CBCS SCHEME

USN

18EE54

Fifth Semester B.E. Degree Examination, Jan./Feb. 2021
Signals and Systems

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Determine whether the following signals are energy or power signals or neither. Justify your answer.
i) $x(t) = e^{j(t+\pi/2)}$ ii) $x(t) = 8\cos(4t) \cdot \cos(6t)$. **(10 Marks)**

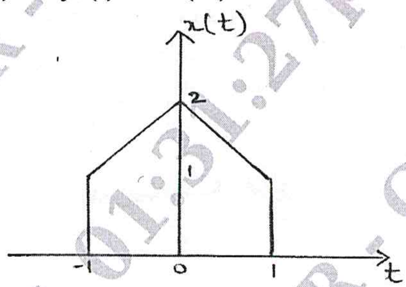
b. Sketch the following signals :
i) $x_1(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$.
ii) $x_2(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$. **(10 Marks)**

OR

2 a. Determine whether the system $y(t) = e^{x(t)}$ is i) Causal ii) Time Invariant iii) Linear iv) Stability v) Memoryless. Justify your answer. **(10 Marks)**

b. For the signal shown in Fig. Q2(b), sketch and label each of the following signals :
i) $y_1(t) = x(t-2)$ ii) $y_2(t) = x(2t-2)$ iii) $y_3(t) = x(\frac{1}{2}t + 2)$
iv) $y_4(t) = x(-2t-1)$ v) $y_5(t) = 3x(2t)$. **(10 Marks)**

Fig. Q2(b)



Module-2

3 a. Evaluate the convolution integral for a system with input x(t) and impulse response h(t). Given $x(t) = u(t-1) - u(t-3)$; $h(t) = u(t) - u(t-2)$. Also sketch y(t). **(10 Marks)**

b. Represent the direct form I and form II realization for the system described by

i) $y[n] + \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] + x[n-1]$.

ii) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = x(t) + 3\frac{d}{dt}x(t)$. **(10 Marks)**

OR

4 a. Determine the complete response of the system describe by the differential equation.

$$\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = \frac{d}{dt}x(t) \text{ with } y(0) = 0 \quad ; \quad \frac{d}{dt}y(t)\Big|_{t=0} = 1 \quad ;$$

For input $x(t) = e^{-2t}u(t)$. **(10 Marks)**

b. Investigate the causality, stability and memory of the LTI system described by the impulse response i) $h(t) = e^{-2|t|}$ ii) $h[n] = 2^n u[n-1]$. **(10 Marks)**

Module-3

5 a. Prove the following properties related to continuous – time Fourier transform :
i) Convolution ii) Parseval's theorem. **(10 Marks)**

b. Determine the Fourier Transform of the following signals :
i) $x(t) = e^{at} u(-t)$ ii) $x(t) = e^{-a|t|}$ iii) $x(t) = e^{-a|t|} \text{sgn}(t)$. **(10 Marks)**

OR

6 a. Determine the Inverse Fourier Transform of the following :
i) $X(jw) = \dfrac{2jw+1}{(jw+2)^2}$ ii) $X(jw) = \dfrac{1}{(a+jw)^2}$. **(10 Marks)**

b. Determine the Fourier transform of the signal $x(t) = e^{-3|t|} \sin(2t)$ using appropriate properties. **(10 Marks)**

Module-4

7 a. Determine the Inverse DTFT of the following :
i) $X(e^{j\Omega}) = 1 + 2\cos\Omega + 3\cos 2\Omega$ ii) $Y(e^{j\Omega}) = j(3 + 4\cos\Omega + 2\cos 2\Omega)\sin\Omega$. **(10 Marks)**

b. Using appropriate properties, determine the DTFT of
i) $x[n] = \left(\dfrac{1}{2}\right)^n u[n-2]$ ii) $x[n] = \sin\left(\dfrac{\pi}{4}n\right)\left(\dfrac{1}{4}\right)^n u[n-1]$. **(10 Marks)**

OR

8 a. Prove the following properties related to DTFT :
i) Frequency differentiation ii) Modulation. **(10 Marks)**

b. Compute the DTFT of the following signals :
i) $x[n] = 2^n u[-n]$ ii) $x[n] = a^{|n|}$; $|a| < 1$. **(10 Marks)**

