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CMR
INSTITUTE
OF TECHNOLOGY

Department of Civil Engineering

Strength of Materials

Third Semester B.E. Degree Examination, Jan/Feb/March 2021

Solution

Module-1

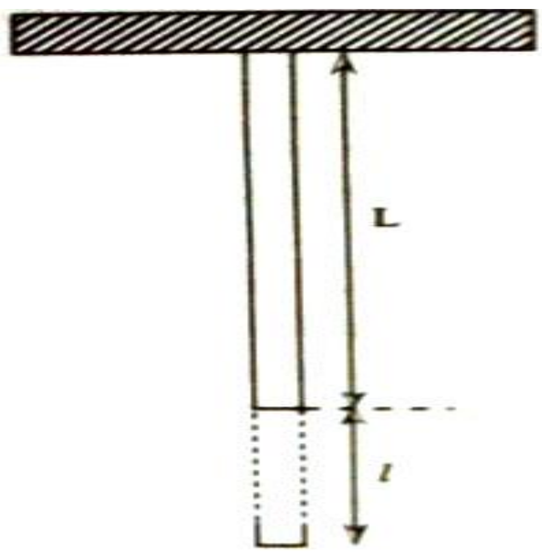
1. a. Explain longitudinal strain and lateral strain (04 Marks)

Solution: Longitudinal strain: If there is a change in the length of a body due to the applied force, then the strain is called longitudinal strain. It is the ratio of the change in length of a body to the original length of the body. It is measured by the change in length per unit length. It is denoted by the Greek letter epsilon $\boldsymbol{\varepsilon}$.

Let the initial length of a body = L

The change in length due to applied force = l (Figure)

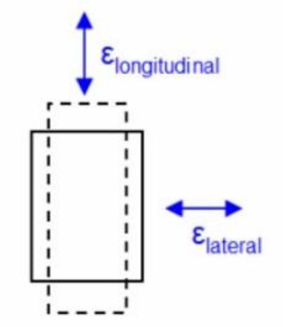
Longitudinal strain = l/L



If the length increases due to tensile stress, the corresponding strain is called tensile strain. If the length decreases due to compressive stress, the strain is called compressive strain.

Lateral strain: Whenever the bar is subjected to the axial load,there will be decrease in the dimensions of the bar in the perpendicular direction of loading. Therefore lateral strain is defined as ratio of decrease in the length of the bar in the perpendicular direction of applied load to that of the original length (gauge length).
i.e, e = dB/B or dD/D

where
e= lateral strain
dd= decrease in depth
D= gauge or original depth
db= decrease in breadth
B = gauge or original breadth



1. b. State and illustrate Saint Venant's principle. (06 Marks)

Solution: In applying the equations for axial loading of members, we have assumed up to this point that we are sufficiently far enough from the point of load application that the distribution of normal stress is uniform. In doing so, we have unknowingly been applying Saint Venant's Principle.

This principle states that: The stresses and strains in a body at points that are sufficiently remote from points of application of load depend only on the static resultant of the loads and not on the distribution of loads.

Point loads on a surface give rise to a stress concentration near the point of application. A stress concentration is an increase in stress along the cross-section that may be caused either by such a point load or by another discontinuity, such as a hole in the material or an abrupt change in the cross-sectional shape. Since we have already shown strain to be proportional to stress, we can get a good idea about the magnitude of normal stress by examining the normal strain in a material as it is being subjected to some loads. To allow this, we can draw lines parallel to the normal plane and see if they remain plane during load application. In each of the following cases, witness how near the discontinuity there is a non-uniform distribution in the strain (and therefore stress) field, while farther away the distribution is linear (ie. the lines remain straight).

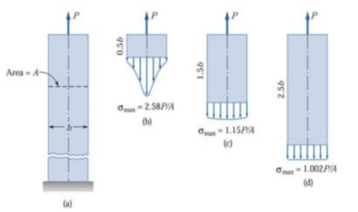


FIG. 1.7 Normal stress distribution in a strip caused by a concentrated load

ILLUSTRATING ST. VENANT'S PRINCIPLE

1. **c. A tension test was conducted on mild steel bar and the following data was obtained from the test:**

   Diameter of the bar = 18mm

   Gauge length of the bar = 82mm

   Load at proportional limit = 75KN

   Extension at a load of 62KN = 0.113mm

   Load at failure = 82 KN

   Final gauf24/ length of the bar = 106mm

   Diameter of the bar at failure = 14mm

   Determine the Young's modulus, proportional limit, true breaking stress, % elongation and percentage reduction in cross sectional area. (10 Marks)

Solution: (i) **Young's Modulus**

Area (A) = $\pi/4(18mm)^2 = 254.46mm^2$

Stress with in proportional limit (σ) = Load / area = $62KN/254.46mm^2 = 243.64\ N/mm^2$

Strain (e) = change in length / original length = 0.11mm / 82mm = 1.34×10^{-3}

Young's Modulus (E) = Stress / Strain = $\sigma/e = 243.64/1.34\text{x}10^{-3} = 1.8 \times 10^{-5}\ N/mm^2$

(ii) Proportionality limit = 75KN

(iii) True breaking stress = Load at failure/Area of cross section = $82KN/254.46\ mm^2 = 322.25\ N/mm^2$

(iii) Percentage elongation = (change in length/ original length)*100 = 29.26%

(iv)Percentage reduction in cross sectional area = $[(d_1^2 - d_2^2)/d_1^2]*100 = [(18^2 - 14^2)/18^2]8100 = 39.50\%$

2. **a. What are the elastic constants and explain them briefly. (06 Marks**

Solution: Elastic Limit: When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed and the body comes back to its origin shape and size, the body is known as elastic body. This property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity. The body will regain its previous shape and size only when the deformation caused by the external force, is within a certain limit. Thus there is a limiting value of force up to and within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material. Hooke's law: It states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit.

