CBCS SCHEME

USN

17CV52

Fifth Semester B.E. Degree Examination, Jan./Feb. 2021
Analysis of Indeterminate Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 Analyze continuous beam shown in Fig. Q1, by Slope deflection method. Draw Bending Moment diagram. Take EI constant. **(20 Marks)**

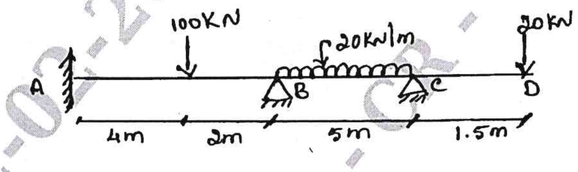

Fig. Q1

OR

2 Analyze the Portal frame shown in Fig. Q2, by Slope Deflection method. Draw bending moment diagram. **(20 Marks)**

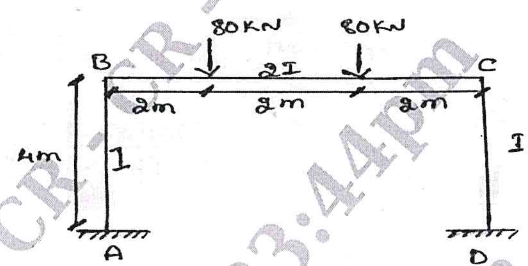

Fig. Q2

Module-2

3 Analyze Continuous beam shown in Fig. Q3, by Moment Distribution method. Draw Bending Moment diagram. Take EI constant. **(20 Marks)**

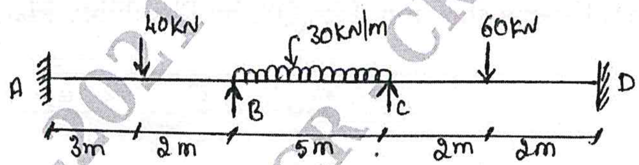

Fig. Q3

OR

4 Analyze Portal frame shown in Fig. Q4, by Moment Distribution method. Draw Bending Moment diagram. **(20 Marks)**

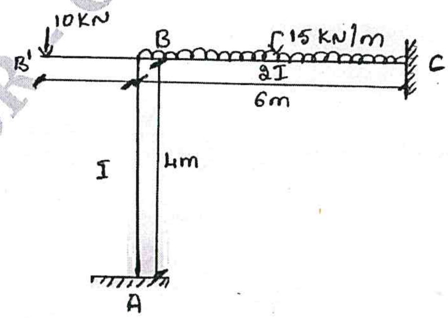

Fig. Q4

Module-3

5 Analyze the Continuous beam shown in Fig. Q5, by Kani's method. Draw Bending Moment diagram. **(20 Marks)**

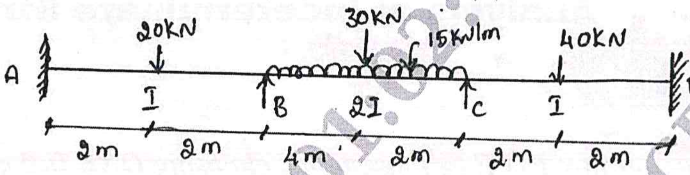

Fig. Q5

OR

6 Analyze the frame shown in Fig. Q6, by Kani's method. Draw Bending Moment diagram. **(20 Marks)**

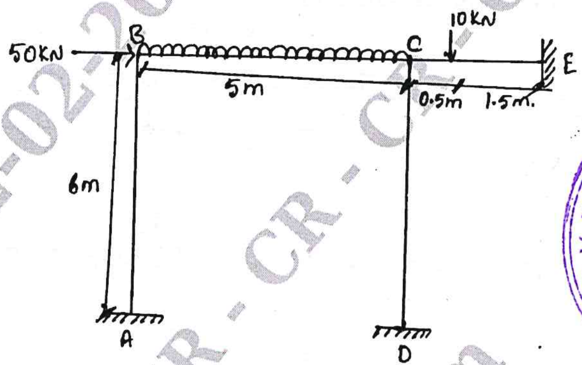

Fig. Q6

Module-4

7 Analyze the beam shown in Fig. Q7, by Flexibility Matrix method. Draw Bending Moment diagram. **(20 Marks)**

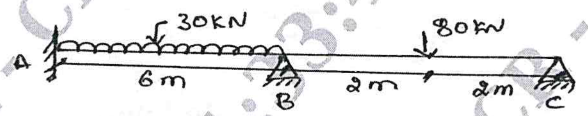

Fig. Q7

OR

8 Analyze Portal frame shown in Fig. Q8, by Flexibility Matrix method. Draw Bending Moment diagram. **(20 Marks)**

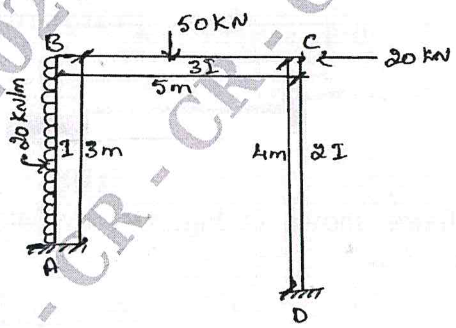

Fig. Q8

Module-5

9 Analyze the beam shown in Fig. Q9, by Stiffness Matrix method. Draw Bending Moment diagram. **(20 Marks)**

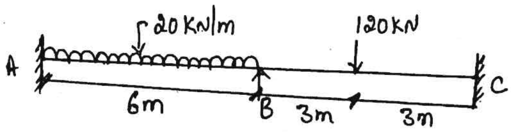

Fig. Q9

OR

10 Analyze Portal frame shown in Fig. Q10, by Stiffness Matrix method. Draw Bending Moment diagram. **(20 Marks)**

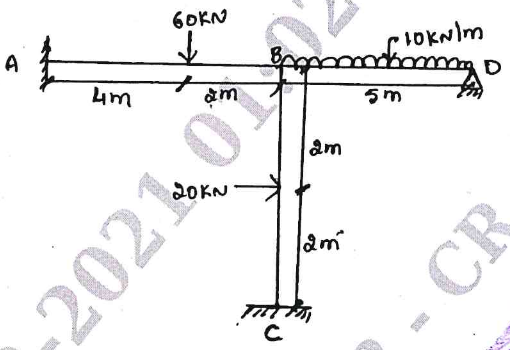

Fig. Q10

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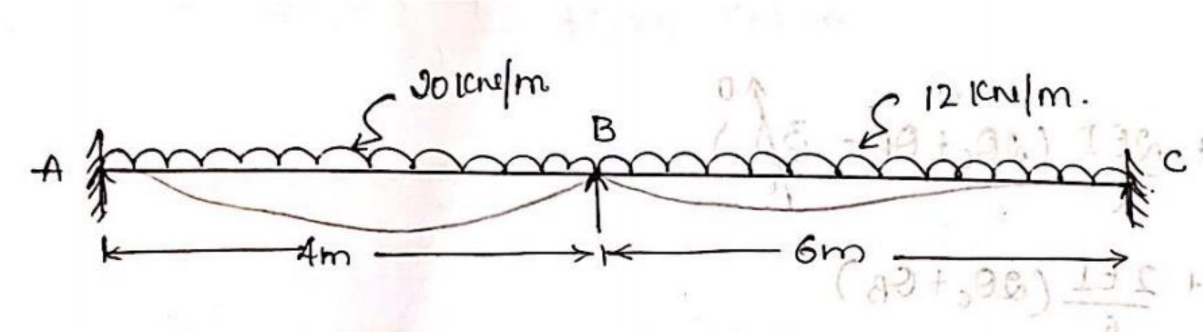
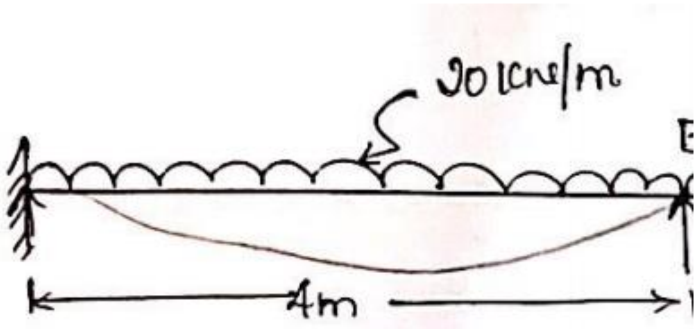
MODULE 1
Slope deflection method

1. A continuous beam ABC is fixed at A and C whereas B is simply supported. The span of AB beam is 4m and BC beam is 6m. The AB member carries an UDL of 20kN/m and span BC carries an UDL of 12kN/m. Find the moment and reaction at supports. Draw the BMD and SFD.



