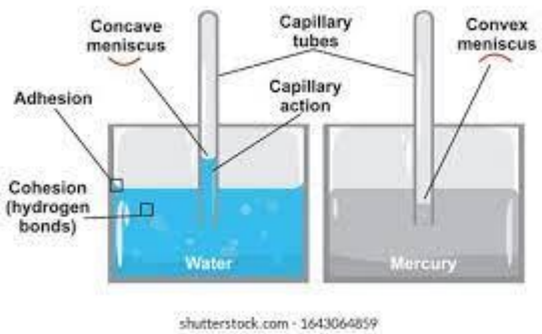
Third Semester B.E. Degree Examination, Dec 2020/Jan 2021

18CV33: Fluid Mechanics

MODULE 1	
1a	**Define the following and mention their units:** (i) **Capillarity (ii) Surface tension (iii) Viscosity** **(06 marks)**

(i) Capillarity

The tendency of a liquid in a capillary tube or absorbent material to rise or fall as a result of surface tension. The more narrow the tube higher will be the capillarity rise/fall. This is due to the surface tension and properties of the medium. The surface of the fluid will be concave or convex surface based on the predominant forces in the fluid. If adhesive force is predominant, the fluid will wet the medium and water will have a concave surface. If cohesive forces are predominant, the fluid will be attracted towards itself, and the meniscus will take a concave surface.



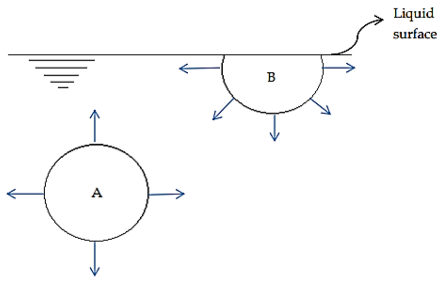
Capillary rise or fall is given as

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

where d be the diameter of the glass tube in m, h be the capillary rise in m, σ is the surface tension of the liquid in N/m, θ is the angle of contact between liquid and glass tube, ρ is the density and g is the acceleration due to gravity

(ii) Surface tension

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with gas or on the surface between two immiscible liquids such that the contact faces behaves like a membrane under tension. Surface tension is due to unbalanced cohesive forces at the interface of liquid, gases or between two immiscible liquids. Unit of the surface tension is N/m



When a liquid possess, relatively, greater affinity for solid molecules or the liquid which have greater adhesion than cohesion, then it will wet a solid surface in contact and tend to rise at the point of contact.

(iii) Viscosity

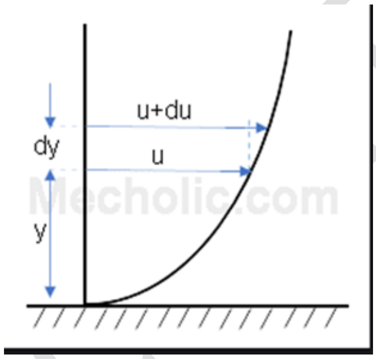
Viscosity is defined as the property of the fluid which offers resistance to the movement of one layer of fluid to the movement of adjacent layer of fluid.

Consider two layers of fluid separated by a distance of u moving at velocities v and u+du as shown.

Movement of these fluid layers induces shear stress and the shear stress is proportional to the rate of change of velocity with respect to y. Mathematically it can be written as

$$\tau \propto \frac{du}{dy}$$

Or $\tau = \mu \frac{du}{dy}$



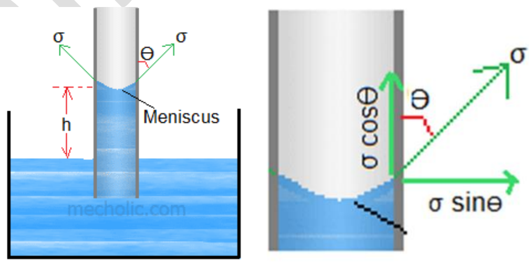
Constant of proportionality is known as coefficient of dynamic viscosity and (du/dy) is known as velocity gradient/rate of shear strain/rate of shear deformation. Or, ratio of shear stress to shear rate is a constant, for a given temperature and pressure, and is defined as the viscosity or coefficient of viscosity. Unit of μ is Ns/m^2 and CGS unit for the same is Poise.

1b Derive an expression for capillary rise/fall of fluid in a tube of small diameter with sketches. (06 marks)

For capillary rise:

Let d be the diameter of the glass tube
h be the capillary rise
σ is the surface tension of the liquid
θ is the angle of contact between liquid and glass tube

Forces acting in vertical direction are:
1. Weight of the liquid column
2. Vertical component of surface tension force



Weight of the liquid column

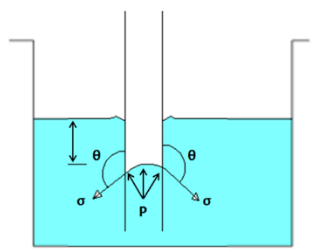
$= \frac{\pi}{4} \times d^2 \times h \times \rho g$ ____________(1)

Vertical component of surface tension = σ × Cosθ × πd_____(2)

$\frac{\pi}{4} \times d^2 \times h \times \rho g = \sigma \times \cos\theta \times \pi d$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

For capillary fall:



Hydrostatic force $= \frac{\pi}{4} \times d^2 \times p$

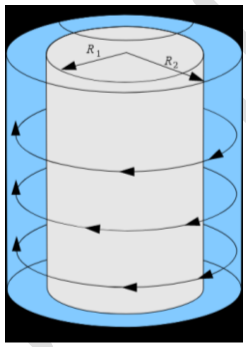
$= \frac{\pi}{4} \times d^2 \times \rho g h$ ____________(1)