Stress /Strain = Constant This constant is known as elastic constant

Types of Elastic Constants:

There are three elastic constants;

Normal stress/ Normal strain = Young'smodulus or Modulus of elasticity (E)

Shear stress/ Shear strain = Shear modulus or Modulus of Rigidity (G)

Direct stress/ Volumetric strain = Bulk modulus (K)

Young'sModulus or Modulus of elasticity (E): It is defined as the ratio of normal stress (σ) to the longitudinal strain (e).

$E = (\sigma n) / (e)$

Modulus of Rigidity or Shear Modulus (G or C): It is the ratio between shear stress (τ) and shear strain (es). It is denoted by G or C.

$G = \tau/es$

Bulk Modulus or Volume Modulus of Elasticity (K): It may be defined as the ratio of normal stress (on each face of a solid cube) to volumetric strain. It is denoted by K. Bulk modulus is a measure of the resistance of a material to change of volume without change of shape or form.

K = Direct Stress / Volumetric strain = σ/ev

2.b. Obtain expression for temperature stress in a bar of uniform cross section when expansion or contraction is prevented. (04 Marks)

Solution: Ordinary materials expand when heated and contract when cooled, hence, an increase in temperature produce a positive thermal strain. Thermal strains usually are reversible in a sense that the member returns to its original shape when the temperature return to its original value.

$\varepsilon_t = L.t\alpha$

Or $\sigma_t = E.\alpha\ t$

α= coefficient of linear expansion for the materialα

L = original Length

t = temp. Change

Thus an increase in temperature produces an increase in length and a decrease in temperature results in a decrease in length except in very special cases of materials with zero or negative coefficients of expansion which need not to be considered here.

If however, the free expansion of the material is prevented by some external force, then a stress is set up in the material. They stress is equal in magnitude to that 37 which would be produced in the bar by initially allowing the bar to its free length and then applying sufficient force to return the bar to its original length.

Change in Length = α L t

Therefore, strain = α L t /L = α t

Therefore, the stress generated in the material by the application of sufficient force to remove this strain = strain x E

or **Stress = αt E**

2.c. A weight of 390KN is supported by a short column of 250mm square in section. The column is reinforced with 8 steel bars of cross sectional area 2500mm^2. Find the stresses in steel and concrete if E_s = 15 E_c.

If stress in concrete must not exceed 4.5 MN/m^2, what area of steel is required in order that column may support a load of 480KN. (10 Marks)

Solution:

$E_s = 15E_c$

A_s= 2500mm^2

A = area of concrete column = 250x250 = 65200mm^2

A_c = 62500 – 2500 = 60000 mm^2

P = 390KN = 390,000N

Condition 1:

Strain in steel = strain in concrete

$$\frac{\sigma s}{Es} = \frac{\sigma c}{Ec}$$

σs = σc x $\frac{Es}{Ec}$ = 15 σc ---- (1) {Es = 15 Ec}

Condition 2: total load = load on steel + load on concrete

P = σsAs + σcAc ---------(2)

390000 = (15σc) x 2500 + 60000σc {substitute σs from (1)}

390000 = 97500σc

σc = 4N/mm^2

σs = 60N/mm^2

If stress in concrete must not exceed 4.5 MN/m^2, what area of steel is required in order that column may support a load of 480KN.

E_s = 15E_c

A = area of concrete column = 250x250 = 65200mm^2

A_c = 62500 – As

P = 480KN = 480,000N

σc = 4.5 MN/m^2 = 4.5 N/mm^2

A_s= ?

σs= 15σc ---- (1)

σs= 15 x 4.5 = 67.5

P = σsAs + σcAc

480000 = 67.5As + 4.5(62500 – As)

⇨ As = 3154.76 mm^2

Module 2

3. a. Derive Lame's equation for the radial and hoop stress for thick cylinder subjected to internal and external pressure. (08 Marks)

Solution:

Lame's Equations are

$\sigma_h = \beta/r^2 + \alpha$
$\sigma_r = \beta/r^2 - \alpha$

Where α and β are Lame's constants

There is a thick vessel subjected to internal fluid pressure p_i at the inner radius r_i. It is under external fluid pressure p_o at the outer radius r_o.
We want to find the equations for σ_h and σ_r at any radius. It is at $r= r_i$, or $r=r_o$ or at any radius between r_i and r_o.

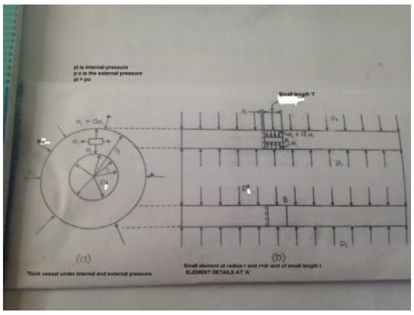


Fig (a)

Consider a long open ended thick walled cylinder in Fig (a) and Fig.(b) subjected inside pressure p_i and external pressure p_o. Inside pressure is high and we are considering the failure of the cylinder lengthwise.

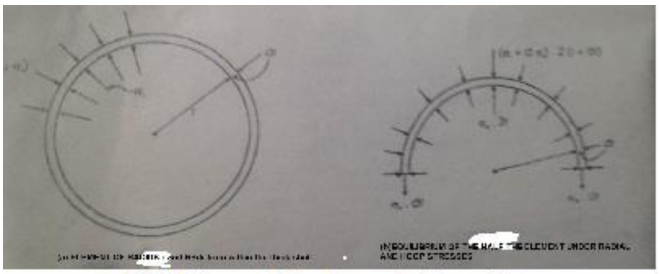


Fig. Element for Lame's Equations

Fig (b)

DERIVATION

Fig.(a) Consider two concentric sections at radius r and r + dr of length 'l' .

