./Feb. 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Verify that, for any three propositions p, q, r the compound proposition $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology or not. **(06 Marks)**

b. Test for validity of following argument:
If Ravi goes out with friends, he will not study
If Ravi do not study, his father becomes angry
His father is not angry
___
$\therefore$ Ravi has not gone out with friends **(07 Marks)**

c. Give direct and indirect proof of following statement "Product of two odd integers is an odd integer". **(07 Marks)**

OR

2 a. For any three propositions p, q, r, prove that $[\sim p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$ **(06 Marks)**

b. Check for validity of following argument,
If a triangle has two equal sides then it is isosceles. If a triangle is isoceles than it has two equal angles.
A certain triangle ABC does not have two equal angles
___
$\therefore$ The triangle ABC does not have two usual sides. **(07 Marks)**

c. Consider the following open statement on set of all real numbers as universe:
$p(x) : x \geq 0 \quad q(x) : x^2 \geq 0 \quad r(x) : x^2 - 3x - 4 = 0 \quad s(x) : x^2 - 3 > 0$
Then find truth value of i) $\exists x\ p(x) \wedge q(x)$ ii) $\forall x,\ p(x) \rightarrow q(x)$ iii) $\forall x,\ q(x) \rightarrow s(x)$ iv) $\forall x,\ r(x) \vee s(x)$ **(07 Marks)**

Module-2

3 a. By mathematical induction prove that
$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{3} n (2n-1)(2n+1)$$ **(06 Marks)**

b. Find coefficient of i) x^0 in the expansion of $\left(3x^2 - \frac{2}{x}\right)^{15}$
ii) $x^{11} y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ **(07 Marks)**

c. A total amount of Rs.1500 is to be distributed to three students A, B, C. In how many ways distribution can be done in the multiples of Rs.100 if
i) Every students sets at least Rs.300
ii) A must get at least Rs.500, B and C must set at least Rs.400 each. **(07 Marks)**

OR

4 a. By mathematical induction prove that for any positive integer n the number $11^{n+2} + 12^{2n+1}$ is divisible by 133 **(06 Marks)**

b. How many positive integers n can be formed from the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000. **(07 Marks)**

c. A certain question paper has 3 parts A, B, C with four questions in Part A, Five In B and Six in C. It is required to answer seven questions by selecting at least two from each part. In how many different ways student can answer seven questions. **(07 Marks)**

Module-3

5 a. Let A = {1, 2, 3, 4, 5, 6}, B = {6, 7, 8, 9, 10} and f be a function from A to B defined by f = {(1, 7) (2, 7), (3, 8) (4, 6) (5, 9) (6, 9)}. Then find $f^{-1}(6)$, $f^{-1}(9)$. If B_1 = {7, 8}, B_2 = {8, 9, 10) find $f^{-1}(B_1)$, $f^{-1}(B_2)$. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and R be a relation on A defined by xRy if and only if x divides y. Then i) Write R as ordered pairs ii) Draw diagram iii) Write matrix of R. **(07 Marks)**

c. If f, g, h are functions from R to R defined by $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2 + 2}$. Then verify that f o(goh) = (fog) oh **(07 Marks)**

OR

6 a. If 30 dictionaries in a library contain total 61,327 pages then prove that at least one of the dictionaries must have at least 2045 pages. **(06 Marks)**

b. For any three nonempty sets A, B, C prove that

i) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

ii) $AX\,(B \cap C) = (A \times B) \cap (A \times C)$ **(07 Marks)**

c. Let A = {1, 2, 3, 4, 6, 8, 12} define a partial order R on A by xRy if and only if x divides y. Draw Hasse diagram of R. **(07 Marks)**

Module-4

7 a. For the integers 1, 2, …n, there are 11660 derangements where 1, 2, 3, 4, 5 appear in first five positions then find value of n. **(06 Marks)**

b. Determine number of integers between 1 and 300 which are i) divisible by exactly two of 5, 6, 8 ii) at least two of 5, 6, 8. **(07 Marks)**

c. Solve $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$ given $a_0 = 1$, $a_1 = 2$ **(07 Marks)**

OR

8 a. Out of 30 students of a hostel 15 study history, 8 study economics, 6 study geography and 3 study all the three subjects. Show that 7 or more study none of the subjects. **(06 Marks)**

b. An apple, a banana, a mango, and an orange to be distributed to 4 boys B_1, B_2, B_3 and B_4. The boys B_1 and B_2 do not wish apple, B_3 does not want banana or mango B_1 refuses orange. In how many ways distribution can be made so that all of them are happy. **(07 Marks)**

c. Solve $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given $a_0 = 2$. **(07 Marks)**

Module-5

9 a. Show that following graphs in the Fig.Q.9(a)(i) and Fig.Q.9(a)(ii) are isomorphic

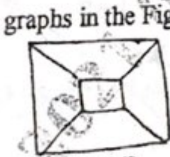

Fig.Q.9(a)(i)

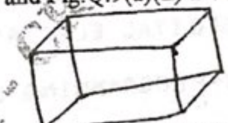

Fig.Q.9(a)(ii)