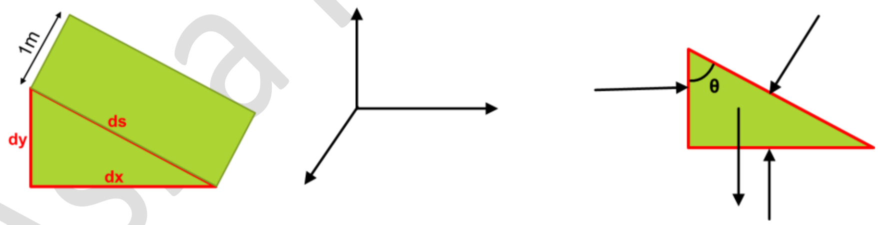
Downward component of surface tension

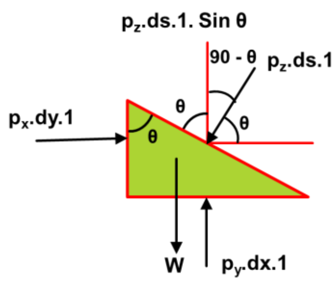
= σ × Cosθ × πd_____(2)

$\frac{\pi}{4} \times d^2 \times h \times \rho g = \sigma \times \cos\theta \times \pi d$

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

<table>
<tr><td>1c</td><td colspan="2">A 100 mm diameter cylinder rotates concentrically inside a 105 mm diameter fixed cylinder. The length of both cylinder is 250 mm. Find the viscosity of the liquid that fills the space between the cylinders, if a torque of 1 Nm is required to maintain a rotating seed of 120 rpm. (08 marks)</td></tr>
<tr><td></td><td>
Given:
$T = F \times distance$
$1 = F \times r$
$12 = F \times 0.05$
$or\ F = 240\ N$
$\frac{F}{A} = \mu \frac{du}{dy}$
$\tau = \frac{F}{A} = \frac{240}{\pi \times 0.1 \times 0.25}$
$\tau = 3055.77\ N/m^2$
$v = r\omega$
$v = 0.050 \times \frac{2 \times \pi \times 120}{60} = 0.628\ m/s$
$\tau = \mu \times \frac{du}{dy}$
$3055.77 = \mu \times \frac{0.628}{2.5 \times 10^{-3}}$
μ = 12.16 Ns/m^2 =121.6 Poise
</td><td>
Given:
dy = 2.5×10^{-3} m
$T = 1\ Nm$
N = 120 rpm
L = 0.25 m


</td></tr>
<tr><td>2a</td><td colspan="2">State and prove Pascal's law for the intensity of pressure at a point in a static fluid 06 marks)</td></tr>
<tr><td></td><td colspan="2">
Pascal's law: It states that the intensity of pressure at a point in a static fluid is equal in all directions. vThe forces acting on the triangular wedge is shown below:


$Weight\ of\ the\ wedge = \rho g \times \frac{1}{2} \times (dx.dy.1)$
Need to resolve the forces in horizontal and vertical directions.
Resolving the forces in X direction:
p_x. dy. 1 $- p_z.\ ds.Cos\theta = 0$
p_x. dy. 1 $- p_z.\ ds.\frac{dy}{ds} = 0$
p_x. dy $= p_z.\ dy$
$p_x = p_z$


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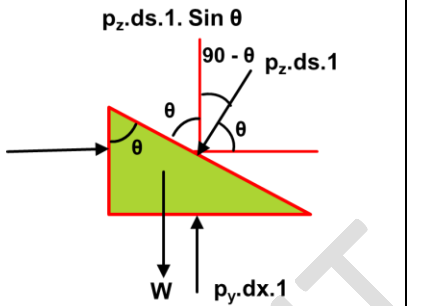
$\boldsymbol{Weight\ of\ the\ wedge} = \boldsymbol{\rho g}\times\mathbf{1/2}\times(\boldsymbol{dx.dy}.\mathbf{1})$



Need to resolve the forces in horizontal and vertical directions.

Resolving the forces in Y direction:

$$p_y.\text{dx}.1 - p_z.\ ds.Sin\theta - \rho g \times \frac{1}{2} \times (dx.dy.1) = 0$$

$$p_y.\text{dx}.1 - p_z.\ ds.\frac{dx}{ds} - \rho g \times \frac{1}{2} \times (dx.dy.1) = 0$$

$$p_y.\text{dx}.1 - p_z.\ dx - \rho g \times \frac{1}{2} \times (dx.dy.1) = 0$$

$$p_y\ = p_z = p_x$$

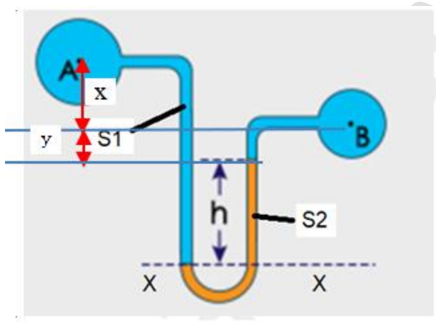
Since the element is very small (dx.dy) will be very small and can be neglected

This infers that fluid pressure is same in all directions

2b **Derive an expression for difference in pressure between two points using a U-tube differential manometer (08 marks)**

Both manometers are used for measuring pressure difference between two points. Differential U tube manometer contains a manometric fluid which is denser than the fluid flowing in pipe.

Inverted U tube manometer is same in construction as like differential manometer but one major difference is that the manometric fluid is lighter than the fluid flowing through pipe because this manometer is mounted inverted in between points for measuring the pressure difference. This is used for measuring the pressure difference between underground pipe lines



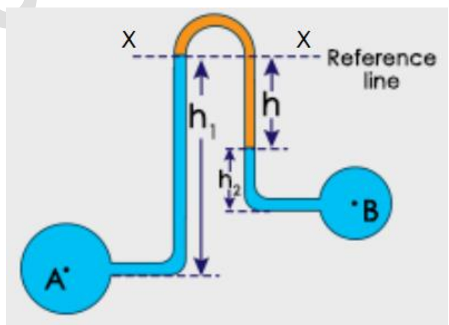
Let the specific gravity of lighter fluid be S1 and that of denser fluid be S2. Let XX be the reference axis.