Module-5

9 a. Determine the Inverse Z – transform if

$$X(z) = \frac{(z^3 - 4z^2 + 5z)}{(z-1)(z-2)(z-3)},$$

with ROCs i) $2 < |z| < 3$ ii) $|z| > 3$ iii) $|z| < 1$. **(10 Marks)**

b. Use Unilateral Z – transform to determine the forced response, natural response and complete response of system described by $y[n] - \frac{1}{2}y[n-1] = 2x[n]$ with input $x[n] = 2\left(\dfrac{-1}{2}\right)^n u[n]$. The initial conditions are $y[-1] = 3$. **(10 Marks)**

OR

10 a. Explain the properties of ROC. **(08 Marks)**

b. A LTI discrete – time system is given by system function

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}.$$ Specify ROC of H(z).

Determine h[n] for the following conditions : i) Stable ii) Causal. **(12 Marks)**

* * * * *

1a.

1 a) i) $x(t) = e^{j(t+\pi/2)}$

$x(t)$ is of infinite duration periodic signal which is combination of sine & cosine signals.

So this can be a power signal.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| e^{j(t+\pi/2)} \right|^2 dt.$$

$$\underline{P = 1\,W}$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} \left| e^{j(t+\pi/2)} \right|^2 dt.$$

$$\underline{E = \infty}$$

$\therefore$ Energy is infinite & average power is finite

Hence $x(t)$ is a power signal.

ii) $x(t) = 8\cos(4t).\cos(6t)$

$\therefore x(t) = 4\cos(10t) + 4\cos(2t)$

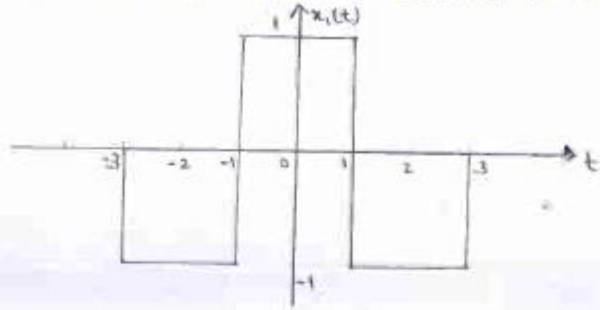
Since $x(t)$ is periodic in nature; it can be a power signal.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt \quad = \underline{16\,W}$$

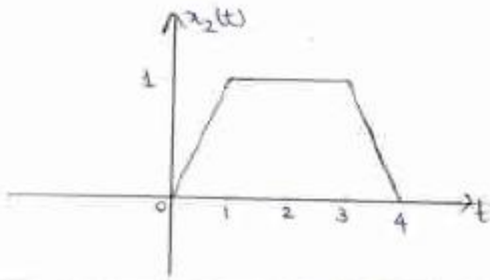
$x(t)$ is power signal.

1b.

(b) i) $x_1(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$



ii) $x_2(t) = r(t) - r(t-1) - r(t-3) + r(t-4)$



2 a) $y(t) = e^{x(t)}$

(i) Since output of system at any instant depends only on present input; system is causal.

(ii) if input is delayed by 't_0' then $y(t) = e^{x(t-t_0)}$

if output is delayed by 't_0' then $y(t-t_0) = e^{x(t-t_0)}$

Since $y(t) = y(t-t_0)$ → time-invariant

(iii) if $x_3(t) = a x_1(t) + b x_2(t)$

then $y_3(t) = e^{[a x_1(t) + b x_2(t)]}$ — (1)