(06 Marks)

b. Define with an example to each i) Complement of a graph ii) Vertex degree iii) Rooted tree iv) Prefix code **(07 Marks)**

c. Apply merge sort to the list -1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3 **(07 Marks)**

OR

10 a. Prove that a tree with n vertices has (n – 1) edges. **(06 Marks)**

b. Determine number of vertices in following graph G:

i) G has 9 edges and all vertices have degree 3

ii) G has 10 edges with 2 vertices of degree 4 and all other have degree 3 **(07 Marks)**

c. Obtain optimal prefix code for the message ROAD IS GOOD. **(07 Marks)**

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Third Sem B.E. Degree Exam. Jan./Feb. 2021 18CS36

Discrete Mathematical Structures

Module 1

[1.a] Let $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ — ($*$)

p	q	r	$p \to q$	$q \to r$	$p \to r$	$p \to (q \to r)$	$(p \to q) \to (p \to r)$	$*$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

From the truth table, we can see that the given proposition ($*$) is always true. ∴ it is a tautology.

[b] Let p: Ravi goes out with friends

q: Ravi will study

r: Ravi's father becomes angry.

Given argument is:

$$\begin{array}{l} p \to \neg q \\ \neg q \to r \\ \neg r \\ \hline \therefore \neg p \end{array} \quad \Rightarrow \quad \begin{array}{l} p \to r \\ \neg r \\ \hline \therefore \neg p \end{array} \quad \text{Rule of syllogism.}$$

This is a valid argument in view of Modus Tollens.

[c] Given statement is

"If x is odd and y is odd then xy is odd"

Let p: x is odd

q : y is odd

r : xy is odd

Given statement in symbolic form : $(p \wedge q) \rightarrow r$

Direct proof: Let $p \wedge q$ be true.

$\Rightarrow$ p is true and q is true.

$\Rightarrow$ x is odd and y is odd

$\Rightarrow x = 2k+1$ & $y = 2l+1$ $\quad k, l \in \mathbb{Z}$

$$\Rightarrow xy = (2k+1)(2l+1) = 4kl + 2k + 2l + 1$$
$$= 2(2kl + k + l) + 1$$
$$= 2m + 1 \quad \text{where } 2kl + k + l = m \in \mathbb{Z}$$

$\Rightarrow$ xy is odd

In-direct proof: wkt, $(p \wedge q) \rightarrow r \Leftrightarrow \neg r \rightarrow \neg(p \wedge q)$

Let $\neg r$ be true. $\Rightarrow$ xy is not odd.

$\Rightarrow$ xy is even.

	or	or
$\Rightarrow$ x is even and y is odd is true and	x is odd & y is even	x is even and y is even.
$\Rightarrow \neg p \wedge q$ is true	p is true and $\neg q$ is true	$\neg p$ is true and $\neg q$ is true.
$\Rightarrow \neg p \vee \neg q$ is true	$\Rightarrow \neg p \vee \neg q$ is true.	$\Rightarrow \neg p \vee \neg q$ is true.

$\Rightarrow \neg(p \wedge q)$ is true.

$\therefore \neg r \rightarrow \neg(p \wedge q)$ is true.

So, by $(p \wedge q) \rightarrow r$ is true.

$\boxed{2a}$ Consider

$$LHS = [\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$$

First consider $\neg p \wedge (\neg q \wedge r) \Leftrightarrow (\neg p \wedge \neg q) \wedge r$ associative law

$\Leftrightarrow [\neg(p \vee q)] \wedge r \Leftrightarrow r \wedge [\neg(p \vee q)]$ De-Morgan's law & commutative law.

and $(q \wedge r) \vee (p \wedge r) \Leftrightarrow (r \wedge q) \vee (r \wedge p)$ Commutative law

$\Leftrightarrow r \wedge (q \vee p)$ Distributive law

$\Leftrightarrow r \wedge (p \vee q)$ Commu law

$\therefore [\neg p \wedge (\neg q \wedge r) \vee (q \wedge r) \vee (p \wedge r)]$

$\Leftrightarrow \{r \wedge [\neg(p \vee q)]\} \vee \{r \wedge (p \vee q)\}$ ~~De-morg~~

$\Leftrightarrow r \wedge \{[\neg(p \vee q)] \vee (p \vee q)\}$ Distributive law

$\Leftrightarrow r \wedge T_0$ Inverse law

$\Leftrightarrow r$

$\boxed{2b}$ Let ~~a is a tri.~~

$p(x)$: x has two equal sides.

$q(x)$: x is isosceles.

$r(x)$: x has two equal angles.

a : triangle ABC.

Given

$$\begin{array}{l} \forall x,\ p(x) \rightarrow q(x) \\ \forall x,\ q(x) \rightarrow r(x) \\ \neg r(a) \\ \hline \therefore \neg p(a) \end{array} \Rightarrow \begin{array}{l} p(a) \rightarrow q(a) \\ q(a) \rightarrow r(a) \\ \neg r(a) \\ \hline \therefore \neg p(a) \end{array}$$

Universal specification

$$\Rightarrow \begin{array}{l} p(a) \rightarrow r(a) \text{ (syllogism)} \\ \neg r(a) \\ \hline \therefore \neg p(a) \end{array}$$

This is a valid argument in view of Modus Tollens.