Equating the pressures for both limbs

$$P_A + S1\rho gx + S1\rho gy + S1\rho gh = P_B + S1\rho gx + S2\rho gh$$

$$P_A + \cancel{S1\rho gx} + S1\rho gy + S1\rho gh = P_B + \cancel{S1\rho gx} + S2\rho gh$$

$$P_A - P_B = (S2 - S1)\rho gh - S1\rho gy$$



Let the specific gravity of lighter fluid be S1 and that of denser fluid be S2. Let XX be the reference axis.

Equating the pressures for both limbs

$$P_A - S1\rho gh1 = P_B - S1\rho gh2 + S2\rho gh$$

$$P_A - P_B = S2\rho gh - S1\rho gh2 + S1\rho gh1$$

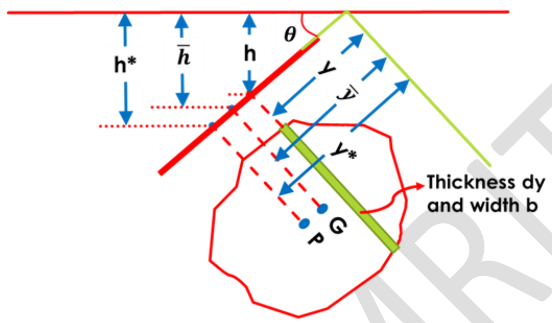
2c **Determine the pressure intensity at the bottom of a tank fille with an oil of specific gravity 0.7 to a height of 10 m. (06 marks)**

Pressure intensity = $S\rho gh = 0.6 = 0.7 \times 1000 \times 9.81 \times 10 = 68.67\ kPa$

MODULE 2

3a **Define: (i) Total pressure (ii) Center of pressure (04 Marks)**

The total pressure is defined as the force exerted by a static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always

	normal to the surface. The centre of pressure is defined as the point of application of the resultant pressure on the surface.
3b	**Derive an expression for total pressure and center of pressure for an inclined plane surface submerged in a liquid. (08 marks)**
	From the figure:  $Sin\theta = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \frac{h}{y}$ $h = ySin\theta$ Total pressure: $Pressure\ intensity\ on\ the\ strip = \rho gh$ $Area\ of\ strip = bdy$ $Total\ force\ on\ the\ strip = \rho gh.bdy = \rho g.dAh$ $Total\ force\ on\ the\ whole\ surface = \int \rho g.dAh$ Total pressure: $Total\ force\ on\ the\ whole\ surface = \int \rho g.dAh$ $Total\ force\ on\ the\ whole\ surface = \int \rho g.dAySin\theta = \rho gSin\theta.\int dAy$ $= \rho gSin\theta.A\frac{\bar{h}}{Sin\theta} = \rho gA\bar{h}$ $Total\ force\ on\ the\ whole\ surface = \rho gA\bar{h}$ Centre of pressure: $Moment\ of\ force\ F\ about\ O-O = F.y^*$________(2) $Moment\ of\ force\ dF\ about\ O-O = \rho gh.bdy.y$ $= \rho gySin\theta.dA.y = \rho gSin\theta.dA.y^2$ $Moment\ of\ force\ about\ O-O = \int \rho gSin\theta.dA.y^2$ $\int dA.y^2 = I_0$ $F.y^* = \rho gSin\theta.I_0$ $y^* = \frac{\rho gSin\theta.I_0}{F} = \frac{\rho gSin\theta.I_0}{\rho gA\bar{h}}$ $y^* = \frac{Sin\theta.I_0}{A\bar{h}}$

$$\frac{h^*}{Sin\theta} = \frac{Sin\theta. I_0}{A\bar{h}}$$

$$h^* = \frac{Sin^2\theta}{A\bar{h}}[I_G + A\bar{y}^2]$$

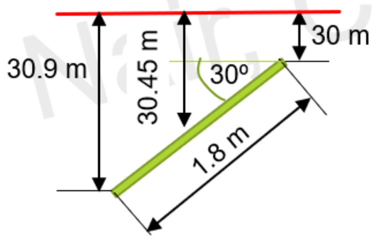
$$h^* = \frac{Sin^2\theta}{A\bar{h}}\left[I_G + A\frac{\bar{h}^2}{Sin^2\theta}\right]$$

$$h^* = \left[\frac{I_G Sin^2\theta}{A\bar{h}} + \bar{h}\right]$$

3c **A 1200 mm 1800 mm size rectangular plate is immersed in water with an inclination of 30° to the horizontal. The 1200 mm side of the plate is kept horizontal at a depth of 30 m below the water surface. Compute the total pressure on the surface and the position of center of pressure. (08 marks)**

$$\boldsymbol{A = 1.2 \times 1.8 = 2.16\, m^2}$$

$$\boldsymbol{I_G = \frac{1.2 \times 1.8^3}{12} = 0.58\, m^4}$$



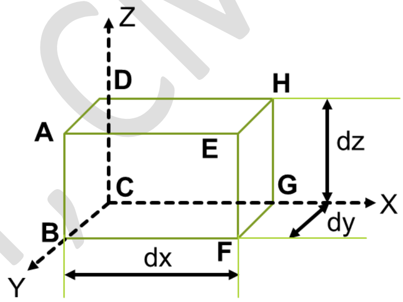
$$\boldsymbol{Total\ pressure = \rho g A\bar{h}}$$
$$\boldsymbol{= 9810 \times 2.16 \times 30.45}$$
$$\boldsymbol{Total\ pressure = 645.22\ kN}$$
$$\boldsymbol{h^* = \left[\frac{I_G Sin^2\theta}{A\bar{h}} + \bar{h}\right]}$$
$$\boldsymbol{h^* = \left[\frac{0.58 \times 0.5^2}{2.16 \times 30.45} + 30.45\right]}$$
$$\boldsymbol{h^* = 30.452\ m}$$