Cut this thin ring into two halves.

Fig. b Consider its equilibrium under the forces acting due to σ_h and σ_r on the element

Applying the equation of equilibrium to the half ring,

Sum of downward forces= Sum of upward forces

σ_h dr l+ σ_h dr l + (σ_r+ dσ_r) 2 (r +dr) l= σ_r 2rl

On simplification, we get

2 σ_h dr = –2dr σ_r –2rdσ_r—2dr dσ_r
Divide by 2dr and neglect drdσ_r

We get

σ_h = — σ_r –r (dσ_r/dr) ------------- (1)

ASSUMPTION

That a transverse plane remains a transverse plane before and after the fluid pressures are applied.

As per this assumption, longitudinal strain has to be constant.

ϵ_{long} = σ_l/E — $\mu\sigma_h$/E +μ σ_r/E = constant
σ_l is constant and σ_l/E is small, therefore can be neglected.
Therefore — $\mu\sigma_h$/E +μ σ_r/E = constant

Taking out –μ/E, we get

σ_h — σ_r = constant= 2α --------------- (2)

(2α has been assumed to be constant because of convenience)

Substitute from eq (1) in eq (2), we get

σ_r + 2α = — σ_r –r (dσ_r/dr)
2 σ_r+ r (dσ_r/dr) =– 2α

Multiply each term by r

2r σ_r+ r^2(dσ_r/dr) =– 2α r

Write it in the below form

$d(r^2 \sigma_r)/dr = -2\alpha r$

Integrating, we get

$r^2 \sigma_r = -\alpha r^2 + \beta$

$\sigma_r = \beta/r^2 - \alpha$ -------------------- (3)

Put this value in eq(1), we get

$\sigma_h = \beta/r^2 + \alpha$ -------------------- (4)

Eqs.(3) and (4) are Lames equations.

3.b. A 2- dimensional element has the tensile stresses of 600 MN/m^2 and compressive stress of 400MN/m^{2}acting on two mutually perpendicular planes and two equal shear stresses of 200MN/m^2 on their planes. Determine

i) Resultant stress on a plane inclined at 30^{o}wrt x – axis

ii)The magnitude and direction of principal stresses

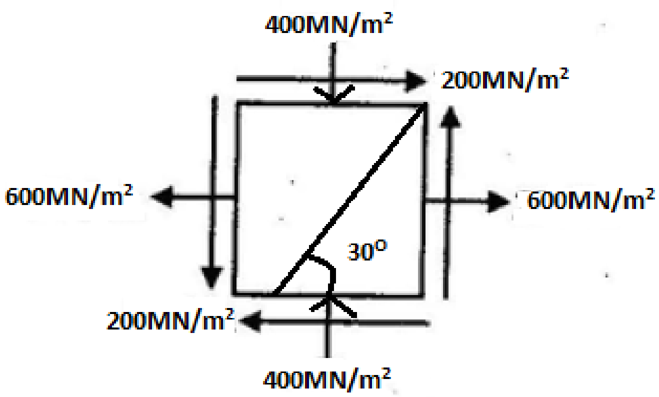
iii) Magnitude and direction of maximum shear stress (12 Marks)

Solution: $\sigma_x = +600\ MN/m^2$

$\sigma_y = -400\ MN/m^2$

$\tau_{xy} = +200\ MN/m^2$

$\varnothing = 90^o - 30^o = 60^o$



i) Resultant stress on a plane inclined at 30°wrt x – axis

$\sigma_n = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2Ø + q \sin 2Ø$

$\sigma_n = \frac{600+(-400)}{2} + \frac{600-(-400)}{2} \cos 2(60^o) + 200 \sin 2(60^o)$

$\boldsymbol{\sigma_n}$ **=**100 -250+173.20 = **23.20MN/m²**

$\sigma_t = \frac{\sigma x - \sigma y}{2} \sin 2Ø + q \cos 2Ø$

$\sigma_t = \frac{600-(-400)}{2} \sin 2(60^o) + 200 \cos 2(60^o)$

$\boldsymbol{\sigma_t}$ **=** 433.012 - 100 = **333.012 MN/m²**

ii) The magnitude and direction of principal stresses

Major Principal Plane = $\frac{\sigma x + \sigma y}{2} + \sqrt{[(\frac{\sigma x - \sigma y}{2})^2 + q^2]}$

$= \frac{600+(-400)}{2} + \sqrt{[(\frac{600-(-400)}{2})^2 + 200^2]}$

$= 100 + \sqrt{[500^2 + 200^2]}$

= 638.51 MN/m²

Minor Principal Plane = $\frac{\sigma x + \sigma y}{2} - \sqrt{[(\frac{\sigma x - \sigma y}{2})^2 + q^2]}$