2c.

(i) $\exists x, p(x) \wedge q(x)$

We note that, there exists a real no $x = 1$ for which both $p(x)$ and $q(x)$ are true.

$\therefore \exists x, p(x) \wedge q(x)$ is a true statement.

It's truth value is 1.

(ii) $\forall x, p(x) \rightarrow q(x)$

for every real no x, $q(x)$ is true.

$\therefore \forall x, p(x) \rightarrow q(x)$ is true. TV is 1.

(iii) $\forall x, q(x) \rightarrow s(x)$

We note that $s(x)$ is false and $q(x)$ is true for $x=1$.

Thus $\forall x, q(x) \rightarrow s(x)$ is false. TV is 0.

(iv) $\forall x, r(x) \vee s(x)$

$r(x)$ is true only for $x=4$ & $x=-1$

$r(x)$ and $s(x)$ are false for $x=1$.

Thus, $r(x) \vee s(x)$ is not always true.

$\therefore \forall x, r(x) \vee s(x)$ is false. TV is 0.

3a. Let $S(n)$: $1^2+3^2+ \cdots + (2n-1)^2 = \frac{1}{3} n(2n-1)(2n+1)$

Basis step: We note that: $S(1): 1^2 = \frac{1}{3} \times 1 \times 3$

which is true.

Basis step: We assume that $S(n)$ is true for $n = k$, where $k \geq 1$.

Then, $1^2+3^2+5^2+ \cdots + (2k-1)^2 = \frac{1}{3} k(2k-1)(2k+1)$

Adding $(2k+1)^2$ on both sides.

USN | | | | | | | | | |

CMRIT

Internal Assessment Test 3 – June 2019

Sub:	C Programming for Problem Solving				Sub Code:	18CPS23	Branch:	1st Yr
Date:	11 Jun 19	Duration:	90 mins	Max Marks: 50	Sem / Sec:	2nd		OBE

	Answer any FIVE FULL Questions	MARKS	CO	RBT
1 (a)	What are structures? Explain definition and declaration of structure with example.	[5]	CO4	L2
(b)	What are pointers? Explain declaration and initialization of pointers with example.	[5]	CO3	L2
2	Write a C program for compute sum, mean and standard deviation of array of integers using pointers.	[10]	CO2	L3
3	What is recursion? Write its advantages and disadvantages. Write a program to print nth term of Fibonacci series using recursion.	[10]	CO4	L3
4	Distinguish between the following types of variables/storage class with examples: auto (automatic), extern (global), static and register.	[10]	CO3	L3
5	What are preprocessor directives? Explain any 5 pre processor directives in C.	[10]	CO3	L1
6	What is an array of structures? Write a C program to store and print Name, USN, Subject and IA Marks of 60 students.	[10]	CO4	L2
7.	How to pass a pointer to a function? Write a C program to implement it.	[10]	CO4	L3
8.	How to pass a structure to a function? Write a C program to implement it.	[10]	CO4	L3

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3c

There are 15 objects (15 hundred Rs notes) to be distributed among 3 students A, B, C.

(i) Every student gets atleast Rs. 300.

Distribute Rs. 300 to every student.

Remaining 6 notes should be distributed among 3 students.

$r = 6, \; n = 3.$

This can be done in ${}^{n+r-1}C_r$ ways

$= {}^{8}C_6$ ways

(ii)

	A	B	C
a)	500	400	600
b)	500	500	500
c)	500	600	400
d)	600	400	500
e)	600	500	400
f)	700	400	400

Direct method.

<u>Using Combination with repetition,</u>

Distribute Rs 500 A, Rs 400 to B & C each.

Remaining 2 notes of 100 should be distributed among 3 students A, B, C.

$r = 2, \; n = 3$

No of was of distributing $= {}^{n+r-1}C_r = {}^{4}C_2 = \frac{4 \times 3}{2} = 6$

$\boxed{4a}$ We note that

$$A_1 = 11^{1+2} + 12^{2+1} = 11^3 + 12^3 = 1331 + 1728 = 3059$$

$3059 = 23 \times 133$ so that 133 divides 3059.

Thus, A_n is divisible by 133 for $n = 1$.

Induction step: Assume that A_n is divisible by 133 for $n = k \geq 1$.

Now, we find that

$$A_{k+1} = 11^{k+3} + 12^{2(k+1)+1}$$

$$= (11^{k+2} \times 11) + (12^{2k+1} \times 12^2)$$

$$= (11^{k+2} \times 11) + (12^{2k+1} \times 144)$$

$$= (11^{k+2} \times 11) + \{12^{2k+1} \times (11 + 133)\}$$

$$= (11^{k+2} + 12^{2k+1}) \times 11 + (12^{2k+1} \times 133).$$

$$= (A_k \times 11) + (12^{2k+1} \times 133)$$

This representation shows that A_{k+1} is divisible by 133 when A_k is divisible by 133.

$\therefore$ By induction, the given result is true.

$\boxed{4b}$ Here n must be of the form

$$n = x_1 x_2 x_3 x_4 x_5 x_6 x_7$$

where $x_1, x_2, \ldots x_7$ are the given digits with $x_1 = 5, 6$ or 7.