4a **Differentiate between:**
(i) Uniform and non-uniform flow ,
(ii) Steady and unsteady flow (04 Marks)

1	Uniform and non-uniform flow	Type of flow in which the velocity at any given point of time does not change with respect to space (along flow direction) is uniform flow. $\left[\frac{\partial V}{\partial s}\right]_{t=constant} = 0;$ Type of flow in which the velocity at any given point of time changes with respect to space (along flow direction) is non uniform flow. $\left[\frac{\partial V}{\partial s}\right]_{t=constant} \neq 0$

	2	Steady and unsteady flow	Type of flow in which the fluid characteristics like velocity, pressure, density etc at a point do not change with time is steady flow. $\left[\frac{\partial V}{\partial t}\right]_{x_0, y_0, z_0} = 0;$ $\left[\frac{\partial \rho}{\partial t}\right]_{x_0, y_0, z_0} = 0;$ $\left[\frac{\partial p}{\partial t}\right]_{x_0, y_0, z_0} = 0;$ Unsteady flow is that type of flow in which the fluid characteristics like velocity, pressure, density etc at a point change with respect to time. $\left[\frac{\partial V}{\partial t}\right]_{x_0, y_0, z_0} \neq 0;$ $\left[\frac{\partial \rho}{\partial t}\right]_{x_0, y_0, z_0} \neq 0;$ $\left[\frac{\partial p}{\partial t}\right]_{x_0, y_0, z_0} \neq 0;$

4b	**Derive continuity equation for a three- dimensional flow in Cartesian coordinates. (08 Marks)**



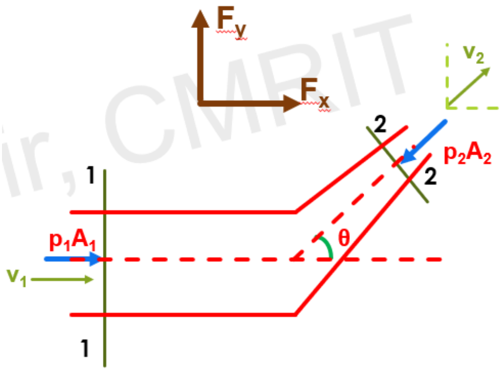
Let u, v and w be the velocity components in x, y and z directions.

$$Mass\ of\ fluid\ entering\ the\ face\ ABCD\ per\ second = \rho u.Area\ of\ ABCD$$
$$Mass\ of\ fluid\ entering\ the\ face\ ABCD\ per\ second = \rho u.(dy.dz)$$
$$Mass\ of\ fluid\ leaving\ the\ face\ EFGH\ per\ second$$
$$= \rho u.(dy.dz) + \frac{\partial(\rho u.(dy.dz).\ dx}{\partial x}$$
$$Gain\ of\ mass\ in\ x - direction = \rho u.(dy.dz) - \rho u.(dy.dz) - \frac{\partial(\rho u.(dy.dz).\ dx}{\partial x}$$
$$Gain\ of\ mass\ in\ x - direction = -\frac{\partial(\rho u).dx.dy.dz}{\partial x}$$
$$Gain\ of\ mass\ in\ y - direction = -\frac{\partial(\rho v).dx.dy.dz}{\partial y}$$
$$Gain\ of\ mass\ in\ z - direction = -\frac{\partial(\rho w).dx.dy.dz}{\partial z}$$
$$Net\ gain\ of\ masses = -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right].dx.dy.dz$$

Net gain in mass = rate of increase of mass of fluid in the element which is given as

$$\frac{\partial \rho}{\partial t}.(dx.dy.dz)$$
$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right].dx.dy.dz = \frac{\partial \rho}{\partial t}.(dx.dy.dz)$$
$$-\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] = \frac{\partial \rho}{\partial t}$$
$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0\ (General\ form)$$

	$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$ $For\ steady\ state,\ \frac{\partial\rho}{\partial t}=0;\ \frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$ $For\ incompressible\ flow, \rho$ is a constant $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$ $For\ 2D\ flow$ $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=\mathbf{0}$
4c	**Evaluate stream function ψ and compute velocity of flow, V, for a two-dimensional flow field given by, u = $4x^3$ and v = - $l2x^2y$ at point (1, 2). Assume ψ= 0 at point (0, 0).**
	$v=\frac{\partial\psi}{\partial x}=-12x^2y$ (1) $-u=\frac{\partial\psi}{\partial y}=-4x^3$ (2) Integrating (1) wrt x $\psi=\frac{-12x^3y}{3}+c1$ (3) Differentiating (3) wrt y $\frac{\partial\psi}{\partial y}=\frac{-12x^3}{3}+\frac{\partial c1}{\partial y}=-4x^3$ (4) Comparing (4) with (2), $\frac{\partial c1}{\partial y}=0$ Or c1 = y Hence $\psi=-4x^3y+y$ $At\ (1,2),\ \ \psi=-4x^3y+y=-4\times1\times2+2=-6$
MODULE 3	
5a	**State Impulse Momentum principle. Give fields where it is applied. (04 Marks)**
	It states that the impulse of a force F acting on a fluid of mass m in short time dt is equal to the rate of change of momentum in the direction of force. This is based on Newton's second law which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. $F=m$a $But\ acceleration,\ a=\frac{dv}{dt}$ $F=\frac{d(mv)}{dt}$___________(1) $F.dt=d(mv)$________(2) Used to determine the resultant force exerted by a flowing fluid in a pipe bend. Used to estimate friction loss in pipes.
5b	**Derive an expression for force exerted by a fluid on a pipe bend. (08 Marks)**