$= \frac{600+(-400)}{2} - \sqrt{[(\frac{600-(-400)}{2})^2 + 200^2]}$

$= 100 - \sqrt{[500^2 + 200^2]}$

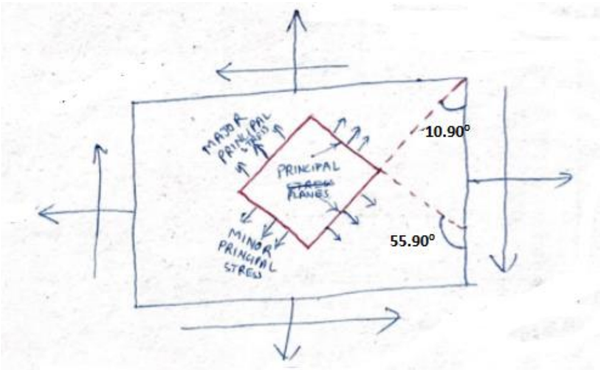
= -438.51 MN/m²

Direction of Principal plane

$\text{Tan} 2\alpha = \frac{2q}{\sigma x - \sigma y}$

$2\alpha = \text{Tan}^{-1}(\frac{2x200}{600-(-400)})$

α = 10.90°, 55.90°



iii) Magnitude and direction of maximum shear stress

Minor Principal Plane = 1/2√[($\sigma_x - \sigma_y$)^2+4q^2]

=1/2√{[(600 – (-400)]^2+4*200^2}

=1/2√{[(**1000**)]^2+160000}

$\sigma_s$max =580000MN/mm^2

Direction of maximum shear stress plane

Tan2β = $\frac{\sigma x - \sigma y}{2q}$

Tan2β = $\frac{600-(-400)}{2*200}$

β = 68.19º

4.a. Obtain expression for volumetric strain in thin cylinder subjected to internal pressure in the form of e_v = $\frac{pd}{2tE}[\frac{5}{2} - \frac{2}{m}]$. (08 Marks)

Solution:

Volumetric Strain(e_v) = change in volume / Original volume

Original Volume (V) = π/4[d^{2}L]

Final volume (V') = π/4[d+d']^2*[L+L']

$$e_v = \frac{\frac{\pi}{4[d+d']2}*[L+L'] - \pi/4[d2L]}{\pi/4[d2L]}$$

$$e_v = \frac{L'}{L} + 2\frac{d'}{d}$$

$$e_v = e_l + 2e_c$$

$$e_v = \frac{pd}{2tE}[\frac{1}{2} - \frac{1}{m}] + \frac{pd}{2tE}[\frac{1}{1} - \frac{1}{2m}]$$

after simplifying

$$e_v = \frac{pd}{2tE}[\frac{5}{2} - \frac{2}{m}].$$

4.b. A cast iron pipe has 200mm internal diameter and 50mm metal thickness and carries water under a pressure of 5N/mm^2. Calculate the maximum and minimum intensities of circumferential stresses and sketch the distribution of circumferential stress intensity and the intensity of radial pressure across the section. (12 Marks)

Solution: given data:

Internal dia. = 200 mm => internal radius r_1=100mm

Thickness = 50mm => external radius r_2=100+50=150mm

Internal pressure p_o= 5N/mm^2

at x = r1, px = p0 = 5N/mm2

The radial pressure (px) is given by equation as

$$p_x = [\frac{b}{x2} - a]$$

now by apply the boundary conditions to the above equation:

at x=r_1=100mm, p_x=5N/mm^2

$$5= [\frac{b}{100^2} - a] \text{---------------------(i)}$$

at x=r_2=150mm, p_x=0N/mm^2

$$0= [\frac{b}{150^2} - a] \text{---------------------(ii)}$$

Solving (i) and (ii) we get

b =90,000

a=4

Now hoop stress at any radius x is given by

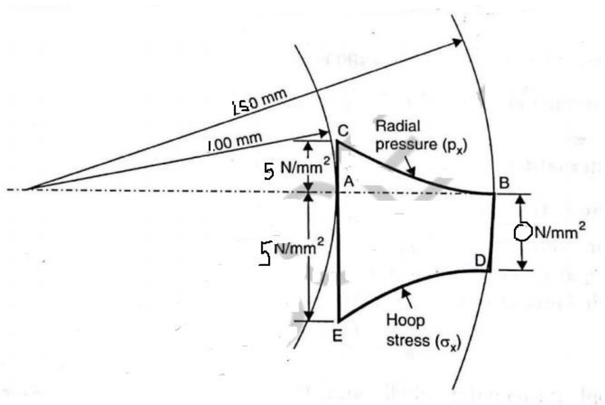
$\sigma_x = [\frac{b}{x2} - a]$

At x=100mm

$\sigma_x = [\frac{90000}{100\hat{}2} - 4]$ = 9-4=5N/mm^2

At x=150mm

$\sigma_x = [\frac{90000}{150\hat{}2} - 4]$ = 4-4=0N/mm^2



5.a. define shear force, bending moment and point of contraflexure. Explain how to calculate them? (06 Marks)

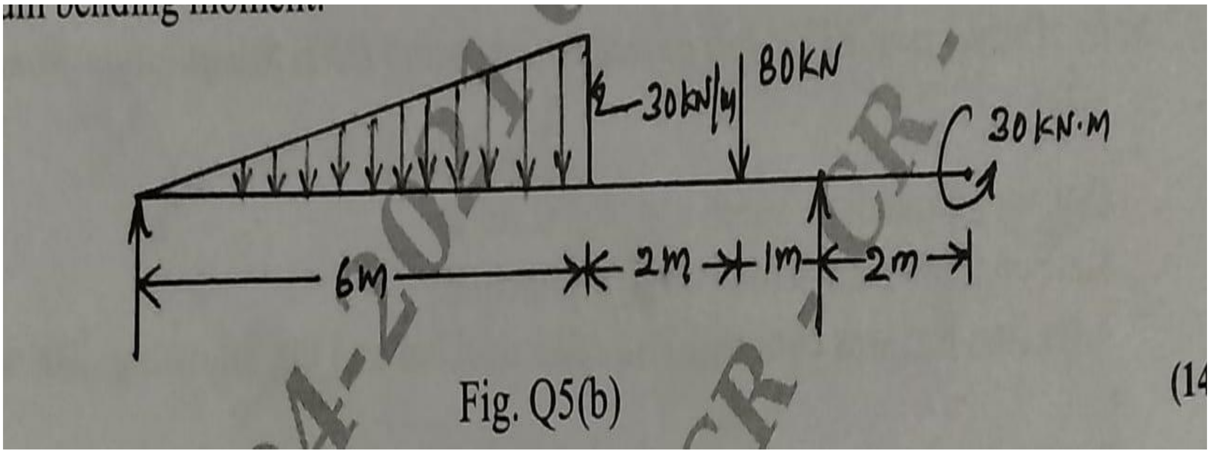
Solution:

Shear Force: The shearing force (SF) is defined as the algebraic sum of all the transverse forces acting on either side of the section of a beam or a frame. The phrase "on either side" is important, as it implies that at any particular instance the shearing force can be obtained by summing up the transverse forces on the left side of the section or on the right side of the section.

Bending Moment: The shearing force (SF) is defined as the algebraic sum of all the transverse forces acting on either side of the section of a beam or a frame. The phrase "on either side" is important, as it implies that at any particular instance the shearing force can be obtained by summing up the transverse forces on the left side of the section or on the right side of the section.

Point of Contraflexure:A point of contraflexure is **a point where the curvature of the beam changes sign**. It is sometimes referred to as a point of inflexion and will be shown later to occur at the point, or points, on the beam where the B.M. is zero.

5.b. develop shear force diagram and bending moment diagram for the beam loaded shown in fig Q5(b) marking the values at salient points. Determine the position and magnitude of maximum bending moment. (14 Marks)



Solution

$R_A + R_D = ½*6*30 + 80 = 170KN$

Take moment about A

$\Sigma M_A = 0 => R_D*9 + 30 = ½*6*30*2/3*6 + 80*8$

$R_D = 107.77$ KN

$R_A = 170 – 107.77 = 62.22$ KN

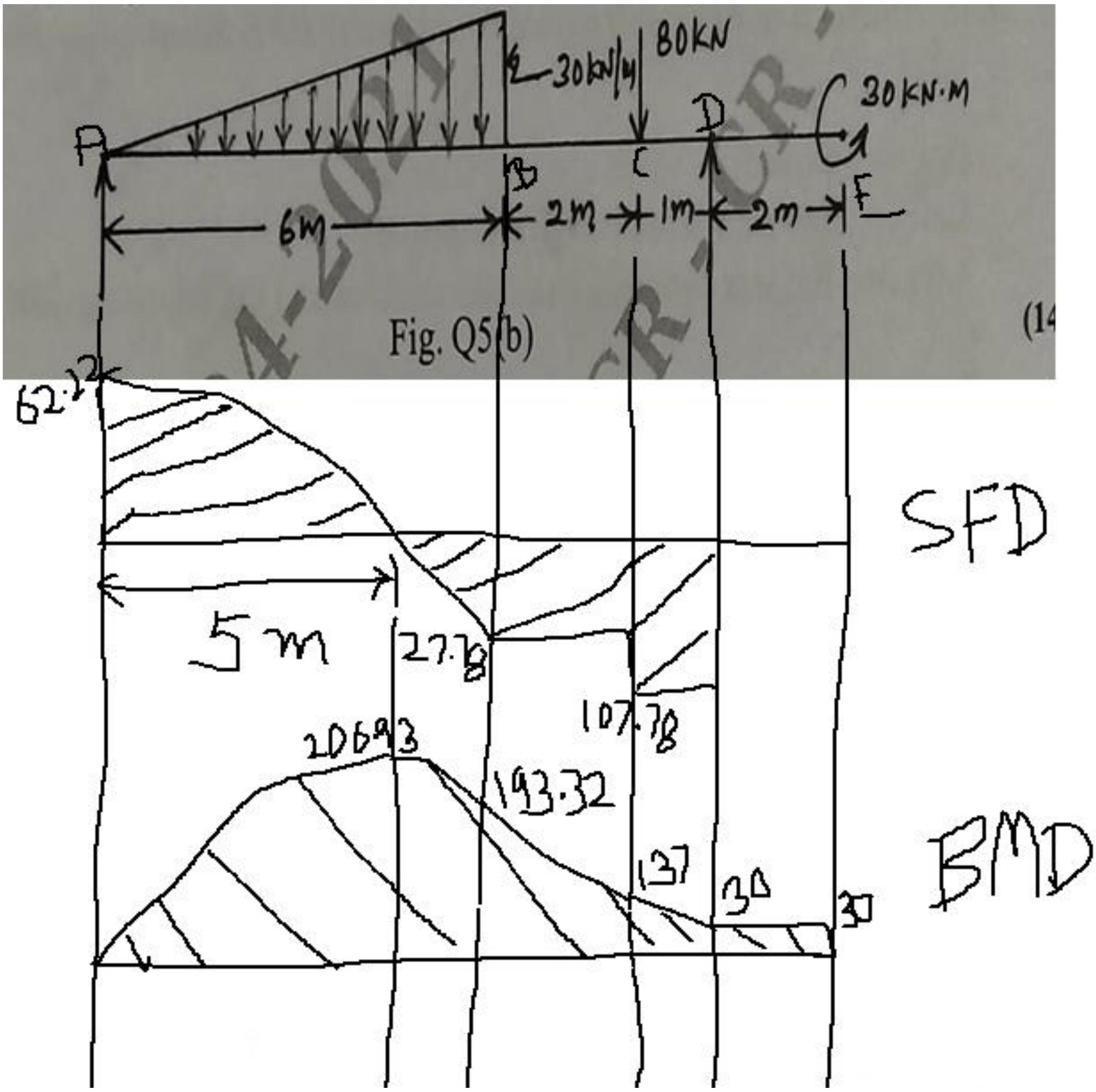
Shear force is zero at

0=62.22 – 1/2x5x => x = 5m

Position and Magnitude of Maximum bending moment

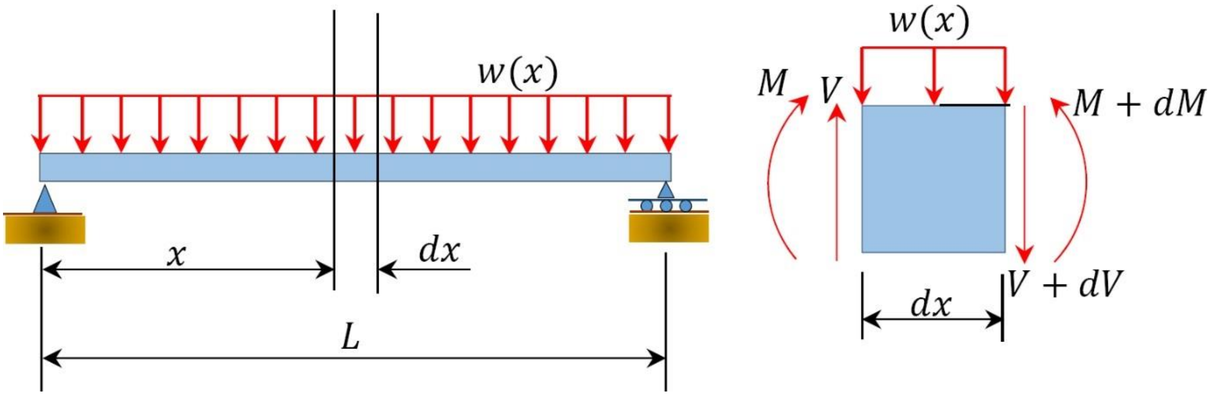
x = 5m

max. bending moment = 206.93 KN-m



6.a. Obtain the relationship between udl, shear force and bending moment. (06 Marks)