Suppose we take $x_1 = 5$. Then it's an arrangement of 6 digits which contains two 4's and one each of 3, 5, 6, 7.

The no of such arrangements $= \dfrac{6!}{2!1!1!1!1!} = 360$

Next suppose $x_1 = 6$.

Then no of arrangements $= \dfrac{6!}{2!\,2!} = 180$

If $x_1 = 7$, then no. of arrangements $= \dfrac{6!}{2!\,2!} = 180$.

Accordingly, by the sum rule,

required answer $= 360 + 180 + 180 = 720$

$\boxed{4c}$ Different possible ways in which a student can make a selection are: (I) 2 questions from part A, 2 from part B & 3 from C.

(II) 2 Qs from A, 3 from B & 2 from part C.

(III) 3 Qs from A, 2 from B and 2 from C.

(I) no of selection $= {}^4C_2 \times {}^5C_2 \times {}^6C_3 = 1200$ ways

(II) no of selection $= {}^4C_2 \times {}^5C_3 \times {}^6C_2 = 900$ ways

(III) No of selection $= {}^4C_3 \times {}^5C_2 \times {}^6C_2 = 600$ ways.

Total no of possible selections $= 1200 + 900 + 600 = 2700$.

Module 3

$\boxed{5a}$

We note that $f^{-1}(6) = \{x \in A \mid f(x) = 6\} = \{4\}$

$f^{-1}(9) = \{x \in A \mid f(x) = 9\} = \{5, 6\}$

For $B_1 = \{7, 8\}$, $f(x) \in B_1$ when $f(x) = 7$ and $f(x) = 8$.

Here $f(x) = 7$ when $x = 1$ and $x = 2$.

$f(x) = 8$ when $x = 3$.

$\therefore f^{-1}(B_1) = \{1, 2, 3\}$

III'y $B_2 = \{8, 9, 10\}$

$f(x) = 8$ when $x = 3$

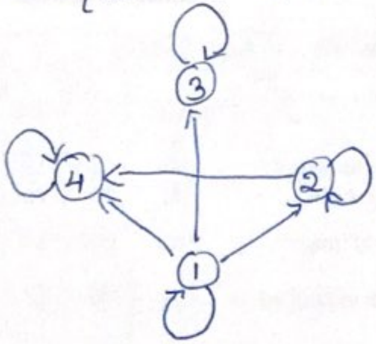
$f(x) = 9$ when $x = 5, 6$

$f(x) = 10$ for no value of x.

$$f^{-1}(B_2) = \{x \in A \mid f(x) \in B_2\} = \{3, 5, 6\}$$

$\boxed{5b}$ $A = \{1, 2, 3, 4\}$

i) $R = \{(1,1) (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$

ii) 

iii) $$M(R) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{5c}$ $(goh)(x) = g[h(x)] = g(\sqrt{x^2+2}) = \sqrt{x^2+2} + 5$

$$\therefore (fo(goh))(x) = f[(goh)(x)] = [(goh)(x)]^2 = (\sqrt{x^2+2} + 5)^2$$

$$= (x^2+2) + 25 + 10\sqrt{x^2+2}$$

$$= x^2 + 27 + 10\sqrt{x^2+2} \quad \text{—— (1)}$$

$$(fog)(x) = f(g(x)) = f(x+5) = (x+5)^2 = x^2 + 25 + 10x$$

$$((fog)oh)(x) = (fog)(h(x)) = [h(x)]^2 + 25 + 10\,h(x)$$

$$= (\sqrt{x^2+2})^2 + 25 + 10\sqrt{x^2+2}$$

$$= x^2 + 25 + 10\sqrt{x^2+2} \quad \text{—— (2)}$$

From (1) & (2) $fo(goh) = (fog)oh$

6a Treating the pages as pigeons and dictionaries as pigeonholes, we find by using the generalized pigeonhole principle that at least one of the dictionaries must contain p+1 or more pages, where

$$p = \left\lfloor \frac{61327-1}{30} \right\rfloor = \lfloor 2044.2 \rfloor = 2044.$$

6b Let $(x,y) \in (A \cup B) \times C$

$\Rightarrow x \in A \cup B$ and $y \in C$

$\Rightarrow x \in A$ or $x \in B$ and $y \in C$

$\Rightarrow (x \in A$ and $y \in C)$ or $(x \in B$ and $y \in C)$

$\Rightarrow (x,y) \in A \times C$ or $(x,y) \in B \times C$

$\Rightarrow (x,y) \in (A \times C) \cup (B \times C)$

$\therefore (A \cup B) \times C \subseteq (A \times C) \cup (B \times C)$ —— (i)