Section 1-1
p_1 ; v_1; A_1
Section 2-2
p_2 ; v_2; A_2
Pressure force exerted at 1-1 = p_1A_1
Pressure force exerted at 2-2 = p_2A_2
Resolving forces in X direction

$$p_1A_1 - p_2A_2Cos\theta + F_x = \rho Q[v_2Cos\theta - v_1]$$

$$F_x = \rho Q[v_2Cos\theta - v_1] - p_1A_1 + p_2A_2Cos\theta$$

Resolving forces in Y direction

$$-p_2A_2Sin\theta + F_y = \rho Q[v_2Sin\theta]$$

Resultant force = $F = \sqrt{F_x^2 + F_y^2}$

$$tan\theta = \frac{F_y}{F_x}$$

$$F_y = \rho Q[v_2Sin\theta] + p_2A_2Sin\theta$$

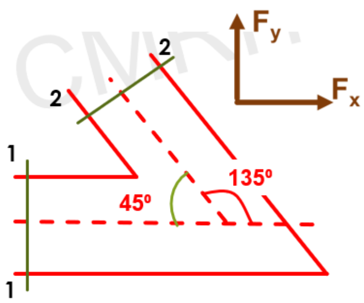
Force exerted by a pipe bend on flowing fluid = $-F_x$ and $-F_y$

c **A pipe of 300 mm diameter, carrying 15000 litres per minute of water is bent by 135°. Find the magnitude and direction of resultant force exerted by the flowing fluid on the bend if the pressure of the flowing water is 39.24 N/cm^2. (08 Marks)**

Given:
Q = 15/60 = 0.25 m^3/s
$d_1 = d_2$ = 0.3 m

$p_1 = 39.24 \times 10^4\ N/m^2$

v A = 0.6



$$0.25 = \frac{\pi}{4} \times 0.3^2 \times v$$

Or $v = 3.54\ m/s$

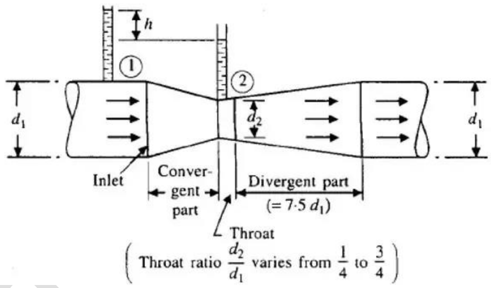
To determine the pressure at 2-2

$$\left[\frac{p}{\rho g} + \frac{v^2}{2g} + z\right]_{1-1} = \left[\frac{p}{\rho g} + \frac{v^2}{2g} + z\right]_{2-2}$$

$Since\ v_1 = v_2\ \ and\ z_1 = z_2, p_1 = p_2 = 39.24 \times 10^4\ N/m^2$

Resolving forces in X direction

$$pA + pACos45 + F_x = \rho Q[-vCos45 - v]$$

	$39.24 \times 10^4 \times 0.071[1 + Cos45] + F_x = -1000 \times 0.25 \times 3.54[1+\text{Cos}45]$ $F_x = -1510.79 - 47560.7 = -49.071$ kN Resolving forces in Y direction $-pASin\theta + F_y = \rho Q[v_2 Sin\theta]$ $F_y = 1000 \times 0.25[3.54Sin45] + 39.24 \times 10^4 \times 0.071Sin45$ $F_y = 19.074\ kN$ Resultant force = $F = \sqrt{F_x^2 + F_y^2} = \sqrt{49.071^2 + 19.074^2} = 52.64\ kN$ $tan\theta = \frac{F_y}{F_x} = \frac{19.074}{-49.071}$ $\theta = 21.24°$
6a	**What is venture effect? Derive an expression for discharge through a venturineter.(08 Marks)**
	The Venturi effect is the reduction in fluid pressure that results when a fluid flows through a constricted section (or choke) of a pipe. The Venturi effect is the reduction in fluid pressure that results when a fluid flows through a constricted section (or choke) of a pipe. The effect utilizes both the principle of continuity as well as the principle of conservation of mechanical energy.  Applying Bernoulli's equation between 1-1 and 2-2 $\left[\frac{p}{\rho g} + \frac{v^2}{2g} + z\right]_{1-1} = \left[\frac{p}{\rho g} + \frac{v^2}{2g} + z\right]_{2-2}$ $z_1 = z_2$ $\left[\frac{p_1}{\rho g} + \frac{v_1^2}{2g}\right]_{1-1} = \left[\frac{p_2}{\rho g} + \frac{v_2^2}{2g}\right]_{2-2}$ $\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$ From the figure, $\frac{p_1 - p_2}{\rho g} = \text{h} = \frac{v_2^2 - v_1^2}{2g}$ Or $v_2^2 - v_1^2 = 2gh$ $a_1 v_1 = a_2 v_2$ $v_1 = \frac{a_2}{a_1} v_2$ Or $v_2^2 - v_1^2 = 2gh$ $v_2^2 - \left[\frac{a_2}{a_1}\right]^2 v_2^2 = 2gh$ $v_2^2 [a_1^2 - a_2^2] = a_1^2 2gh$ $v_2^2 = \frac{a_1^2}{[a_1^2 - a_2^2]} 2gh$ $v_2 = \sqrt{\frac{a_1^2}{[a_1^2 - a_2^2]} 2gh}$