if $y_3'(t) = a y_1(t) + b y_2(t)$

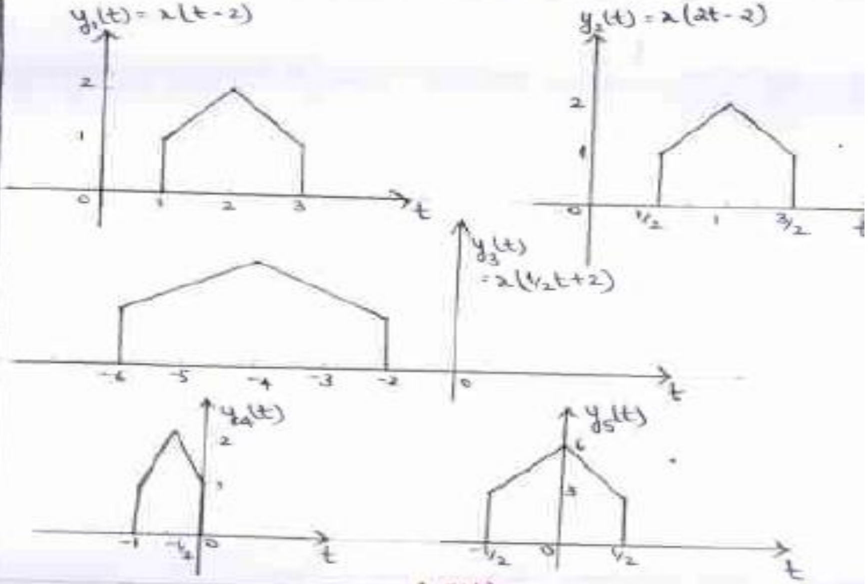
$y_3'(t) = a e^{x_1(t)} + b e^{x_2(t)}$ — (2)

Since $y_3(t) \neq y_3'(t)$ → non-linear

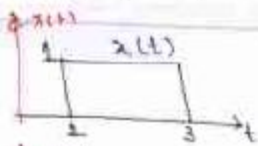
(iv) if input $x(t)$ is bounded and is a finite value; then system is <u>stable</u>

(v) Since $y(t)$ depends only on present value of input; system is <u>memoryless</u>.

b)



3 a) $x(t) = u(t-1) - u(t-3)$



$h(t) = u(t) - u(t-2)$

$y(t) = x(t) * h(t)$

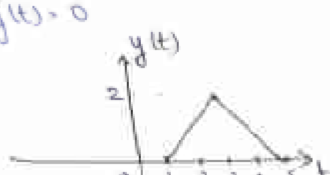
$y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau) \cdot d\tau$

(i) $t < \phi$; $y(t) = 0$ (2m)

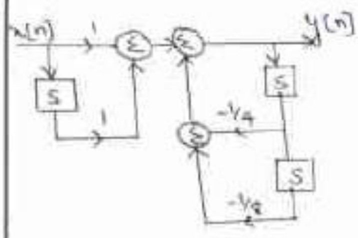
(ii) $1 < t < 3$; $y(t) = t-1$

(iii) $3 < t < 5$; $y(t) = -t+5$

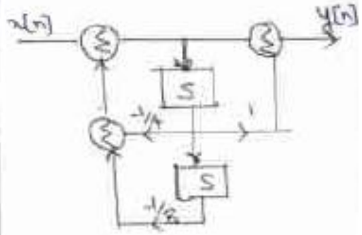
(iv) $t > 5$; $y(t) = 0$



b) i) $y[n] + \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] + x[n-1]$



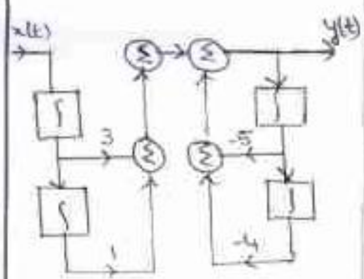
Direct form-I



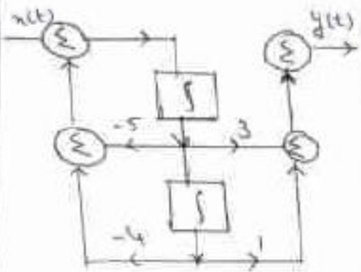
Direct form-II

ii) $\frac{d^2}{dt^2}y(t) + 5\frac{d}{dt}y(t) + 4y(t) = x(t) + 3\frac{d}{dt}x(t)$

$y(t) + 5\int y(t)\,dt + 4\iint y(t)\,dt = \iint x(t)\,dt + 3\int x(t)\,dt$



Direct form-I



Direct form-II

4 a) Homogeneous soln:

$$s^2 y(t) + 5 s y(t) + 4 y(t) = 0$$

$$\therefore y^h(t) = c_1 e^{-t} + c_2 e^{-4t}$$

Particular soln:

$$y^p(t) = k \cdot e^{-2t} u(t)$$

$$\frac{d}{dt} y^p(t) = -2k e^{-2t} u(t)$$

$$\frac{d^2}{dt^2} y^p(t) = 4k \cdot e^{-2t} u(t)$$

To find value of k:

$$4 k e^{-2t} u(t) + 5\left[-2k e^{-2t} u(t)\right] + 4\left[k e^{-2t} u(t)\right] = -2 e^{-2t} u(t)$$

$$\therefore k = 1$$

$$y^p(t) = 1 \cdot e^{-2t} u(t)$$

Total response: $y(t) = y^h(t) + y^p(t)$

$$y(t) = c_1 e^{-t} + c_2 e^{-4t} + e^{-2t} u(t)$$

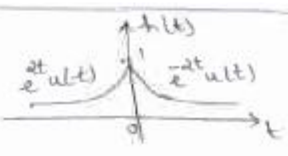
$$\therefore y(0) = c_1 + c_2 + 1 = 0 \Rightarrow c_1 + c_2 = -1$$

$$\frac{d}{dt} y(0) = -c_1 - 4c_2 - 2 = 1 \Rightarrow c_1 + 4c_2 = -3$$

$$c_1 = -1/3; \quad c_2 = -2/3;$$

$$y(t) = \left(-\tfrac{1}{3} e^{-t} - \tfrac{2}{3} e^{-4t} + e^{-2t}\right) u(t) \quad ; \quad t \geq 0$$

b) (i) $h(t) = e^{-2|t|}$



h(t) is non-causal; since $h(t) \neq 0$ for $t < 0$

not memoryless since $h(t) \neq c \cdot \delta(t)$

$$S = \int_{-\infty}^{\infty} |h(t)| dt \Rightarrow S = 1 \text{ finite}$$

$\therefore$ h(t) is stable

(ii) $h[n] = 2^n u[n-1]$

$h[n]$ is causal since $h[n] = 0$ for $n < 0$

$h[n]$ is not memoryless since $h[n] \neq C\,\delta[n]$

$S = \sum_{k=-\infty}^{\infty} |h[k]|$

$= \sum_{k=1}^{\infty} 2^k \cdot 1 = \infty$ unstable.

5 a) i) convolution

if $x(t) \xleftrightarrow{FT} X(\omega)$

$y(t) \xleftrightarrow{FT} Y(\omega)$

then $z(t) = x(t) * y(t) \xleftrightarrow{FT} Z(\omega) = X(\omega) \cdot Y(\omega)$

convolution operation is transformed to modulation in frequency domain.

Proof –

(ii) Parseval's Theorem

if $x(t) \xleftrightarrow{FT} X(\omega)$

then $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |X(f)|^2 df$

Energy of the signal can be obtained by interchanging its energy spectrum

Proof –

b) i) $x(t) = e^{at} u(-t)$

$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$

$X(\omega) = \int_{-\infty}^{\infty} e^{at} u(-t) \cdot e^{-j\omega t} dt$

$X(\omega) = \int_{0}^{\infty} e^{at} e^{-j\omega t} \cdot dt$

$X(\omega) = \frac{1}{a - j\omega}$

ii) $x(t) = e^{-a|t|}$

$$X(t) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$X(t) = \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t} \cdot dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} \cdot dt$$

$$X(t) = \frac{2a}{a^2 + \omega^2}$$

(iii) $x(t) = e^{-a|t|} \operatorname{sgn}(t)$; $\operatorname{sgn}(t) = \begin{cases} 1 & ; t > 0 \\ -1 & ; t < 0 \end{cases}$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$X(\omega) = \int_{-\infty}^{0} -e^{at} \cdot e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} \cdot dt$$

$$X(\omega) = \frac{-2j\omega}{a^2 + \omega^2}$$

6 a) i) $X(j\omega) = \dfrac{2j\omega + 1}{(j\omega + 2)^2}$

$$X(j\omega) = \frac{A}{(j\omega+2)} + \frac{B}{(j\omega+2)^2} \quad \text{using partial fraction}$$

$$X(j\omega) = \frac{2}{(j\omega+2)} - \frac{3}{(j\omega+2)^2}$$

using standard fourier transform

$$x(t) = 2 \cdot e^{-2t} u(t) - 3t \cdot e^{-2t} u(t)$$

(ii) $X(j\omega) = \dfrac{1}{(a + j\omega)^2} = \dfrac{1}{(a+j\omega)} \dfrac{1}{(a+j\omega)}$