Now let $(x,y) \in (A \times C) \cup (B \times C)$

$\Rightarrow (x,y) \in A \times C$ or $(x,y) \in B \times C$

$\Rightarrow (x \in A$ and $y \in C)$ or $(x \in B$ and $y \in C)$

$\Rightarrow x \in A$ or $x \in B$ and $y \in C$

$\Rightarrow x \in A \cup B$ and $y \in C$

$\Rightarrow x \in (A \cup B) \times C$

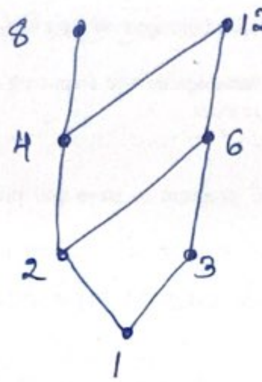
$\therefore (A \times C) \cup (B \times C) \subseteq (A \cup B) \times C$ —— (ii)

From (i) & (ii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

6 c) $A = \{1, 2, 3, 4, 6, 8, 12\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (1,8), (1,12), (2,2), (2,4), (2,6), (2,8), (2,12), (3,3), (3,6), (3,12), (4,4), (4,8), (4,12), (6,6), (6,12), (8,8), (12,12)\}$

Hasse diagram:



Module 4

7a The integers 1, 2, 3, 4 and 5 can be deranged in the first five places in d_5 ways; the last $n-5$ integers in d_{n-5} ways. Hence, the no of derangements is $d_5 \times d_{n-5}$. This is given as 11660. Thus, we have $d_5 \times d_{n-5} = 11660$, so that

$$d_{n-5} = \frac{11660}{d_5} = \frac{11660}{44} = 265$$

But $265 = d_6$. Thus, $n-5 = 6$ so that $n = 11$.

7b Let $S = \{1, 2, \cdots 300\}$ and A_1, A_2, A_3 be subsets of S whose elements are divisible by 5, 6, 8 respectively.

$S_0 = |S| = 300$, $|A_1| = \lfloor 300/5 \rfloor = 60$, $|A_2| = \lfloor 300/6 \rfloor = 50$, $|A_3| = \lfloor 300/8 \rfloor = 37$,

$|A_1 \cap A_2| = \lfloor 300/30 \rfloor = 10$, $A_1 \cap A_3 = \lfloor 300/40 \rfloor = 7$, $|A_2 \cap A_3| = \lfloor 300/24 \rfloor = 12$

$|A_1 \cap A_2 \cap A_3| = \lfloor 300/120 \rfloor = 2$

(i) $E_2 = S_2 - {}^3C_1 S_3 = 29 - ({}^3C_1) \times 2 = 23$

(ii) $L_2 = S_2 - {}^2C_1 S_3 = 29 - (2 \times 2) = 25$

7c $a_n = 2(a_{n-1} - a_{n-2})$

Characteristic eq^n is : $k^2 - 2k + 2 = 0$

$$k = 1 \pm i$$

$\therefore$ the general sol^n is:

$a_n = r^n [A \cos n\theta + B \sin n\theta]$ where $r = \sqrt{1^2 + 1^2} = \sqrt{2}$

& $\theta = \tan^{-1} 1 = \pi/4$

$$\therefore a_n = (\sqrt{2})^n \left[A \cos \frac{n\pi}{4} + B \sin \frac{n\pi}{4}\right] \quad \text{—— (1)}$$

Given $a_0 = 1$ & $a_1 = 2$

$\Rightarrow 1 = A$ & $2 = (\sqrt{2}) \left[A \cos \frac{\pi}{4} + B \sin \frac{\pi}{4}\right] = A + B$

$\Rightarrow A = 1$ & $B = 1$

$\therefore$ sol^n is $a_n = (\sqrt{2})^n \left[\cos \frac{n\pi}{4} + \sin \frac{n\pi}{4}\right]$ using (1).

8a Let S denote the set of all students in a hostel.

$A_1, A_2, A_3 \rightarrow$ set of all students who study History, Economics, Geography, respectively.

$S_1 = \sum |A_i| = 15 + 8 + 6 = 29$

$S_3 = |A_1 \cap A_2 \cap A_3| = 3$

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| = |S| - \sum |A_i| + \sum |A_i \cap A_j| - \sum |A_1 \cap A_2 \cap A_3|$$

$$= |S| - S_1 + S_2 - S_3$$

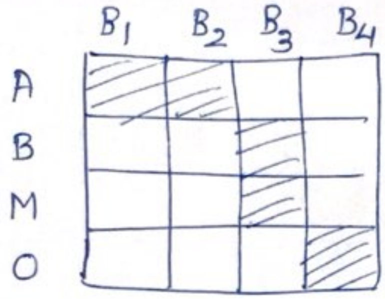
$$= 30 - 29 + S_2 - 3 \quad = S_2 - 2$$

$(A_1 \cap A_2 \cap A_3) \subseteq (A_i \cap A_j)$ for $i, j = 1, 2, 3$

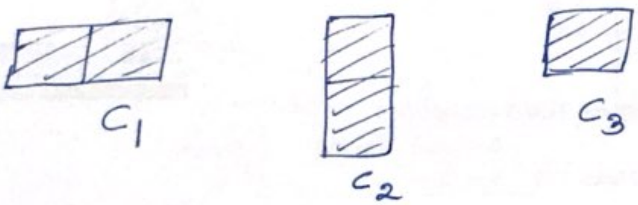
$$S_2 = \sum |A_i \cap A_j| \geq 3 |A_1 \cap A_2 \cap A_3| = 9$$