For the derivation of the relations among *w*, *V*, and *M*, consider a simply supported beam subjected to a uniformly distributed load throughout its length, as shown in Figure 4.3. Let the shear force and bending moment at a section located at a distance of *x* from the left support be *V* and *M*, respectively, and at a section $x + dx$ be $V + dV$ and $M + dM$, respectively. The total load acting through the center of the infinitesimal length is wdx.



To compute the bending moment at section *x* + *dx*, use the following:

$$M_{x+dx} = M + Vdx - wdx.dx/2$$

$$= M + Vdx \text{ (neglecting the small second order term } wdx^2/2)$$

$$M + dM = M + Vdx$$

or $$\frac{dM}{dx} = V(x) \quad (4.1)$$

Equation 4.1 implies that the first derivative of the bending moment with respect to the distance is equal to the shearing force. The equation also suggests that the slope of the moment diagram at a particular point is equal to the shear force at that same point. Equation 4.1 suggests the following expression:

$$\Delta M = \int V(x)dx \quad (4.2)$$

Equation 4.2 states that the change in moment equals the area under the shear diagram. Similarly, the shearing force at section *x* + *dx* is as follows:

$$V_{x+dx} = V - wdx$$

$$V + dV = V - wdx$$

or

$$\frac{dV}{dx} = -w(x) \quad (4.3)$$

Equation 4.3 implies that the first derivative of the shearing force with respect to the distance is equal to the intensity of the distributed load. Equation 4.3 suggests the following expression:

$$\Delta V = \int w(x)dx \qquad (4.4)$$

Equation 4.4 states that the change in the shear force is equal to the area under the load diagram. Equation 4.1 and 4.3 suggest the following:

$$\frac{d^2M}{dx^2} = -w(x) \qquad (4.5)$$

Equation 4.5 implies that the second derivative of the bending moment with respect to the distance is equal to the intensity of the distributed load.

6.b. construct SFD and BMD for the beam loaded shown in Fig Q6(6). Also locate the point of contraflexure. (14 Marks)

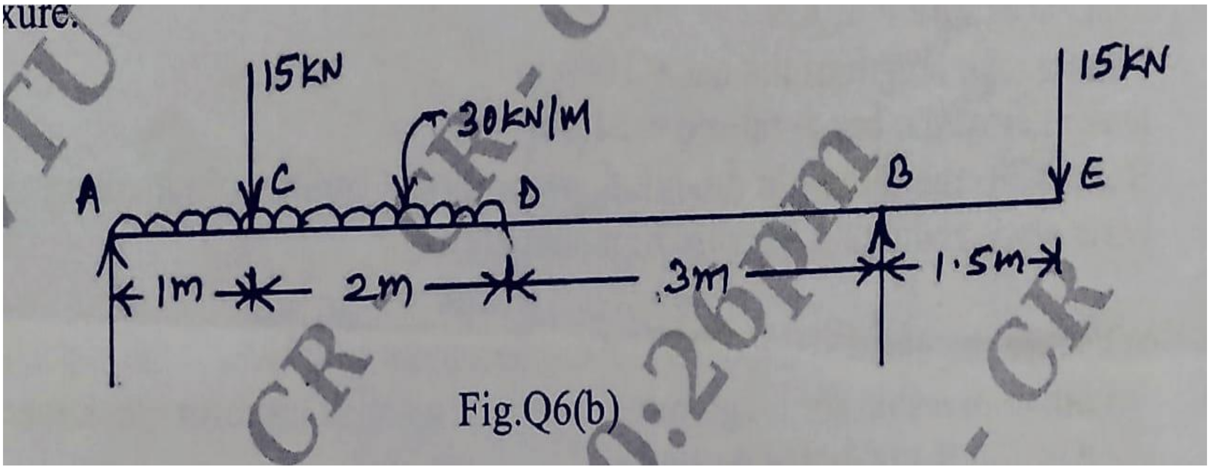


Fig.Q6(b)