b	**A pitot tube fixed in a pipe of 300 mm diameter is used to measure the velocity and rate of flow. If the stagnation and static pressure heads are 6.0 m and 5.0 m respectively, compute the velocity and rate of flow. Assume Cv= 0.98 for the pitot tube. (06 Marks)**
	h = 6 – 5 = 1 m $c_v = 0.98$ $v = c_v\sqrt{2gh}$ $v = 0.98 \times \sqrt{2 \times 9.81 \times 1} = 4.34\ m/s$ $Q = a\ v = \frac{\pi}{4} \times 0.3^2 \times 4.34 = 0.31\ m^3/s$
c	**A 20 cm x 10 cm venturimeter is used to measure the flow of water in a horizontal pipe. The pressure at the inlet of venturimeter is 17.658 N/cm^2 and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through the venturimeter assuming Cd=0.98. (06 Marks)**
	Given: $d_1 = 0.2$ m; $d_2 = 0.1$ m $c_d = 0.98$ $Q_{act} = c_d a_1 a_2 \sqrt{\frac{2gh}{[a_1^2 - a_2^2]}}$ $a_1 = \frac{\pi}{4} \times 0.2^2 = 31.42 \times 10^{-3}\ m^2$ $a_2 = \frac{\pi}{4} \times 0.1^2 = 7.85 \times 10^{-3}\ m^2$ $\sqrt{a_1^2 - a_2^2} = \sqrt{31.42 \times 10^{-3^2} - 7.85 \times 10^{-3^2}} = 30.42 \times 10^{-3} m^2$ $a_1 a_2 = 31.42 \times 10^{-3} \times 7.85 \times 10^{-3} = 246.65 \times 10^{-6} m^2$ $\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = \frac{17.658 \times 10^4}{9810} + \frac{0.30 \times 13.6 \times 9810}{9810}$ $\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 22.08\ m$ $Q_{act} = c_d a_1 a_2 \sqrt{\frac{2gh}{[a_1^2 - a_2^2]}}$ $Q_{act} = 0.98 \times 246.65 \times 10^{-6} \times \sqrt{\frac{19.62 \times 22.08}{30.42 \times 10^{-3}}}$ $Q_{act} = 28.85 \times 10^{-3}\ m^3/s$
MODULE 4	
7a	**Define hydraulic coefficients for an orifice and give the relation between them. (06 Marks)**
	Hydraulic coefficients are: ➢ Coefficient of velocity, c_v It is the ratio of the velocity of the jet at the vena contracta to the theoretical velocity of the jet. ➢ Coefficient of contraction, cc It is the ratio of the area of the jet at the vena contracta to the area of the orifice.

$$c_c = \frac{a_2}{a_0}$$

- Coefficient of discharge, cd

  It is the ratio of the actual discharge from an orifice to theoretical discharge from orifice.

$$c_d = \frac{Q_{act}}{Q_{th}}$$

Prove that $[c_c \times c_v] = c_d$

$$c_d = \frac{Q_{act}}{Q_{th}}$$

$$Q_{act} = c_d \times Q_{th} = a_{act} \times v_{act}$$

$$Q_{act} = c_d \times Q_{th} = [c_c \times a_{th}] \times [c_v \times v_{th}]$$

$$Q_{act} = c_d \times Q_{th} = [c_c \times c_v] \times [v_{th} \times a_{th}]$$

$$[c_c \times c_v] \times [v_{th} \times a_{th}] = [c_c \times c_v] \times Q_{th} = c_d \times Q_{th}$$

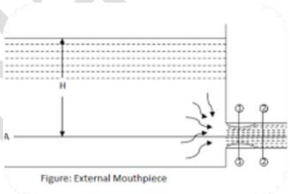
Hence $[c_c \times c_v] = c_d$

b **Give classification of mouth pieces with suitable sketches. (06 Marks)**

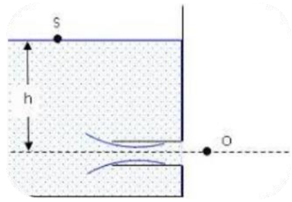
Mouthpiece - It is the short length of a pipe which is two to three times its diameter in length, fitted in a tank/vessel containing the fluid. It is used to measure the rate of flow of fluid

Mouthpieces are classified on the basis of their position with respect to the tank

On the basis of their position with respect to the tank or vessel to which they are fitted, mouthpieces are classified as external mouthpieces and internal mouthpieces.



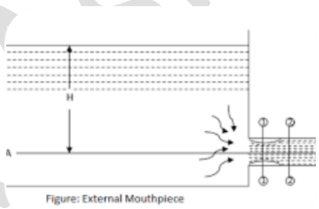
External mouthpiece



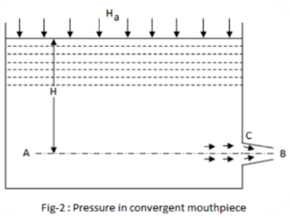
Internal mouthpiece/ Borda's mouthpiece/re-entrant mouthpiece

Mouthpieces are classified on the basis of their shape

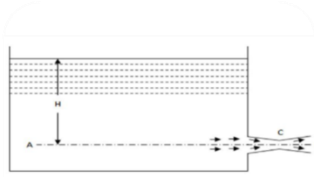
On the basis of their shape, mouthpieces are classified as cylindrical mouthpieces, convergent mouthpieces and convergent-divergent mouthpieces.



Cylindrical mouthpiece



Convergent mouthpiece



Convergent - Divergent mouthpiece

Mouthpieces are classified on the basis of nature of discharge

On the basis of nature of discharge at the outlet of mouthpiece, mouthpieces are classified as mouthpieces running full and mouthpieces running free.

If the jet of liquid after contraction does not touch the sides of mouthpiece, mouthpiece will be termed as running free.

If the jet of liquid after contraction expands and fills the whole mouthpiece, mouthpiece will be termed as running full.