$$|\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3| \geq 9 - 2 = 7$$

$\boxed{8b}$



Let



$$r(C, x) = r(C_1, x) \times r(C_2, x) \times r(C_3, x)$$

$$= (1 + 2x) \times (1 + 2x) \times (1 + x)$$

$$= 1 + 5x + 8x^2 + 4x^3$$

Here, $r_1 = 5$, $r_2 = 8$, $r_3 = 4$

$S_0 = n! = 4! = 24$, $S_k = (n-k)!\, k!$

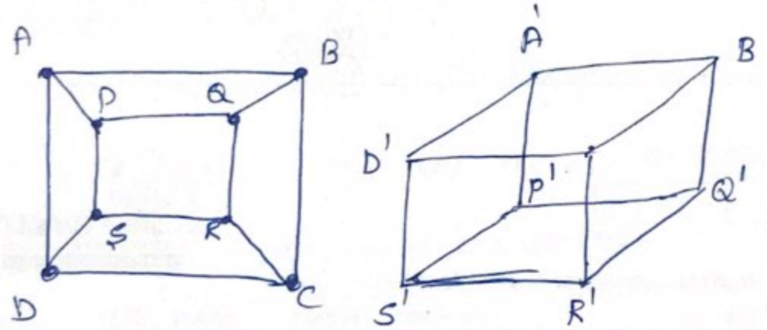
$S_1 = (4-1)! \times r_1 = 30$. $S_2 = (4-2)! \times r_2 = 16$, $S_3 = (4-3)! \times r_3$
$= 4$

$$\bar{N} = S_0 - S_1 + S_2 - S_3 = 6$$

$\boxed{8c}$

$$a_{n+1} = 3a_n + 5 \times 3^{n+1}$$
$$= 3a_n + f(n+1)$$
$$a_n = 3^n a_0 + \sum_{k=1}^{n} 3^{n-k} f(k)$$
$$= 3^n a_0 + 3^{n-1} f(1) + 3^{n-2} f(2) + \dots + 3^0 f(n)$$
$$= 2 \times 3^n + 3^{n-1} \times (5 \times 3^1) + 3^{n-2} \times (5 \times 3^2) + \dots + 3^0 \times (5 \times 3^n)$$
$$= 2 \times 3^n + 5 \times (n 3^n)$$
$$= (2 + 5n) 3^n$$

$\boxed{9a}$



Let's consider the one-to-one correspondence b/w the vertices of the two graphs under which the vertices A, B, C, D, P, Q, R, S of the first graph correspond to the vertices A', B', C', D', P', Q', R', S' respectively of the second graph, and vice-versa.

The edges determined by corresponding vertices correspond so that the adjacency of vertices is retained.

Both the graphs have 8 vertices, 12 edges and are cubic graphs.

$\therefore$ They are isomorphic.

9b

i) If G is a simple graph of order n, then the complement of G in K_n is called the complement of G. It is denoted by $\bar{G}$.

ii) Rooted tree - A directed tree T is called a rooted tree if (i) T contains a unique vertex, called the root, whose in-degree is equal to 0,

(ii) the in-degrees of all ~~the~~ other vertices of T are equal to 1.

(iii) vertex degree - ~~Number of edges~~ Let $G = (V, E)$ be a graph and v be a vertex of G. Then, the no of edges of G that are incident on v with loops counted twice is called the degree of the vertex.

iv) Prefix code - Let P be a set of binary set of binary sequences that represent a set of symbols. Then P is called a prefix code if no sequence in P is the prefix of any other sequence in P.

9C $\{-1, 7, 4, 11, 5, -8, 15, -3, -2, 6, 10, 3\}$

$\{-1, 7, 4\}$ $\{11, 5, -8\}$ $\{15, -3, -2\}$, $\{6, 10, 3\}$

$\{-1, 7\}$, $\{4\}$ $\{11, 5\}$, $\{-8\}$ $\{15, -3\}$, $\{-2\}$ $\{6, 10\}$ $\{3\}$

$\{-1\}$, $\{7\}$ 4 $\{11\}$ $\{5\}$ -8 $\{15\}$, $\{-3\}$ -2 $\{6\}$ $\{10\}$ 3

-1 7 11 5 15 -3 6 10

Now these sublists are to be merged

-1 7 11 5 15 -3 6 10

$\{-1, 7\}$ 4 $\{5, 11\}$ -8 $\{-3, 15\}$ -2 $\{6, 10\}$ 3

$\{-1, 4, 7\}$ $\{-8, 5, 11\}$ $\{-3, -2, 15\}$ $\{3, 6, 10\}$

$\{-8, -1, 4, 5, 7, 11\}$ $\{-3, -2, 3, 6, 10, 15\}$

$\{-8, -3, -2, -1, 3, 4, 5, 6, 7, 10, 11, 15\}$

The sorted-out version of the given list is :

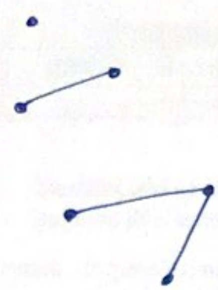
$-8, -3, -2, -1, 3, 4, 5, 6, 7, 10, 11, 15$

[10a]

$n=1$:

$n=2$:

$n=3$:



Result is true for $n=1, 2, 3$.