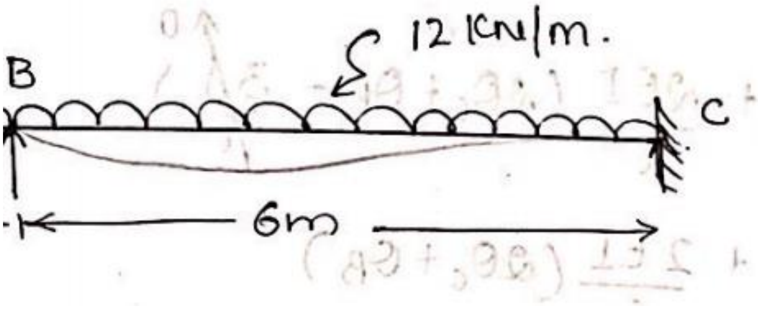
1. Find the FEM



For Span AB

M_{FAB}= - $\frac{wl^2}{12}$ = - $\frac{20\ x\ 4^2}{12}$ = -26.67kN.m

M_{FBA}= $\frac{wl^2}{12}$ = $\frac{20\ x\ 4^2}{12}$ = 26.67kN.m



For span BC

M_{FBC}= - $\frac{wl^2}{12}$ = - $\frac{12\ x\ 6^2}{12}$ = -36kN.m

$M_{FCB} = \frac{wl^2}{12} = \frac{12\ x\ 6^2}{12} = 36$kN.m

2. <u>Apply slope deflection equation :</u>

<u>For Span AB =</u> *l*= 4m, I= I, E= E

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\partial}{l})$

$M_{AB} = M_{FAB} + \frac{2EI}{4} x\theta_B = -26.67 + 0.5EI\theta_B$……………….1

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A - \frac{3\partial}{l})$

$= M_{FBA} + \frac{2EI}{4} x2\theta_B = 26.67 + EI\ \theta_B$…………………….2

<u>For Span BC =</u> *l*= 6m, I= I, E= E

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\partial}{l})$

$= -36 + \frac{2EI}{6} X2\theta_B = -36 + 0.67EI\ \theta_B$……………………3

$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B - \frac{3\partial}{l})$

$= 36 + \frac{2EI}{6}(\theta_B) = 36 + 0.34\ EI\theta_B$ … … … … … … … … .4

3. <u>Joint equilibrium condition</u>

The joints are B

At joint B moments developed are $M_{BA}+M_{BC} = 0$

$26.67 + EI\,\theta_B - 36 + 0.67EI\,\theta_B = 0$

$\theta_B = 5.58/EI$

4. Final moments :

Substitute the value of $\theta_B = 5.58/EI$ in equation 1, 2, 3 and 4

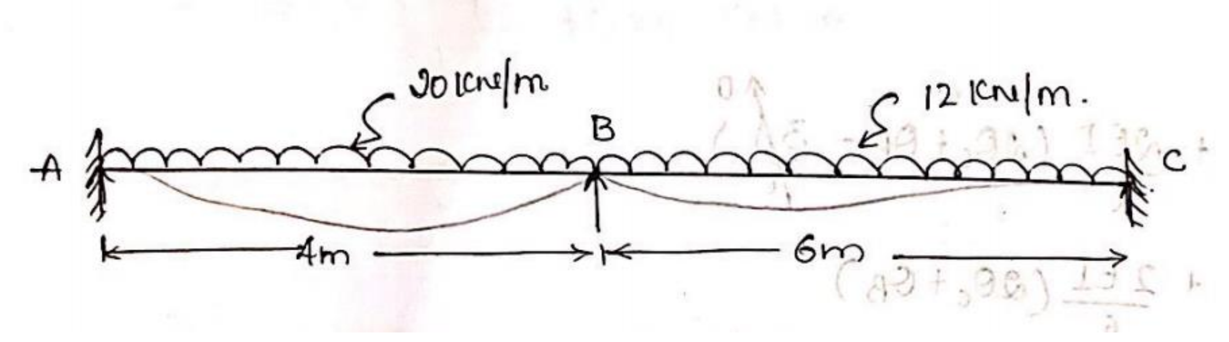
$M_{AB} = -26.67 + 0.5EI\theta_B = -23.81$ kN.m

$M_{BA} = 26.67 + EI\,\theta_B = 32.25$ Kn.m

$M_{BC} = -36 + 0.67EI\,\theta_B = -32.25$Kn.m

$M_{CB} = 36 + 0.34\,EI\theta_B = 37.87$ kN.m

5. Shear force



Consider span AB and span BC

$\sum F_y = V_A + V_B - 20X4 = 0$

LHS of joint B i.e AB span

$\sum M_B = 0 = V_A X\ 4 - 23.81 - 20X\ 4\ X\ 4/2 + 32.25$

$V_A = 37.885$kN

RHS of joint B i.e BC span

$\sum M_B = 0 = -V_c X\ 6 + 37.87 + 12\ X\ 6X6/2 - 32.25$

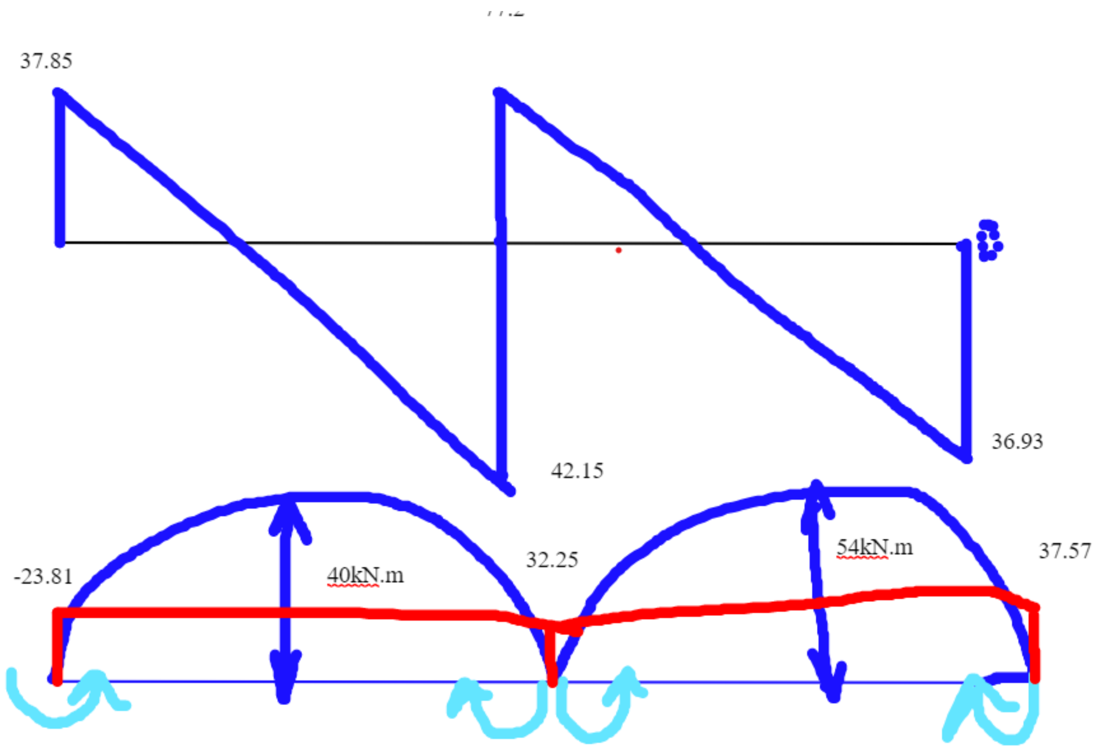
$V_C = 36.93$ kN

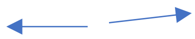
Substitutibg the value of Va and Vc in below eq get Vb

$V_A + V_B + V_c - 20X4 - 12X\ 6 = 0$

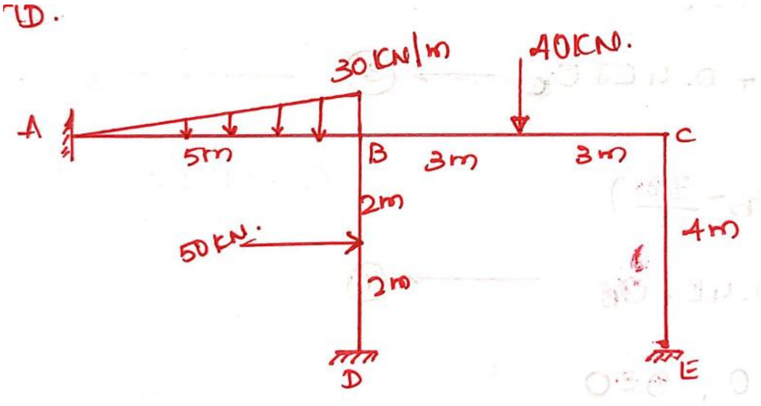
$V_B = 77.2$Kn, $V_A = 37.885$Kn, $V_C = 36.93$ kN,