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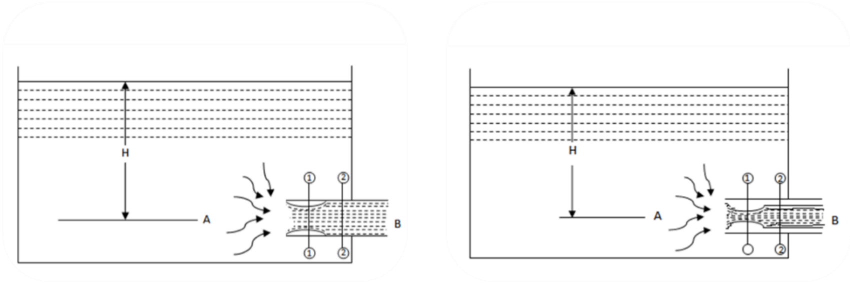

Mouthpiece running full   Mouthpiece running free

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<tr><td>c</td><td>

A jet of water issuing from an orifice 25 mm diameter under a constant head of 1.50 m, falls 0.915 m vertically before it strikes the ground at a horizontal distance of 2.288 m from vena - contracta. The discharge is found to be 102 litres per minute. Calculate the hydraulic coefficients of the orifice. (08 marks)

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Given:

d = 0.025 m

H = 1.5 m

Q = 0.102/60 = 1.7×10^{-3} m^3/s

x = 2.288 m

y = 0.915 m

$$c_v = \frac{x}{\sqrt{4yH}}$$

$$c_v = \frac{2.288}{\sqrt{4 \times 0.915 \times 1.5}}$$

$$c_v = 0.976$$

$$Q_{act} = 1.7 \times 10^{-3}\ m^3/s$$

$$Q_{th} = \frac{\pi \times 0.025^2 \times \sqrt{19.62 \times 1.5}}{4}$$

$$Q_{th} = 2.66 \times 10^{-3} m^3/s$$

$$c_d = \frac{Q_{act}}{Q_{th}}$$

$$c_d = \frac{1.7 \times 10^{-3}}{2.66 \times 10^{-3}}$$

$$c_d = 0.64$$

$$c_c = \frac{c_d}{c_v} = \frac{0.643}{0.976} = 0.66$$

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<tr><td>8a</td><td>

Enumerate advantages of triangular notches over rectangular notches. (04 Marks)

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Triangular notch will provide more precise results during measuring low discharge as compare to result obtained from rectangular notch. This is due to the space for the water to flow through the notch. Only one reading of H will be required for determination of discharge in case of triangular notch. For rectangular notch, the space of water it is constant for every depth.

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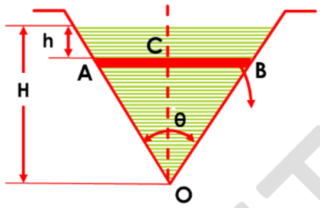
b **Derive the expression for discharge through a triangular notch. (08 Marks)**



$\tan(\theta/2) = \frac{AC}{OC} = \frac{AC}{H-h}$

$AC = (H-h).\tan\theta/2$

Width of the strip = $2(H-h).\tan\theta/2$

Area of strip = $2(H-h).\tan\theta/2$ .dh

Theoretical velocity = $\sqrt{2gh}$

$dQ = c_d \times a_{th} \times v_{th}$

$dQ = c_d \times (2(H-h).\tan\theta/2 \ \text{.dh})\sqrt{2gh}$

$dQ = c_d \times (2(H-h).\tan\theta/2 \ \text{.dh})\sqrt{2gh}$

$\int dQ = \int c_d \times (2(H-h).\tan\theta/2 \ \text{.dh})\sqrt{2gh}$

$\int dQ = 2c_d \times \tan\theta/2 \times \sqrt{2g} \times \int \left[Hh^{\frac{1}{2}} - h^{\frac{3}{2}}\right] dh$

$Q = 2c_d \times \tan\theta/2 \times \sqrt{2g} \times \left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right]_0^H$

$Q = 2c_d \times \tan\theta/2 \times \sqrt{2g} \times \left[\frac{H^{5/2}}{3/2} - \frac{H^{5/2}}{5/2}\right]_0^H$

$Q = 2c_d \times \tan\theta/2 \times \sqrt{2g} \times \frac{2 \times 2 \times H^{5/2}}{15}$

$Q = \frac{8}{15} c_d \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}$

For a right angled notch, θ = 90°, c_d = 0.6

$\tan\theta/2$ = 1

$Q = 1.417 \times H^{5/2}$

c **A river 60 m wide has vertical banks and 1.50 m depth of flow. The velocity of flow is 1.20 m/s. A broad crested weir 2.40 m high is constructed across the river. Find the head on the weir crest considering the velocity of approach. Assume Cd - 0.90. (08 Marks)**

Given:

L = 60 m

H = 1.5 m

c_d = 0.9

A = 60×1.5 = 90 m^2

Q = 90×1.2 = 108 m^3/s

Without considering velocity of approach

$Q_{act} = 1.705 \times c_d \times L \times H^{3/2}$

$108 = 1.705 \times 0.9 \times 60 \times H^{3/2}$

$H = 1.11$ m

By considering velocity of approach

$h_a = \frac{v_a^2}{2g} = \frac{1.2^2}{19.62} = 0.073\ m$

$$108 = 1.705 \times 0.9 \times 60 \times [((H + h_a)^{3/2}) - (h_a)^{\frac{3}{2}}]$$

$$108 = 1.705 \times 0.9 \times 60 \times [((H + 0.073)^{3/2}) - (0.073)^{\frac{3}{2}}]$$

$$[((H+0.073)^{3/2})-(0.073)^{\frac{3}{2}}]=1.17$$

Let H = 1.05

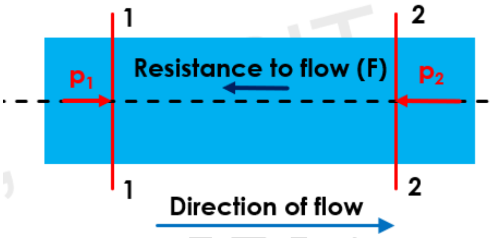
LHS = RHS

Hence H = 1.05 m

MODULE 5

9a **Derive Darcy-Weisbach equation for head loss due to friction in a pipe. (08 Marks)**