Assume that the result is true for $n=k$.

Consider a tree T with $k+1$ vertices. Remove an edge e (say) from the tree. Now there are 2 components, both of which are trees, say T_1 and T_2.

Let the no of edges in T_1 & T_2 be k_1 and k_2 resp'ly.

$$k_1, k_2 \leq k+1.$$

No of edges in $T_1 = k_1 - 1$

— " — $T_2 = k_2 - 1$

Total no of edges in T_1 and T_2 (taken together) $= k_1 + k_2 - 2$

$$= (k+1) - 2$$

$$= k - 1$$

Keeping the edge e back in its place,

No of edges in T_1 & T_2 (together) $= (k-1) + 1 = k$.

So, the result is true for $k+1$ also.

Hence, by M.I., the result is true for all $+^{ve}$ in n.

$\boxed{10b}$ (i) Suppose G is of order n.

Sum of the ~~vertices~~ degrees of all vertices $= 3n$.

Since G has 9 edges, we have $3n = 2 \times 9$

$$\Rightarrow n = 6$$

Thus, the order of G is 6.

(ii) Suppose the order of G is n. Since 2 vertices are of deg 4 & remaining are of degree 3,

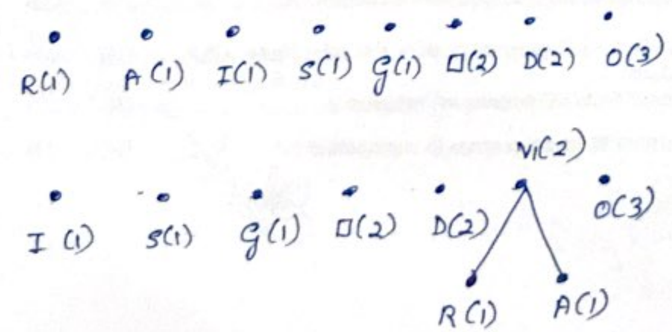
the sum of deg of all vertices $= (2 \times 4) + (n-2) \times 3$

$$\Rightarrow 2 \times 4 + (n-2) \times 3 = 2 \times 10$$

$$\Rightarrow n = 6$$

$\boxed{10c}$ The given message consists of letters R, O, A, D, I, S, G with frequencies 1, 3, 1, 2, 1, 1, 1 resp'ly. Further, there is a blank space (□) occuring twice.

Arrang the letters & □ in non-decreasing order of their weights.



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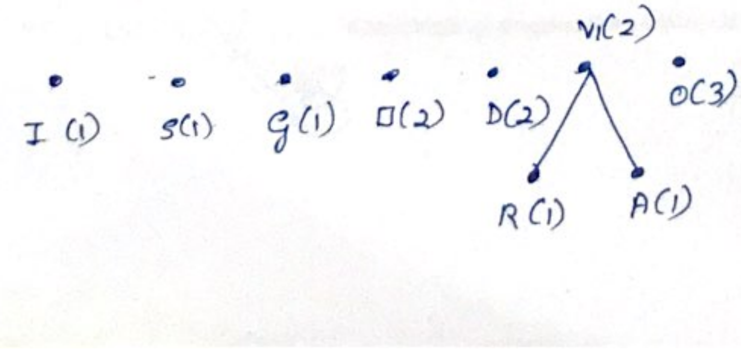
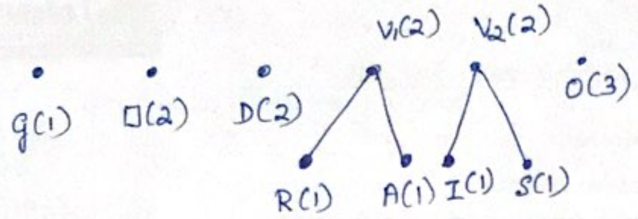
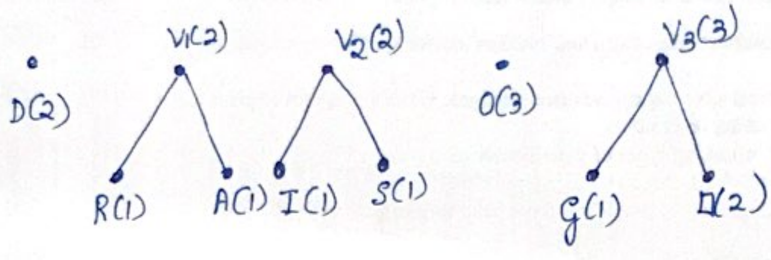
Arrang the letters & □ in non-decreasing order of their weights.