$V_B = 77.2$kN





P10. Analyse the given FRAME shown in fig using SDM. Draw BMD and SFD



1. Fixed end moments

<u>For span AB</u>

M_{FAB}= -wl^2/30 = -30*5^2/30= -25kN.m

M_{FBA}= wl^2/20 = 30*5^2/20= 37.5kN.m

<u>For span BC:</u>

M_{FBC}= - $\frac{wl}{8}$ =$\frac{40*6}{8}$ = -30kN.m

M_{FCB}= $\frac{wl}{8}$ = 30kN.m

<u>For span BD:</u>

M_{FBD}=wl/8= 25kN.m

M_{FDB}= -wl/8= -25kN.m

2. Slope deflection equation:

For span AB = l= $5m$, $\theta_A = 0$ $becz\ it\ is\ fixed$

M_{AB} = M_{FAB}+ $\frac{2EI}{l}(2\theta_A+\theta_B-\frac{3\partial}{l})$

M_{AB} = -25+0.4EIθ_B…………..1

$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B - \frac{3\partial}{l})$

$M_{BA} = 37.5 + 0.8EI\theta_B$……………….2 4228.7

For Span BC, *l=6m,*

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_c - \frac{3\partial}{l})$

$M_{BC} = -30 + 0.67EI\theta_B + 0.34EI\ \theta_C$……………3

$M_{CB} = M_{FCB} + \frac{2EI}{l}(\theta_B + 2\theta_c - \frac{3\partial}{l})$

$M_{CB} = 30 + 0.34EI\ \theta_B + 0.67EI\theta_C$………………..4

For Span BD , *l=4m,* $\theta_D = 0\ becz\ it\ is\ fixed$

$M_{BD} = M_{FBD} + \frac{2EI}{l}(2\theta_B + \theta_D - \frac{3\partial}{l})$

$M_{BD} = 25 + EI\theta_B$……………3

$M_{DB} = M_{FDB} + \frac{2EI}{l}(2\theta_D + \theta_B - \frac{3\partial}{l})$

$M_{DB} = -25 + 0.5EI\theta_B$………………..4

For Span CE , *l=m,* $\theta_E = 0\ becz\ it\ is\ fixed$

$M_{CE} = M_{FCE} + \frac{2EI}{l}(2\theta_C + \theta_E - \frac{3\partial}{l})$

$M_{CE} = 0 + EI\theta_C$……………5

$M_{EC} = M_{FEC} + \frac{2EI}{l}(2\theta_E + \theta_C - \frac{3\partial}{l})$

$M_{EC} = 0 + 0.5EI\ \theta_C$………………..6

3. Equilibrium conditions at joint

Joints B, C

Let us consider joint B

M_{BA}+ M_{BC}+M_{BD}= 0

37.5+$0.8EI\theta_B - 30 + 0.67EI\theta_B +$
$0.34EI\ \theta_C$ +25+$EI\theta_B$........7

Let us consider joint C

M_{CB} + M_{CE} = 0

30+0.34EI θ_B + $0.67EI\theta_C$+ $EI\theta_C$ 8

Solving eq 7 and 8

θ_B =-10.99/EI

θ_c = -15.72/EI

4. Final moments

Substitute the value θ_B =-10.99/EI, θ_c =-15.72/EI in eq. 1, 2, 3 and 4, 5, 6

M_{AB}= -25+0.4EI$\theta_B = -25 - 0.4EI * 10.99$=-29.4kN.m
M_{BA}= 28.70kN.m
M_{BC}=-42.61kN.m
M_{CB}= 15.78kN.m
M_{BD}= 14.01kN.m
M_{DB}=-30.49kN.m
M_{CE}= -15.72Kn.M
M_{EC}= -7.89

5. Shear force:

All vertical forces
Va-30*5*0.5-40+Vb+Vc=0
=Va+vb+Vc=115
Taking moment about B LHS
Mb=0= Va*5-(30*5/2*(5/3))+28.70-29.4=0

Va= 25.13kN

Taking moment about B RHS

Mc=0=-Vc*6+40*3+15.78-42.61=0

Vc= 15.52

Vb= 74.35

Take moment about B LHS

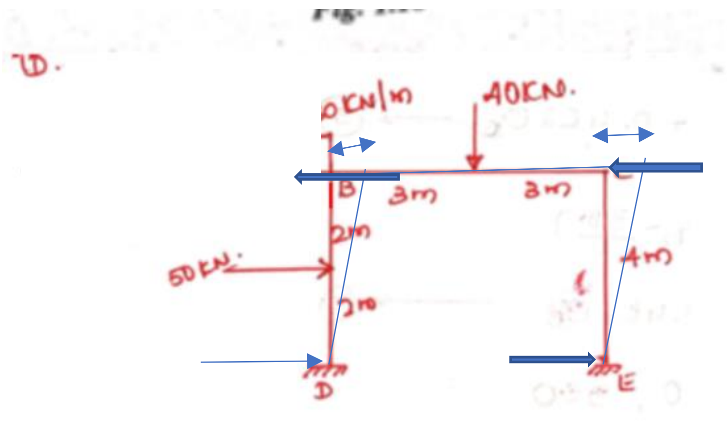
Mb=0= Hd*4-50*2+14.01-30.49=

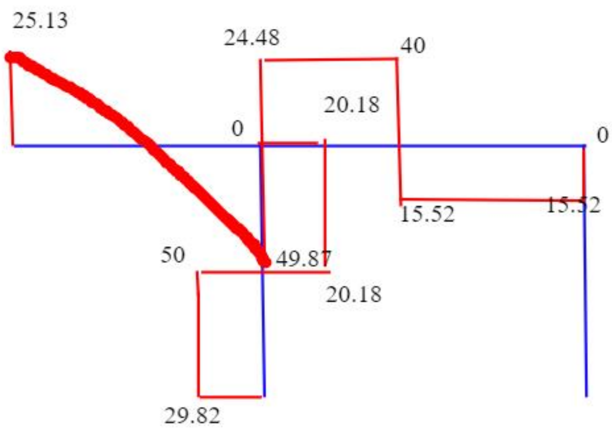
Hd=29.12kN

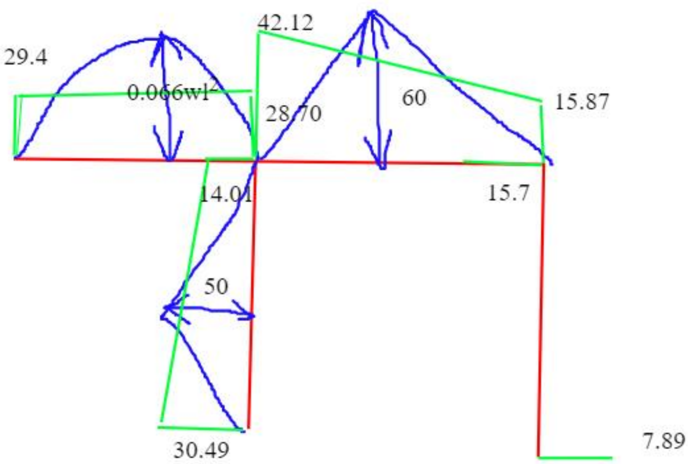
Hb+Hd=50

Hb=b= 50-29.12

Hb= 20.87kN



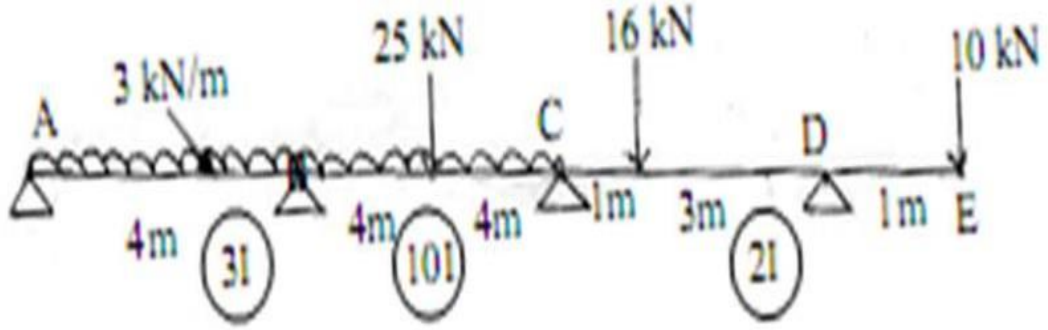




Module 2

MOMENT DISTIBUTION METHOD

Problem 5: Analyse the given continuous beam using MDM



Steps

1. Fixed end moments:
   Mfab= $-wl^2/12$= -4
   Mfba= 4kNm
   Mfbc= $-wl^2/12-wl/8$= -41kN.m
   Mfcb= 41kN.m
   Mfcd= -9
   Mfdc= 3
   Mfde= -10*1=-10kN.m

2. Distribution factor

Joint	*Member*	Relative Stiffness factor (K)	$\sum K$	Distribution factor DF= $\frac{K}{\sum K}$
B	BA	3E3I/4= 2.25EI	7.25EI	0.31
	BC	4E10I/8= 5EI		0.68
C	CB	4E10I/8= 5EI	6.5EI	0.76
	CD	3E2I/4= 1.5EI		0.23

3. Moment distribution table

Joint	A	B	C	D

Member	AB	BA	BC	CB	CD	DC	DE
DF	-	0.31	0.68	0.76	0.23	-	-
FEM	-4	4	-41	41	-9	3	-10
releasing the jt. A Adjusting the overhanging moment at DE	4					7	
		4/2=2			7/2=3.5		
Initial moment.	0	6	-41	41	-5.5	10	-10
Balance	-	10.85	23.8	-26.98	-8.16	-	-
Carry over factor	-	…	-13.49	11.9	….	-	-
Balance	-	4.18	9.17	-9.04	-2.73	-	-
Carry over factor	-	…	-4.52	4.58	…	-	-
Balance	-	1.4	3.07	-3.48	-1.05	-	-
Carry over factor	-	…	-1.74	1.53	…	-	-
Balance	-	0.53	1.18	-1.16	-0.35	-	-
Carry over factor	-	…	-0.58	0.59	…	-	-
Balance	-	0.17	0.39	-0.44	-0.13	-	-
Carry over factor	-	…	-0.22	0.19	…	-	-
Balance	-	0.06	0.14	-0.14	-0.04	-	-
Final moment	0	23.19	-23.8	18.55	-17.96	10	-10