Solution:

$R_A + R_B$ = 30*3+15+15 =120

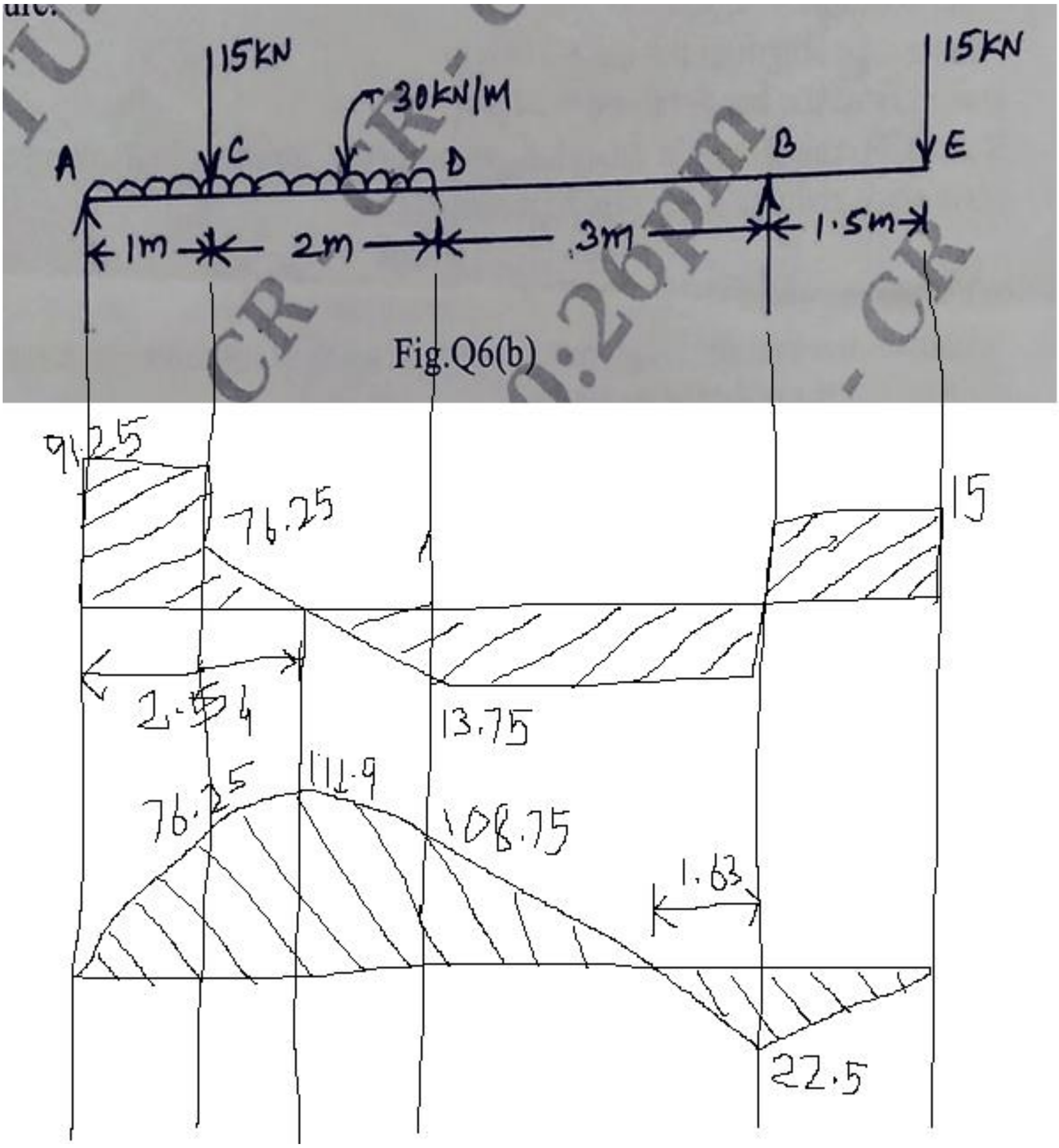
R_B *6 = 15*1+30*3/2 +15*7.5

R_B=28.75 KN

R_{A} = 91.25KN

SF is zero at

91.25 -15-30x = 0 => x=2.54m from 'A'



7.a. derive torsional equation with usual notations. (06 Marks)

Torsion Equation Derivation

There are some assumptions made for the derivation of the torsion equation, those assumptions are as follows.

- The material should be homogeneous and should have elastic property throughout.
- The material should follow the theory of Hooke's law.
- The material should have shear stress that is proportional to the shear strain.

- There should be a plane cross-sectional area.
- The section should be circular.
- Every diameter of the material must rotate at the same angle.
- The stress of the material must not exceed the limit of its elasticity.

Consider a solid circular shaft having radius R which is exposed to a torque T at one end and the other end is also under the same torque.

Angle in radius = arc/ radius

Arc Ab = Rθ = LY

γ = Rθ/L

Where,

A and B: these are considered as the two fixed points present in the circular shaft

Y: the angle subtended by AB

G = τ/(modulus of rigidity)

Where,

τ : shear stress

: shear strain

τ/G = □

∴ R / L = τ/G

Consider a small strip of the radius with thickness dr that is subjected to shear stress.