R(1) A(1) I(1) S(1) G(1) □(2) D(2) O(3)

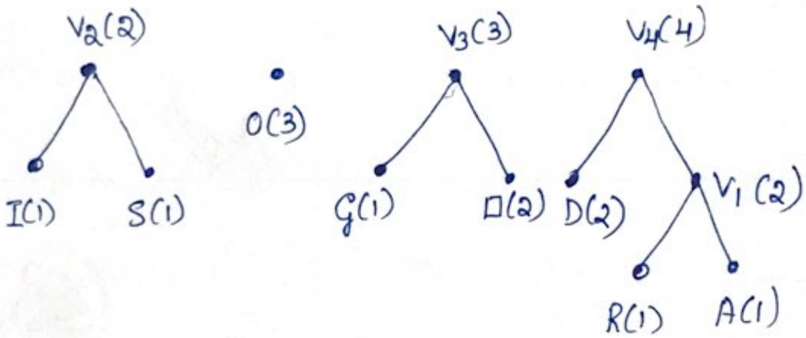




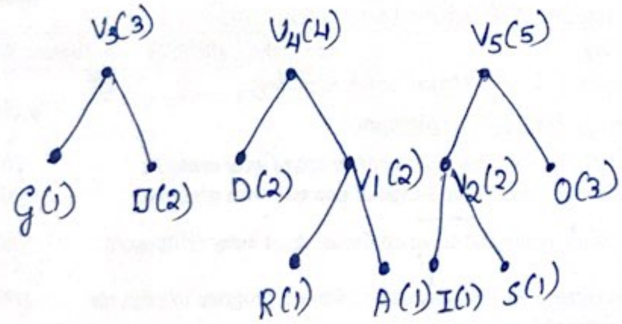
g(1)
□(2)
D(2)
V1(2)
R(1)
A(1)
V2(2)
I(1)
S(1)
O(3)



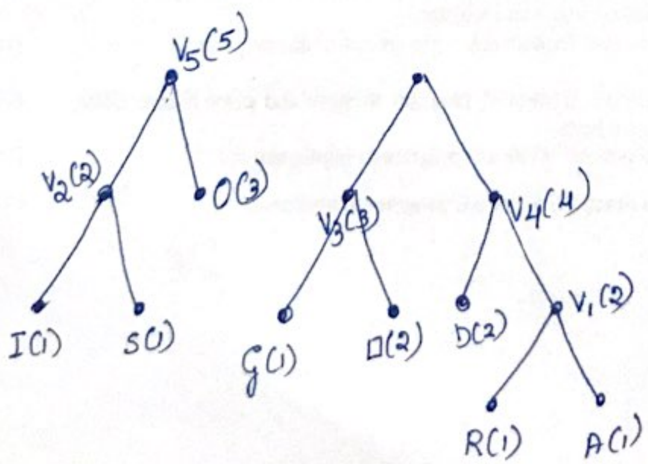
D(2)
V1(2)
R(1)
A(1)
V2(2)
I(1)
S(1)
O(3)
V3(3)
G(1)
□(2)



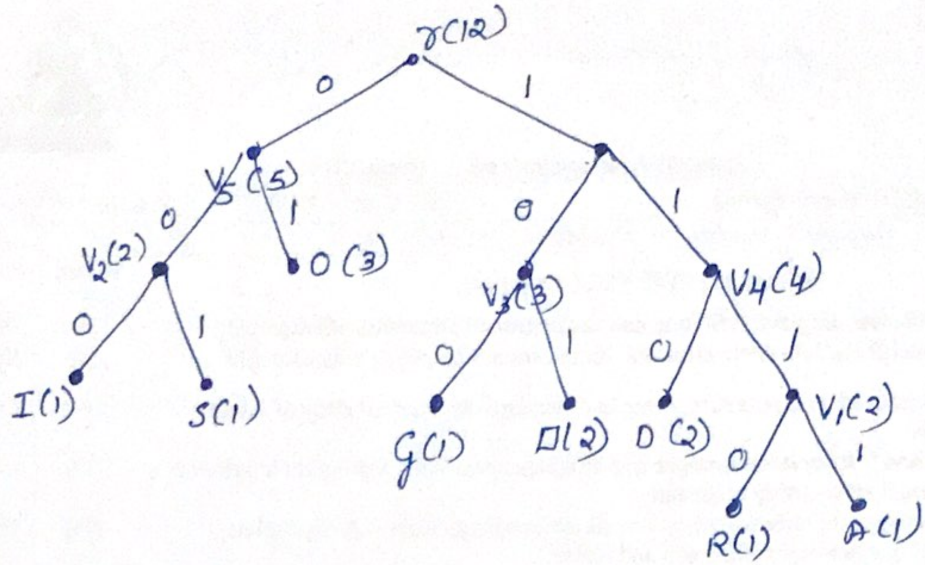
V2(2)
I(1)
S(1)
O(3)
V3(3)
G(1)
□(2)
V4(4)
D(2)
V1(2)
R(1)
A(1)



V3(3)
G(1)
□(2)
V4(4)
D(2)
V1(2)
R(1)
A(1)
V5(5)
V2(2)
I(1)
S(1)
O(3)



V5(5)
V2(2)
O(3)
I(1)
S(1)
V3(3)
V4(4)
G(1)
□(2)
D(2)
V1(2)
R(1)
A(1)



R : 1110 , A : 1111 , I : 000 , S : 001

G : 100 , □ : 101 , D : 110 , O : 01

Code for ROAD IS GOOD:

111001 111110 101 00000 110/ 100 0101110