4. Shear force:

$\sum V_F$= V_A+ V_B+ V_C+ V_D= 3*12+25+16+10

MOMENT WRT TO **B** LHS

$\sum M_B$= V_A*4-3*4*2+0+23.19=0

V_A= 0.2kN

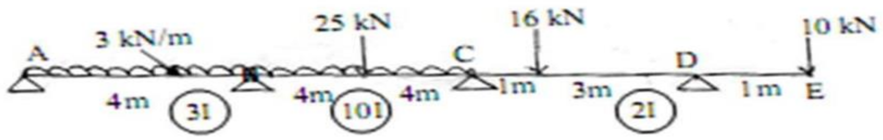
MOMENT WRT TO C LHS

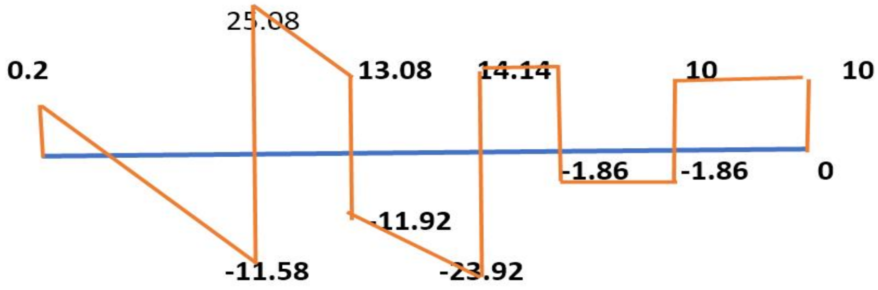
$\sum M_C$= 0= 0.2*12 + V_B*8-3*12*6-25*4+0+18.55

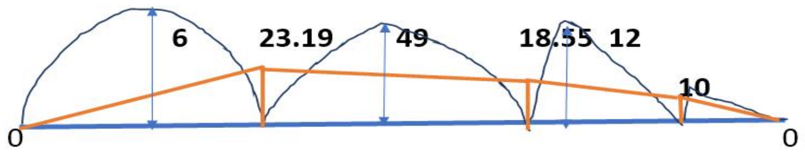
MOMENT WRT TO D LHS

$\sum M_D$= 0= 0.2*16 + 36.88*12-(3*12*(6+4))-25*8+0+10+ V_c*4-16*3

V_c= 38.06kN, V_B= 36.88kN, V_A= 0.2kN V_D= 11.86kN







Final moment	0	23.19	-23.8	18.55	-17.96	10	-10

Problem 9

Fig.Q4(a)

b. Analyse the portal frame shown in Fig.Q4(b) using moment distribution method. Draw bending moment. **(08 Marks)**

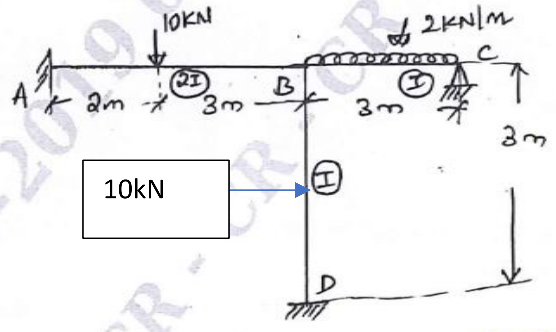


Fig.Q4(b)

Steps

1. Fixed end moments:
   Mfab= -10*2*3*3/5*5= -7.2
   Mfba= 10*2*2*3/5*5= 4.8
   Mfbc= 2*3*3/12= -1.5
   Mfcb= 1.5
   Mfbd= -3.75

Mfdb= 3.75
Distribution factor

Joint	*Member*	Relative Stiffness factor (K)	$\sum K$	Distribution factor DF= $\frac{K}{\sum K}$
B	BA	4E2I/5=1.6EI	3.9EI	1.6/3.9= 0.41
	BC	3EI/3=1EI		1/3.9= 0.25
	BD	4EI/3=1.34EI		1.34/3.9= 0.33

2. Moment distribution table

Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
DF	-	0.41	0.25	0.33	-	-
FEM	-7.2	4.8	-1.5	-3.75	1.5	3.75
Release jt C					-1.5	
			-0.75			
Initial mome	-7.2	4.8	-2.25	-3.75	0	3.75
Balance		0.49	0.3	0.39		
Carry over factor	0.24					0.19
Balance	0	0	0	0	0	0
Final moment	-6.96	5.29	-1.95	-3.36	0	3.94

Shear force:

$\sum V_F$= V_A+ V_B+ V_C= 10+2*3

MOMENT WRT TO B LHS

$\sum M_B$= V_A*5-10*3-6.96+5.29=0

V_A= 6.33

V_B=7.32

MOMENT WRT TO B RHS

$\sum M_B$= - V_c*3+2*3*1.5-1.95+0 = 0

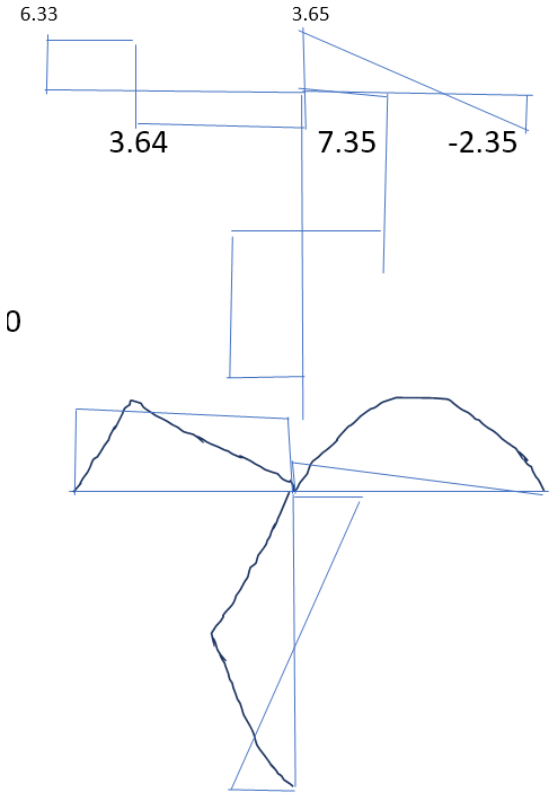
V_c= 2.35

Horizontal loads

H_B+ H_D= 10

Taking the mome wrt B H_D*3-10*1.5-3.36+3.94

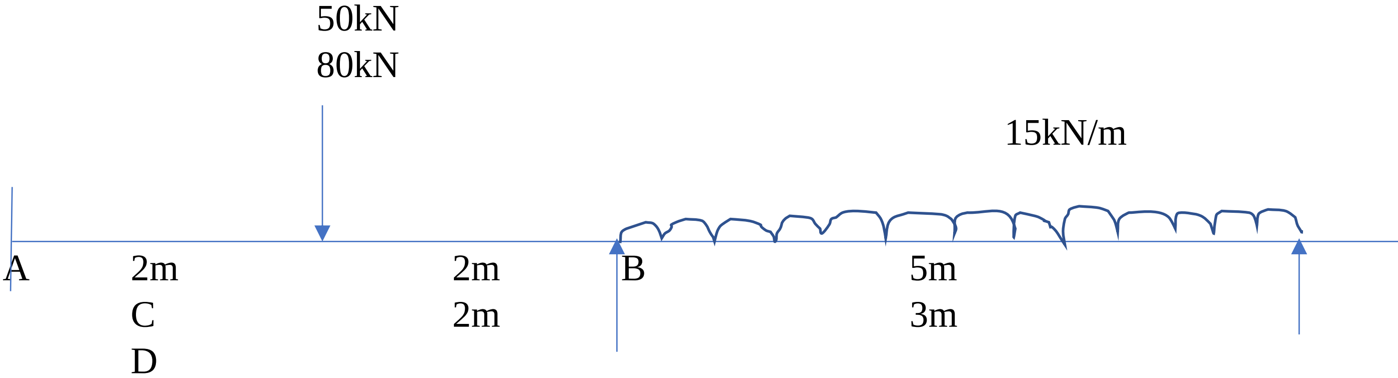
H_D= 5.19kN



MODULE 3

Kani's Method

Problem 1: Analyse the given beam by Kani's method, draw BMD and SFD.