Consider uniform horizontal flow of pipe. Applying Bernoulli's equation between (1) and (2)



$$\left[\frac{p}{\rho g}+\frac{v^2}{2g}+z\right]_{1-1}=\left[\frac{p}{\rho g}+\frac{v^2}{2g}+z\right]_{2-2}+loss$$

$z_1=z_2=0; v_1=v_2$

$$\left[\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1\right]=\left[\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2\right]+Losses$$

$$\frac{p_1}{\rho g}-\frac{p_2}{\rho g}=h_f$$

$p_1-p_2=h_f\rho g$_(1)

According to Froude

Frictional force, F is given as

F = frictional resistance/wetted area/unit velocity × wetted area × square of velocity

According to Froude

Frictional force, F is given as

F = frictional resistance/wetted area/unit velocity × wetted area × square of velocity

$F = f' \times \pi dL \times v^2$

Considering equilibrium of forces:

$p_1a_1-p_2a_2=F= f' \times \pi dL \times v^2$

$$p_1-p_2=\frac{f' \times \pi dL \times v^2}{A}\_\_\_\_(2)$$

Substituting (2)*in* (1)

$$p_1-p_2=h_f\rho g=\frac{f' \times \pi dL \times v^2}{A}$$

$$\frac{P}{A}=\frac{\pi d}{\pi d^2/4}=\frac{4}{d}$$

$$h_f=\frac{f' \times 4L \times v^2}{\rho g d}$$

$$\frac{f'}{\rho}=\frac{f}{2}$$

$$h_f=\frac{4fL\,v^2}{2gd}$$

Called as Darcy Weisbach equation

b **List major and minor losses in a pipe flow. (04 Marks)**

Friction loss

	➢ Darcy Weishbach eqn ➢ Chezy's equation Minor losses: ➢ Loss of head due to sudden enlargement ➢ Loss of head due to sudden contraction ➢ Loss of head at the entrance of a pipe ➢ Loss of head at the exit of a pipe ➢ Loss of head due to sudden obstruction in a pipe ➢ Loss of head due to bend in the pipe ➢ Loss of head in various pipe fittings
c	**Water is required to be supplied to a colony of 4000 residents at a rate of 180 litres per person from a source 3 km away. If half the daily requirement needs to be pumped in 8 hours against a friction head of 18 in, find the size of the main pipe supplying water. Assume friction factor as 0.028. (08 Marks)**
	Q = 4000×0.180/(2×8×3600) Q = $12.5 \times 10^{-3} m^3/s$ $18 = \frac{0.028 \times 3000 \times (12.5 \times 10^{-3})^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times d^5}$ d = 0.143 m
10 a	**What is an equivalent pipe? Derive an expression for diameter of an equivalent pipe. (08 Marks)**
	This is defined as a pipe of uniform diameter having loss of head and discharge equal to the loss of head and the discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called as Equiavlent Size of the Pipe. Neglecting minor losses, Total head loss $= \frac{4f_1L_1v_1^2}{2gd_1} + \frac{4f_2L_2v_2^2}{2gd_2} + \frac{4f_3L_3v_3^2}{2gd_3}$ $Q = a_1v_1 = a_2v_2 = a_3v_3$ $Q = \frac{\pi}{4} \times d_1^2 \times v_1 = \frac{\pi}{4} \times d_2^2 \times v_2 = \frac{\pi}{4} \times d_3^2 \times v_3$ $v_1 = \frac{Q}{\frac{\pi}{4} \times d_1^2}$ $H = \frac{4f_1L_1Q^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times d_1^5} + \frac{4f_2L_2Q^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times d_2^5} + \frac{4f_3L_3Q^2}{2g \times \left(\frac{\pi}{4}\right)^2 \times d_3^5}$ $If\ f\ is\ uniform,$ $H = \frac{64fQ^2}{2g \times \pi^2}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$ $Considering\ an\ equivalent\ pipe\ of\ length\ L\ and\ diamter\ d$ $\frac{64fQ^2}{2g \times \pi^2}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] = \frac{64fQ^2}{2g \times \pi^2}\left[\frac{L}{d^5}\right]$ $\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} = \frac{L}{d^5}$ This equation is called as Dupit's equation
b	**Explain phenomenon of water hammer in pipes. (04 Marks)**

	Water hammer is a phenomenon that can occur in any piping system where valves are used to control the flow of liquids or steam. Water hammer is the result of a pressure surge, or high-pressure shockwave that propagates through a piping system when a fluid in motion is forced to change direction or stop abruptly. The hammer effect (or water hammer) can harm valves, pipes, and gauges in any water, oil, or gas application. It occurs when the liquid pressure is turned from an on position to an off position abruptly. When water or a liquid is flowing at full capacity there is a normal, even sound of the flow. Pressure rise is dependent upon: ➢ Velocity of flow of water in the pipe ➢ Length of the pipe ➢ Time taken to close the valve ➢ Elastic properties of the material of the pipe
c	**Water is flowing in a pipe of 150 mm diameter with a velocity of 2.5 m/s, when it is suddenly brought to rest by closing the valve. Find the pressure rise in the pipe assuming it to be elastic with E = 206 GN/m^2 and Poisson's ratio of 0.25. The bulk modulus of water, K = 206 GN/m^2. Thickness of pipe wall is 5 mm. (08 Marks)**
	Elastic pipe: $p = v\sqrt{\frac{\rho}{\left[\frac{D}{Et}+\frac{1}{K}\right]}}$ $p = 2.5\sqrt{\frac{1000}{\left[\frac{0.15}{206\times10^{9}\times5\times10^{-3}}+\frac{1}{206\times10^{9}}\right]}}$ $= 2.5\sqrt{\frac{1000}{[1.46\times10^{-10}+4.85\times10^{-12}]}} = 6.44\ MPa$