$$X(j\omega) = X_1(j\omega) \cdot X_2(j\omega)$$

$$x_1(t) = e^{-at} u(t); \quad x_2(t) = e^{-at} u(t)$$

using convolution property

$$x(t) = x_1(t) * x_2(t)$$

$$x(t) = \int_{-\infty}^{\infty} e^{-a\tau} u(\tau) \cdot e^{-a(t-\tau)} \cdot u(t-\tau) \cdot d\tau$$

$$x(t) = t \cdot e^{-at} u(t)$$

b) $x(t) = e^{-3|t|} \sin 2t$

$$x(t) = \frac{e^{j2t} - e^{-j2t}}{2j} e^{-3|t|}$$

$$x(t) = \frac{1}{2j}\left[e^{j2t} \cdot e^{-3|t|} - e^{-j2t} \cdot e^{3|t|}\right]$$

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$$

$$\therefore e^{-3|t|} \xleftrightarrow{FT} \frac{6}{9+\omega^2}$$

Using shifting property of frequency

$$e^{j2t} e^{-3|t|} \xleftrightarrow{FT} \frac{6}{9+(\omega-2)^2}$$

$$\therefore e^{-j2t} \cdot e^{-3|t|} \xleftrightarrow{FT} \frac{6}{9+(\omega+2)^2}.$$

$$\therefore x(t) \xleftrightarrow{FT} \frac{1}{2j}\left[\frac{6}{9+(\omega-2)^2} - \frac{6}{9+(\omega+2)^2}\right]$$

$$= \frac{3}{j}\left[\frac{1}{9+(\omega-2)^2} - \frac{1}{9+(\omega+2)^2}\right]$$

7 a) i) $X(e^{j\Omega}) = 1 + 2\cos\Omega + 3\cos 2\Omega$

$$X(e^{j\Omega}) = 1 + 2\left[\frac{e^{j\Omega} + e^{-j\Omega}}{2}\right] + 3\left[\frac{e^{j2\Omega} + e^{-j2\Omega}}{2}\right]$$

Inverse DTFT

$$\therefore x[n] = \delta[n] + \delta[n+1] + \delta[n-1] + \frac{3}{2}\delta[n+2] + \frac{3}{2}\delta[n-2]$$

$$\therefore x[n] = \{3/2, 1, \underset{\uparrow}{1}, 1, 3/2\}$$

(ii) $Y(e^{j\Omega}) = j(3 + 4\cos\Omega + 2\cos 2\Omega)\sin\Omega$

$Y(e^{j\Omega}) = \frac{1}{2}e^{j3\Omega} + e^{j2\Omega} + e^{j\Omega} - e^{-j\Omega} - e^{-j2\Omega} - \frac{1}{2}e^{-j3\Omega}$

Inverse DTFT

$y[n] = \{\frac{1}{2}, 1, 1, \underset{\uparrow}{0}, -1, -1, -1/2\}$

b) (i) $x[n] = \left(\frac{1}{2}\right)^n u[n-2]$

$x[n] = (1/2)^2 \cdot \left(\frac{1}{2}\right)^{n-2} u[n-2]$

$(1/2)^n u[n] \xleftrightarrow{DTFT} \frac{1}{1 - 1/2\, e^{-j\Omega}}$

time-shift

$(1/2)^{n-2} u[n-2] \xleftrightarrow{DTFT} e^{-j2\Omega}\frac{1}{1 - 1/2\, e^{-j\Omega}}$

Linearity property

$\left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{n-2} u[n-2] \xleftrightarrow{DTFT} \left(\frac{1}{2}\right)^2 e^{-j2\Omega}\frac{1}{1 - 1/2\, e^{-j\Omega}}$

$X(e^{j\Omega}) = \frac{1}{4} e^{-j2\Omega}\frac{1}{1 - 1/2\, e^{-j\Omega}}$

(ii) $x[n] = \sin\left(\frac{\pi}{4}n\right)(1/4)^n u[n-1]$

$x[n] = \left[\frac{e^{j\pi/4 n} - e^{-j\pi/4 n}}{2j}\right](1/4)\,(1/4)^{n-1} u[n-1]$

Time-shift property

$(1/4)^{n-1} u[n-1] \xleftrightarrow{DTFT} e^{-j\Omega}\frac{1}{1 - 1/4\, e^{-j\Omega}}$

Frequency-shift.