Solution:

1. FEM:

   M_{FAB}= -25

   M_{FBA}= 25

   M_{FBC}= -31.25

   M_{FCB}= 31.25

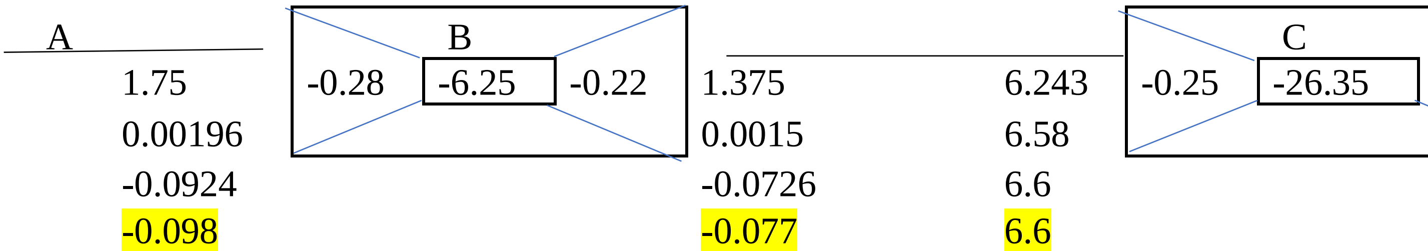
   M_{FCD}= -57.6

   M_{FDC}= 38.44kN.m

2. Rotational factor(U):

Joint	Member	Relative stiffness-k	Total Stiffness- $\sum$k	Rotational factor= U= $-\frac{1k}{2\sum k}$
-B	BA	4EI/L= 4/4=1	1.8	-0.28
	BC	4EI/L=4/5= 0.8		-0.22
C	CB	4EI/L=4/5=0.8	1.6	-0.25
	CD	4EI/L=4/5=0.8		-0.25

3. Rotational Contribution:



Rotational moment: M`= Rotational factor ($\sum M_F$ + far end joint rotational moment)

Traial 1: ba= -0.28(-6.25+0) = 1.750

bc= -0.22(-6.25+0) = 1.375

cb= -0.25(-26.35+1.375)= 6.243

cd= -0.25(-26.35+1.375) =6.243

Traial 2: ba= -0.28(-6.25+6.243) = 0.0028

bc= -0.22(-6.25+6.243) = 0.0015

cb= -0.25(-26.35+1.375) = 6.243

cd= -0.25(-26.35+1.375) =6.243

4. Final moment
    a. M= M_F+2M` (Near end rotational moment) +M` (Far end rotational moment)

M_{AB}= -25+2*0+1*-0.098=

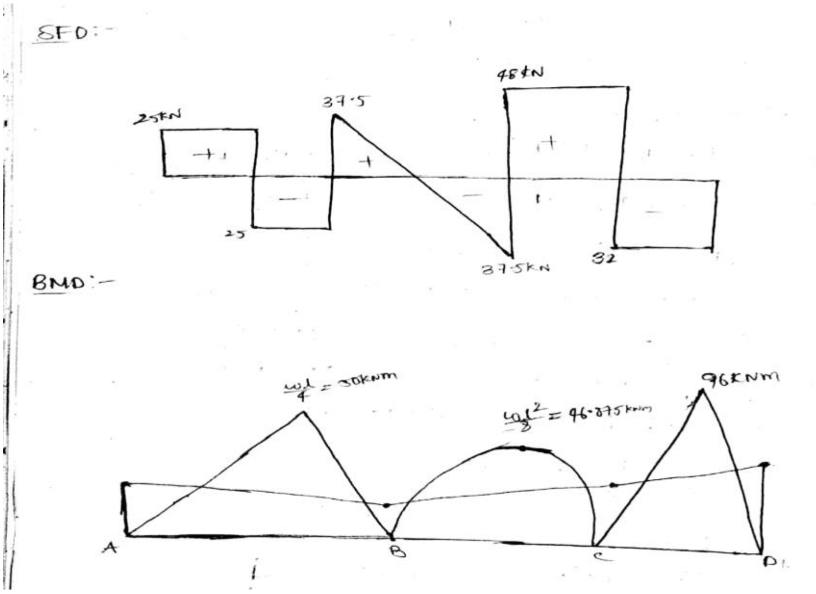
M_{BA}= 25+2*-0.098+1*0=

M_{BC}= -31.25+2*-0.0077+1*6.6=

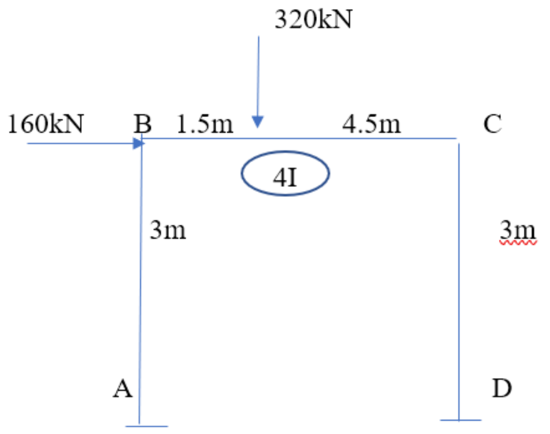
M_{CB}= 31.25+2*6.6-1*0.0077=

M_{CD}= -57.6+2*6.6-1*0=

M_{DC}= 38.44+2(0) +6.6=



Analyse the frame shown in Fig.Q6 by Kani's method. Draw bending moment diagram.



Solution:

1. FEM:
   M_{FAB}= 0
   M_{FBA}= 0
   M_{FBC}= -270kN.m
   M_{FCB}= 90kN.m
   M_{FCD}= 0
   M_{FDC}= 0

2. Rotational factor(U):

Joint	Member	Relative stiffness-k	Total Stiffness- ∑k	Rotational factor= U= $-\frac{1k}{2\sum k}$
B	BA	4EI/L= 4/3= 1.33	4	-0.5*1.33/4= -0.16
	BC	4EI/L= 4*4/6= 2.67		-0.5*2.67/4=-0.33
c	CB	4EI/L= 4*4/62.67	4	-0.5*2.67/4=-0.33
	CD	4EI/L= 4/3= 1.33		-0.5*1.33/4= -0.16

3. Displacement factor

VERTICAL Member	Relative stiffness-k	Total Stiffness- ∑k	Distribution factor = $\frac{k}{\sum k}$	Displacement factor = -3/2 *DF
AB	4EI/L=4/3= 1.33	2.66	0.5	-0.75
CD	4EI/L=4/3=1.33		0.5	-0.75

4. Rotational Contribution:

	B						C	
	-270	-0.33	-270		90	-0.33	90	
	-0.16		89.1		-59.10		-0.16	
		43.2	151.80		-36.59	-28.65		
		73.60	154.59		-27.29	-17.74		
		74.95	152.29		-25.08	-13.23		
		74.17				-12.16		
	-0.75	-130.91				-130.91	-0.75	
		-161.89				-161.89		
		-166.29				-166.29		
		-166.50				-166.50		
		0				0		
	A						D	

Trail 1: Rotational moment: M\`= Rotational factor (∑M_F + far end joint rotational moment)

Storey moment = load *column height/3 = 160*3/3 = 160kN.m

Displacement contribution= Displacement factor*(storey moment+ Rotational moment at top+ bottom)

CD and AB= -0.75(160+(43.2-28.65)+(0+0))= -130.91

Trial 2: for BA= -0.16(-270-59.10-130.91) =

BC= -0.33(-270-59.10-130.91)

CB= -0.33(90+151.80 -130.91)=-36.59

CD= -0.16(90+151.80-130.91)= -17.74

Displacement contribution= Displacement factor*(storey moment+ Rotational moment at top+ bottom)

CD and AB= -0.75(160+(73.6-17.74)+(0+0))= -161.89

Trial 3: for BA= -0.16(-270-36.59-161.89) =74.95

BC= -0.33(-270-36.59-161.89)= 154.59

CB= -0.33(90+154.95 -161.89)=-27.29

CD= -0.16(90+154.59-161.89)= -13.23

Displacement contribution= Displacement factor*(storey moment+ Rotational moment at top+ bottom)

CD and AB= -0.75(160+(74.95-13.23)+(0+0)= -166.29

Trial 4: for BA= -0.16(-270-27.29-166.29) =74.17

BC= -0.33(-270-27.29-166.29)= 152.98

CB= -0.33(90+152.98-166.29)=-25.08

CD= -0.16(90+152.98-166.29)= -12.16

Displacement contribution= Displacement factor*(storey moment+ Rotational moment at top+ bottom)

CD and AB= -0.75(160+(74.17-12.16)+(0+0)= -166.50

5. Final moment
   a. M= M_F+2M` (Near end rotational moment) +M` (Far end rotational moment) +displacement moment

M_{AB}= 0+2*0+1*74.17-166.50 = -92.33

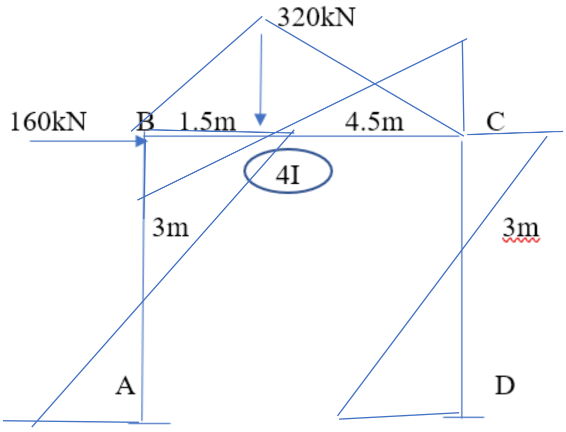
M_{BA}= -270+2*74.17+1*0-166.50= -288.22

M_{BC}= -270+2*74.17+1*-25.08+0=146.8

M_{CB}= 90+2*-25.08+1*74.17+0=114.01

M_{CD}= 90+2*-12.16+1*0-166.50=-100.82

M_{DC}= 0+2*0+1*-12.16-166.50= -178.66



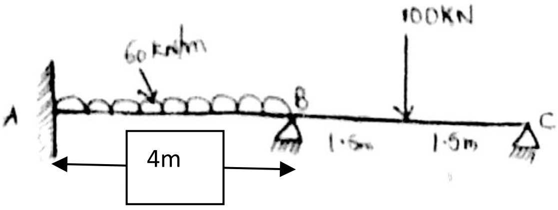
MODULE 4

MATRIX METHOD OF ANALYSIS

FLEXIBILTY METHOD

1. Force method of analysis (also known as flexibility method of analysis,

Problem 1: Analyze the beam by flexibility matrix method.



Step 1:

DOSI = 5-3= 2

Therefore 2 redundant = unknows= M_A and M_B

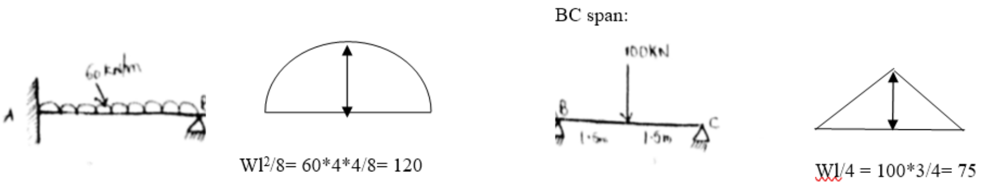
Step 2:

Selection of Redundant : A and B are coordinates 1 and 2

Step 3: Applying conjugate beam method

Split the beam as AB and BC

AB span :



Δ_{1L}= Rotation at A- Shear at A` for conjugate beam = Area of BMD/ EI

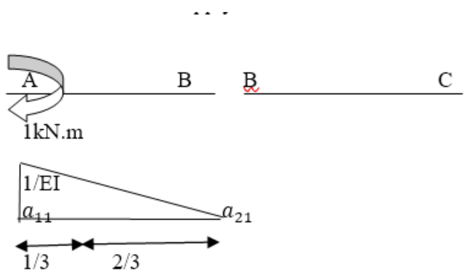
= ½ (2/3* 4*120)/EI = 160/EI

Δ_{2L}= Rotation at B- Shear at B` for conjugate beam = Area of BMD/ EI

= ½ (2/3* 4*120)/EI + ½ (1/2*3*75)/EI = 216.25/EI

Step 4: Getting Flexibility matrix :

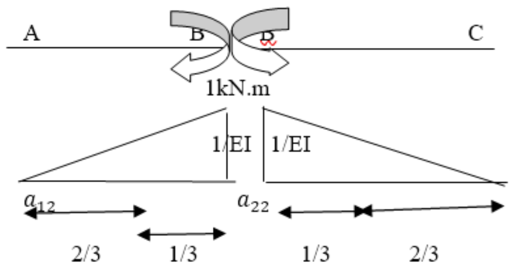
Apply unit moment at coordinate 1



a_{11}= ½*4*1/EI*2/3 = 1.33/EI

a_{21} = ½*4*1/EI*1/3= 0.67/EI

Apply unit moment at coordinate 2



a_{12}= ½ *4*1/EI*1/3 = 0.67/EI

a_{22} = (1/2*4*1/EI*2/3)+(1/2*3*1/EI*2/3)= 2.33/EI

$$[P]= [a]^{-1}[\Delta - \Delta_l]$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} \begin{bmatrix} \Delta - \Delta_{1l} \\ \Delta - \Delta_{2l} \end{bmatrix}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \text{EI} \begin{bmatrix} 1.33 & 0.67 \\ 0.67 & 2.33 \end{bmatrix}^{-1} \frac{1}{EI} \begin{bmatrix} 0 - 160 \\ 0 - 216.25 \end{bmatrix}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{vmatrix} -86.00 \\ -68.08 \end{vmatrix}$$

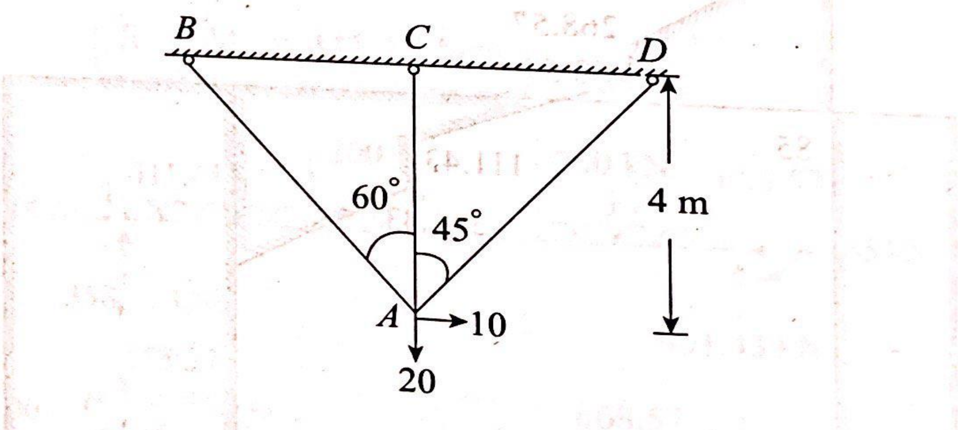
M_{AB}= -86.0kN.m

M_{BA}= 68.08kN.m

M_{BC}=-68.08kN.m

M_{CB}= 0kN.m

Example 4.18 Analyse the truss shown in Fig. 4.129 by the flexibility matrix meth choosing the force member *AD* is redundant. Assume *EA* constant for all membe



Steps:

1. **Determine the static indeterminacy**

3-2=1

2. **Selection of redundant: member AD considered as redundant**

   Length of the members

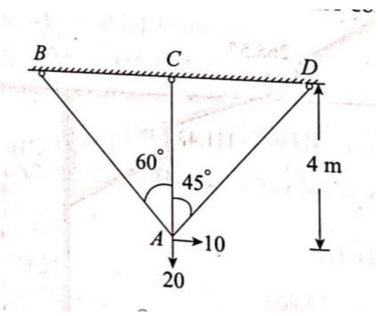
   AB= 8m

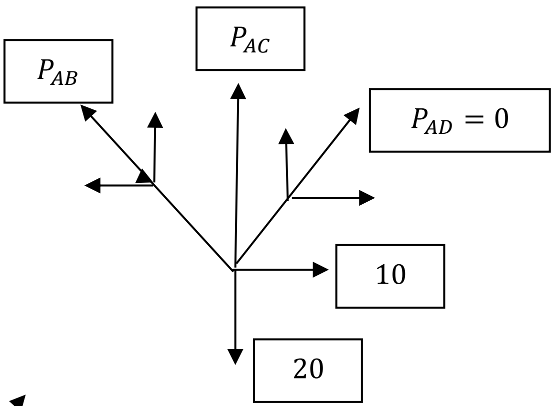
   AC= 4m

   AD= 5.65m

3. **Computing the axial force- actual force (P): by method of joint = P_{AD}=0**

   Consider the joint A





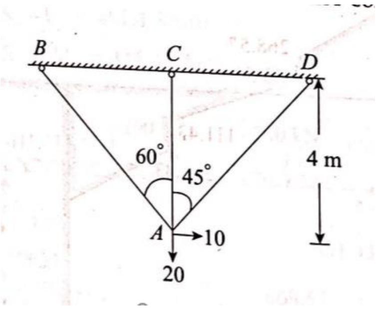
Horizontal forces $\sum H = 0$,= **P_{AD}Sin 45+10- P_{AB}Sin60=0**

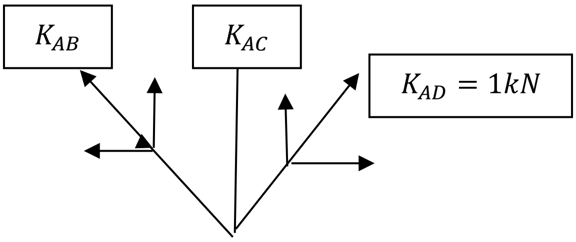
10= P_{AB}Sin60

P_{AB}= 11.54(T)

$\sum V$= 0=,= **P_{AD}Cos 45-20+P_{AB}Cos60+ P_{AC} =0**

4. Computing the axial force – Unit load application at redundant (K)





Horizontal forces = 0,= **K_{AD}Sin 45- K_{AB}Sin60=0**

1Sin 45- K_{AB}Sin60=0

K_{AB}= 0.816kN

$\sum V$= 0=,= **K_{AD}Cos 45+K_{AB}Cos60+ K_{AC} =0**

1Cos 45+0.816Cos60+ K_{AC} =0

5. Force in the member – actual load and unit load

Member	Length(L) m	Area (mm^2)	E (kN/mm^2)	P (kN)	K(kN)	$\frac{PKL}{AE}$	$\frac{K^2L}{AE}$
AB	8	A	E	11.54	0.816	75.33/AE	5.32/AE
AC	4	A	E	14.22	-1.11	-63.13/AE	4.92/AE
AD	5.65	A	E	00	1	0	5.65/AE
						12.26/AE	15.89/AE

6. Calculation of deflection

$$\Delta_L = \sum \frac{PKL}{AE} = 12.26/AE$$

7. Compatibility equation

$$F= \sum \frac{K^2L}{AE} = 15.89/AE$$

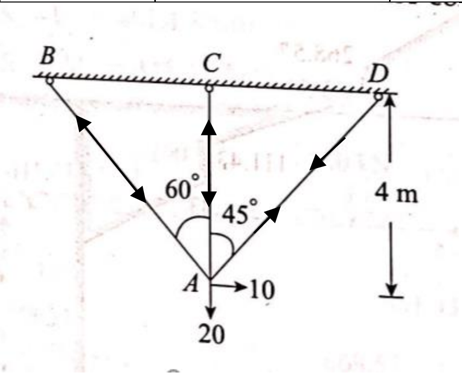
$$\{\Delta - \Delta_L\} = [F]\ \{R\}$$

$\{0\text{-}12.26\} * [15.89]^{-1} = \{R\}$

$$\frac{-12.26}{15.89} = -0.77$$

8. Final moment

Member	P (kN)	K(kN)	R(kN)	P_F=P+KR (kN)	Nature of force
AB	11.54	0.816	-0.77	10.91	T
AC	14.22	-1.11	-0.77	15.07	T
AD	0	1	-0.77	-0.77	C