$e^{j\pi/4 n}(1/4)^{n-1} u[n-1] \xleftrightarrow{DTFT} \frac{1}{e^{j(\Omega - \pi/4)} - 1/4}$

$X(e^{j\Omega}) = \frac{1}{8j}\left[\frac{1}{e^{j(\Omega - \pi/4)} - 1/4} - \frac{1}{e^{j(\Omega + \pi/4)} - 1/4}\right]$

8 a) i) Frequency differentiation

If $x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$

Then $-jn\, x[n] \xleftrightarrow{DTFT} \frac{d}{d\Omega} X(e^{j\Omega})$

Proof:

(ii) Modulation

If $x[n] \xleftrightarrow{DTFT} X(e^{j\Omega})$

and $y[n] \xleftrightarrow{DTFT} Y(e^{j\Omega})$

Then $z[n] = x[n] \cdot y[n] \xleftrightarrow{DTFT} Z(e^{j\Omega})$

$$= \frac{1}{2\pi}\left[X(e^{j\Omega}) \circledast Y(e^{j\Omega})\right]$$

$\circledast$ periodic convolution

Proof:

b) (i) $x[n] = 2^n u[-n]$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\Omega n}$$

$$\therefore X(e^{j\Omega}) = \sum_{n=-\infty}^{0} 2^n \cdot e^{-j\Omega n}$$

$$\therefore X(e^{j\Omega}) = \frac{2}{2 - e^{j\Omega}}$$

(ii) $x[n] = a^{|n|}$; $|a| < 1$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} a^{-n} \cdot e^{-j\Omega n} + \sum_{n=0}^{\infty} a^{n} \cdot e^{-j\Omega n}$$

$$X(e^{j\Omega}) = \frac{1 - a^2}{1 - 2a\cos\Omega + a^2}$$

9 a) $X(z) = \dfrac{z(z^2-4z+5)}{(z-1)(z-2)(z-3)}$

ROCs: (i) $2 < |z| < 3$

$\dfrac{X(z)}{z} = \dfrac{z^2-4z+5}{(z-1)(z-2)(z-3)}$

$\dfrac{X(z)}{z} = \dfrac{A}{z-1} + \dfrac{B}{z-2} + \dfrac{C}{z-3}$

$\dfrac{X(z)}{z} = \dfrac{1}{z-1} + \dfrac{1}{z-2} + \dfrac{1}{z-3}$

$\therefore x[n] = -(3)^n u[-n-1] + (1)^n u[n] - (2)^n u[n]$

(ii) $|z| > 3$

$x[n] = 3^n u[n] + (1)^n u[n] - (2)^n u[n]$

(iii) $|z| < 1$

$x[n] = -(3)^n u[-n-1] - (1)^n u[-n-1] + 2^n u[-n-1]$

b) $y[n] - \frac{1}{2} y[n-1] = 2x[n]$

natural response: $y[n] - \frac{1}{2} y[n-1] = 0$

$Y(z) = \dfrac{\frac{3}{2} \cdot z}{(z-\frac{1}{2})}$

$\therefore y^h[n] = \frac{3}{2} \cdot \left(\frac{1}{2}\right)^n u[n]$

Forced response: $x[n] = 2 \cdot \left(-\frac{1}{2}\right)^n u[n]$

$Y(z) = 2 \dfrac{z}{(z+\frac{1}{2})} + 2 \dfrac{z}{(z-\frac{1}{2})}$

$\therefore y^f[n] = 2 \cdot \left(-\frac{1}{2}\right)^n u[n] + 2\left(\frac{1}{2}\right)^n u[n]$

complete response: $y[n] = y^h[n] + y^f[n]$

$y[n] = \frac{7}{2}\left(\frac{1}{2}\right)^n u[n] + 2 \cdot \left(-\frac{1}{2}\right)^n u[n]$

10 a) Properties of ROC

b) $$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$

By partial fraction expansion

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})} + \frac{2}{(1 - 3z^{-1})}$$

location of poles $z = \frac{1}{2}$ and $z = 3$

(i) stable system

$\frac{1}{2} < |z| < 3$

$\therefore h[n] = (\frac{1}{2})^n u[n] - 2(3)^n u[-n-1]$

(ii) causal system

$|z| > 3$

$\therefore h[n] = (\frac{1}{2})^n u[n] + 2(3)^n u[n]$