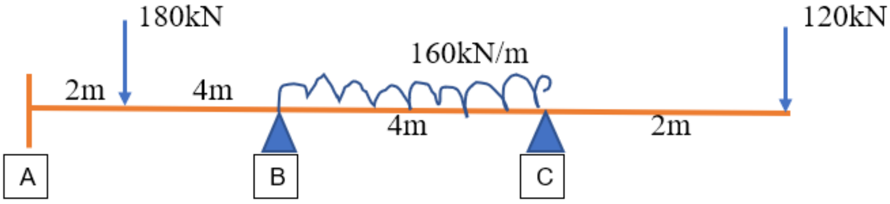
MODULE 5

MATRIX METHOD OF ANALYSIS

STIFNESS MATRIX METHOD

Problem 3:

Analyze the beam shown in fig by displacement method. AB= 2I, BC= I



1. Fixed end moment:
   Mfab= -160kNm
   Mfba= 80kN.m
   Mfbc= -213.33kN.m
   Mfcb= 213.33kN.m

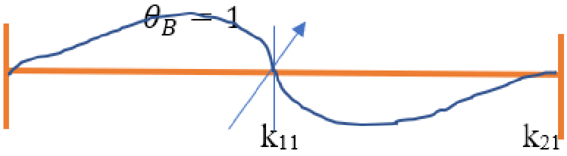
2. [Δ], [P], [P_L]
   a. $[\Delta] = unknown\ displacment\ matix = \begin{bmatrix}\theta_B \\ \theta_C\end{bmatrix}$

   [P]= moments acting -External $= \begin{bmatrix} 0 \\ 120 * 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 240 \end{bmatrix} =$

   [P_L]= joint moments $= \begin{bmatrix} M_{FBA} + M_{FBC} \\ M_{FCB} \end{bmatrix} = \begin{bmatrix} -133.33 \\ 213.33 \end{bmatrix}$

3. Stiffness matrix= to find the θ_B, θ_c

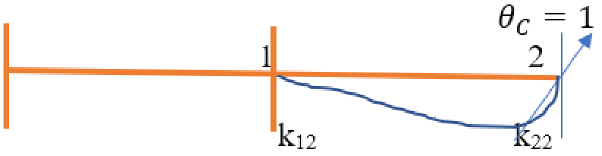
Applying the unit rotation along the co-ordinate 1



$k_{11} = \frac{4E2I}{6} + \frac{4EI}{4} = 2.33EI$

$k_{21} = \frac{2EI}{4} = 0.5EI$

Apply unit rotation at B = coordinate 2



$k_{12} = \frac{2EI}{4} = 0.5EI$

$k_{22} = \frac{4EI}{4} = EI$

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = [EI] \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$[\Delta] = [k]^{-1} [P - P_L]$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 2.33 & 0.5 \\ 0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 - (-133.33) \\ 240 - 213.33 \end{bmatrix}$$

$$\theta_B = \frac{57.68}{EI}, \theta_C = -\frac{2.16}{EI}$$

4. Substitute the above values in the slope deflection equation

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\partial}{l})$

$= -160+2E2I/6[57.68/EI] = -121.54$kN.m

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A - \frac{3\partial}{l})$ = 80+2EI*2/6(2*57.68/EI)= 156.9kn,m

M_{BC}= -156.73 kN.m

M_{CB} = 240 kN.m

5. Shear force and Bending moment diagram

$V_A + V_B + V_C$ = 180+160*4+120

Taking moment about B LHS

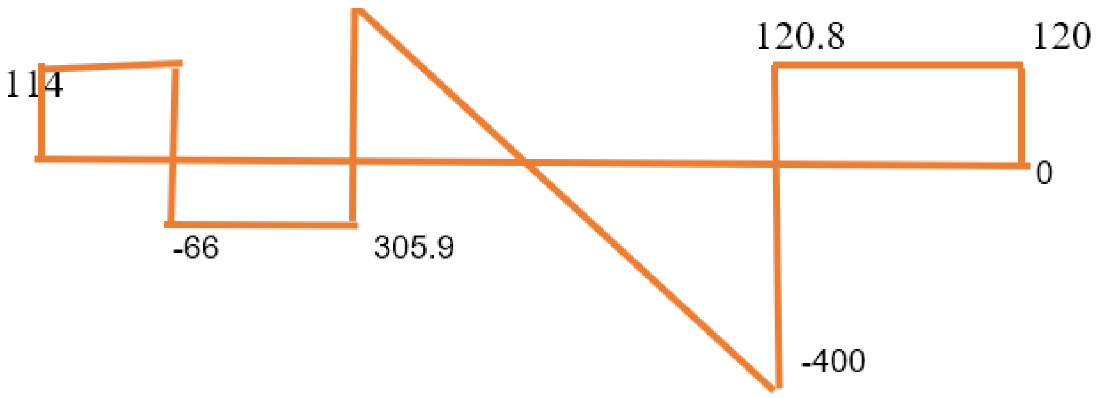
M_B= V_{A*}6-180*4-121.54+156.9=0

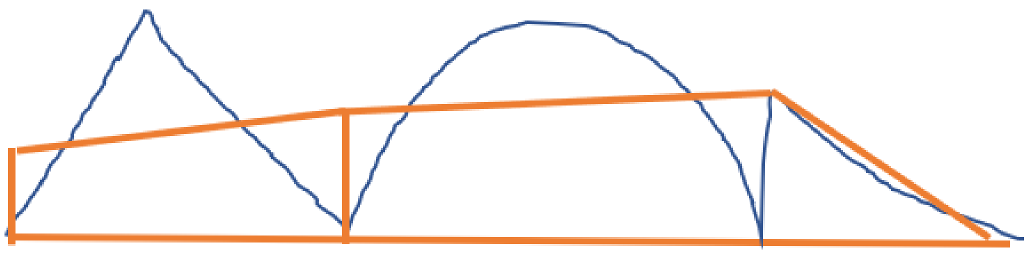
V_A= 114.10

Taking moment about B RHS

M_B= 120*6+160*4*2-156.3+240-V_{C*}4=0

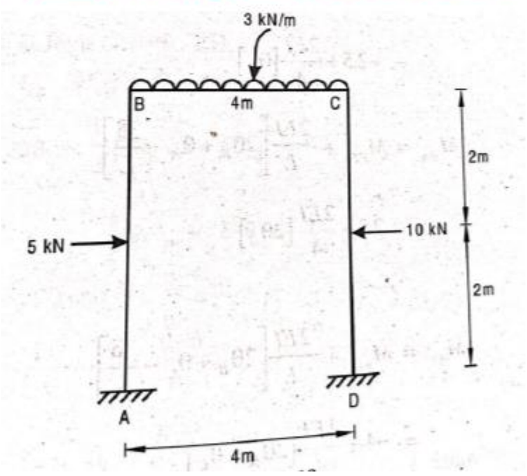
V_C= 520.8kNV_B= 305.9kN





Problem 6:

Analyze the given FRAME shown in fig using SMM. Draw BMD



1. Fixed end moments

For span AB

M_{FAB}= -wl/8 = -5*4/8= -2.5kN.m

M_{FBA}= wl/8 = 5*4/8=2.5kN.m

M_{FBC}= - $\frac{wl^2}{12}$ =-3*4^2/12= -4kN.m

M_{FCB}= $\frac{wl^2}{12}$ = 4kN.m

For span CD:

M_{FCD}=-wl/8= 10*4/8 =-5kN.m

M_{FDC}=5kN.m

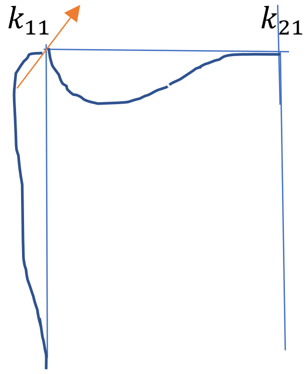
2. [Δ], [P], [P_L]

$$[\Delta] = unknown\ displacment\ matix = \begin{bmatrix}\theta_B\\ \theta_c\end{bmatrix}$$

[P]= moments acting -External $=\begin{bmatrix}0\\0\end{bmatrix}$=

[P_L]= joint moments $=\begin{bmatrix}M_{FBA} + M_{FBC}\\ M_{FCB} + M_{FCD}\end{bmatrix}$= = $\begin{bmatrix}2.5-4\\ 4-5\end{bmatrix}$= $\begin{bmatrix}-1.5\\ -1\end{bmatrix}$

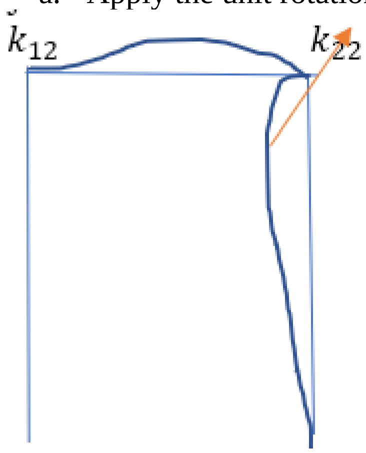
3. Apply the unit rotation at the Coordinate 1 = joint B



k_{11}= $\frac{4EI}{4}$+$\frac{4EI}{4}$= 2EI

k_{21}=$\frac{2EI}{4}$=0.5EI

a. Apply the unit rotation at the Coordinate 2 = joint C



k_{12}= $\frac{2EI}{4}$=0.5EI

k_{22}= $\frac{4EI}{4}$+$\frac{4EI}{4}$= 2EI

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = [EI]\begin{bmatrix} 2 & 0.5 \\ 0.5 & 2 \end{bmatrix}$$

$[\Delta] = [k]^{-1} [P-P_L]$

$$\begin{bmatrix}\theta_B\\ \theta_C\end{bmatrix}=\frac{1}{EI}\begin{bmatrix}2 & 0.5\\ 0.5 & 2\end{bmatrix}^{-1}\begin{bmatrix}0-(-1.5)\\ 0-(-1)\end{bmatrix}$$

$$\theta_B = \frac{0.66}{EI}, \theta_C = \frac{0.33}{EI}$$

4. Substitute $\theta_B = \frac{0.66}{EI}$, $\theta_C = \frac{0.33}{EI}$ in the SDE

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\partial}{l})$

M_{AB} = -2.5+0.5EIθ_B…………..1

$M_{BA} = M_{FBA} + \frac{2EI}{l}(\theta_A + 2\theta_B - \frac{3\partial}{l})$

M_{BA} = 2.5+EIθ_B………………..2

For Span BC , *l=4m,*

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_c - \frac{3\partial}{l})$

M_{BC}= $-4+\frac{2EI}{4}(2\theta_B + \theta_c) = -4 + EI\theta_B + 0.5EI\ \theta_c$……………3

$M_{CB} = M_{FCB} + \frac{2EI}{l}(\theta_B + 2\theta_c - \frac{3\partial}{l})$

M_{CB}=$4 + EI\theta_C + 0.5EI\theta_B$ ……………….4

For Span CD , *l=4m,*

$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D - \frac{3\partial}{l})$

M_{CD}= $-5+\frac{2EI}{4}(2\theta_C) = -5 + EI\theta_C$……………3

$M_{DC} = M_{FDC} + \frac{2EI}{l}(2\theta_D + \theta_C - \frac{3\partial}{l})$

M_{DC}=$5 + 0.5EI\theta_C$……………….4

Substitute the value θ_B =0.67, θ_c =0.33 in eq. 1, 2, 3 and 4

M_{AB}= -2.5+0.5EIθ_B = -2.17kN.m

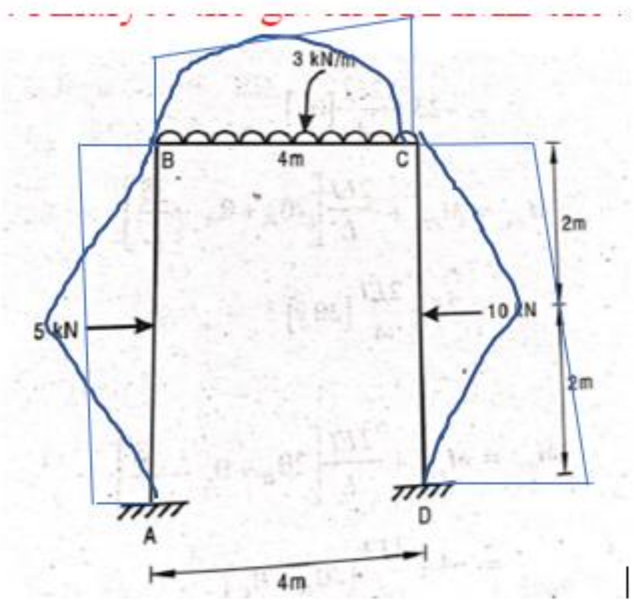
M_{BA}= 2.5+EIθ_B = 3.16kN.m

M_{BC}= $-4 + EI\theta_B + 0.5EI\ \theta_c$ = -3.16kN.m

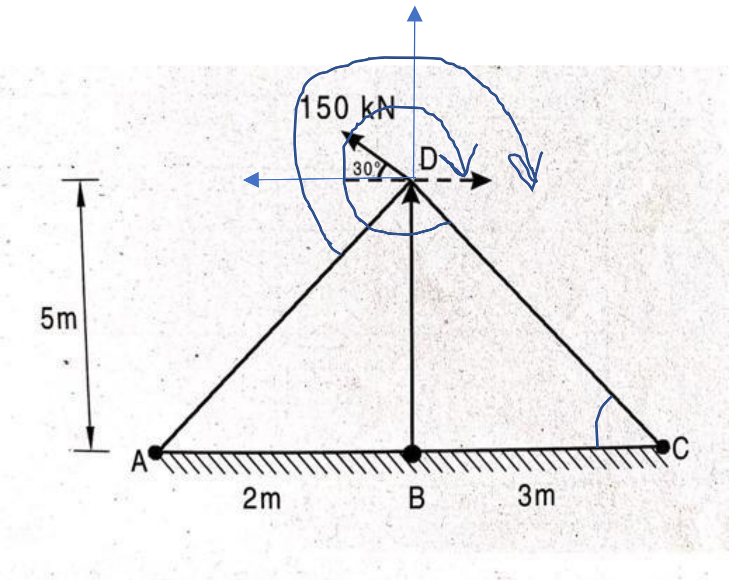
M_{CB}= $4 + EI\theta_C + 0.5EI\theta_B$ = 4.66kN.m

M_{CD} = $-5 + EI\theta_C$ =-4.66kN.m

M_{DC}= $5 + 0.5EI\theta_C$ = 5.16kN.m



1. Analyse the pin jointed truss shown in fig by stiffness method.



B

HY= Sqrt (3*3+5*5)= 5.83m = CD

AD= Sqrt (2*2+5*5)= 5.38m

Horizontal component= 150COS 30= -129.9KN

Vertical component= 150SIN30 = 75KN

Tan θ = BD/BC= 5/3= 60^0

Tan θ= BD/AB= 5/2= 68.19^0

1. Determine the KI or DOF
   a. The joint A, B, C and D are fixed and they cannot move. But joint O is free and can move horizontally and vertically.
      KI= (0+0+0+0+2) = 2

Member	AE	L(mm)	AE/L (kN/mm)	θ	$\frac{AE}{L}cos^2\theta$	$\frac{AE}{L}\cos\theta sin\theta$	$\frac{AE}{L}sin^2\theta$
AD	AE	5380	AE/5380	180+68.19=248.2	$2.36*10^{-5}$AE	$6.41*10^{-5}$AE	$1.60*10^{-4}$AE
BD	AE	5000	AE/5000	270	0AE	0	$2*10^{-4}$AE
CD	AE	5830	AE/5830	360-60=	$4.28*10^{-5}$AE	$-7.42*10^{-5}$AE	$1.28*10^{-4}$AE
Total				$\sum$	k_{11}= $6.84*10^{-5}$AE	k_{12}=k_{21}= -$1.01*10^{-5}$AE	k_{22}= $4.8*10^{-4}$AE

2. [Δ], [P]

$$[\Delta] = unknown\ displacment\ matix = \begin{bmatrix}\Delta_1\\ \Delta_2\end{bmatrix}$$

[P]= External forces $=\begin{bmatrix}-129.9\\ 75\end{bmatrix}$

$$[P] = [\text{k}]\ [\Delta]$$

$$\begin{bmatrix} -129.9 \\ 75 \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix}$$

$$= \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} 6.84 * 10^{-5} & -1.01 * 10^{-5} \\ -1.01 * 10^{-5} & 4.8 * 10^{-4} \end{bmatrix}^{-1} \begin{bmatrix} -129.9 \\ 75 \end{bmatrix}$$

$$\begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} = \frac{1}{AE} \begin{bmatrix} -188.04 * 10^3 \\ 116.55 * 10^3 \end{bmatrix}$$

3. Calculate the member forces

P_{AB}= AE/L($\{\Delta_{Bx}$- $\Delta_{Ax}\}$cos θ_{AB}+$\{\Delta_{By}$- $\Delta_{Ay}\}$sin θ_{AB})=

= $\frac{AE}{5380}$$((0 - (-188.04 * 10^3)$cos248.19) +(0-($116.55 * 10^3$)*sin248.19))/AE

= -106.44kN

P_{Ac}= AE/L($\{\Delta_{cx}$- $\Delta_{Ax}\}$cos θ_{Ac}+$\{\Delta_{cy}$- $\Delta_{Ay}\}$sin θ_{Ac})

= $\frac{AE}{5000}$$((0 - (-188.04 * 10^3)$cos270) +(0-($116.55 * 10^3$)*sin270))/AE

= 22.11kN

P_{AD}= AE/L($\{\Delta_{Dx}$- $\Delta_{Ax}\}$cos θ_{AD}+$\{\Delta_{Dy}$- $\Delta_{Ay}\}$sin θ_{AD})=

=$\frac{AE}{5830}$$((0 - (-188.04 * 10^3)$cos300) +(0-($116.55 * 10^3$)*sin300))/AE

= 176.75kN.

END