□' * 2πr dr

Where,

r: radius of the small strip

dr: the thickness of the strip

y: shear stress

2πτ′r2dr2πτ′r2dr

(torque at the center of the shaft)

T=∫R02πτ′r2drT=∫0R2πτ′r2dr

T=∫R02πGθ/Lr3drT=∫0R2πGθ/Lr3dr

(substituting for τ')

T=(2πGθ/L)∫R0r3vdr=Gθ/L[(πd4)/32]T=(2πGθ/L)∫0Rr3vdr=Gθ/L[(πd4)/32]

(after integrating and substituting for R)

(Gθ/L)J (substituting for the polar moment of inertia)

∴ T/J = τ/r = Gθ/L

7.b. A T- section of flange 120mmx12mm and overall depth 200mm with 12mm web thickness is loaded such that at a section it has a bending moment of 20KN.m and shear force of 120KN. Sketch the bending and shear stress distribution diagram marking the salient values. (14 Marks)

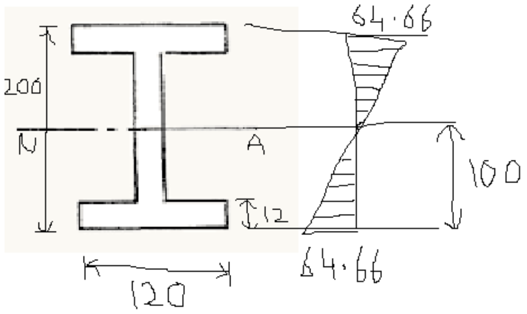
Solution:

1. **Position of N.A.:** as it is a symmetric section the position of NA is at a height of 200/2=100mm from bottom.
2. **MI about NA:**

$$I = \frac{12*200^3}{12} - 2[\frac{54*176^3}{12}] = \mathbf{30.93x10^6 mm^4}$$

3. **bending stress diagram**

$$\sigma_b = \frac{M}{I}y = \frac{20{,}000*1000}{30.93x10^6} * 100 = \mathbf{64.66\ N/mm^2}$$



4. **shear Stress diagram**
   a. **shear stress distribution at bottom of flange**

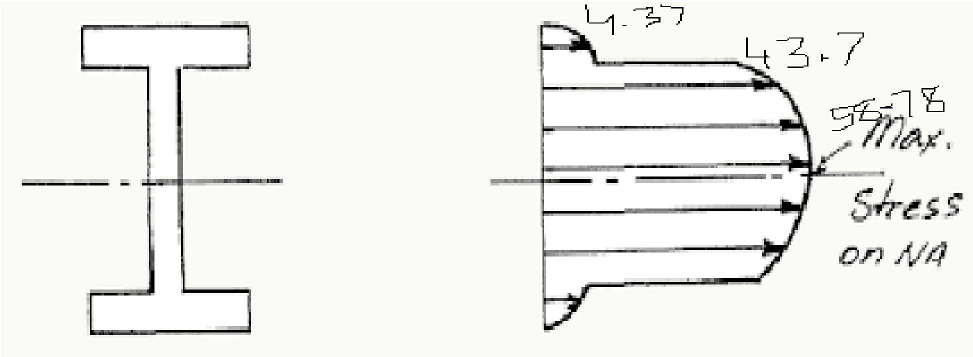
$$\tau = \frac{F(ay)}{Ib} = \frac{120000(120*12*94)}{30.93x10^6*120} = 4.37\ N/mm^2$$

   b. **shear stress distribution at top of web**

$$\tau = \frac{F(ay)}{Ib} = \frac{120000(120*12*94)}{30.93x10^6*12} = \mathbf{43.7\ N/mm^2}$$

c. shear stress distribution at NA

$$\tau = \frac{F(ay)}{Ib} = \frac{120000[(120*12)(100-6)+(12*88)\left(\frac{88}{2}\right)]}{30.93x10^{6}*12} = \textbf{58.78 N/mm}^{2}$$



8.a. Derive Bernoulli-euler bending equation with usual notations. (08 Marks)

Solution:

Bending theory is also known as flexure theory is defined as the axial deformation of the beam due to external load that is applied perpendicularly to a longitudinal axis which finds application in applied mechanics.

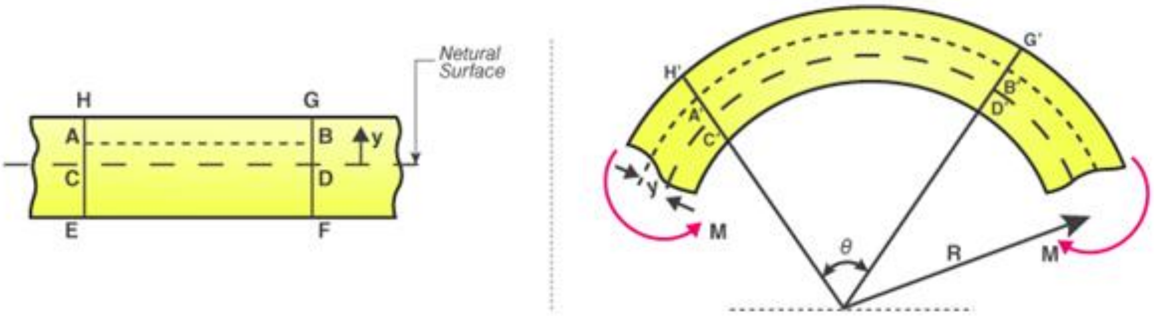
For a material, flexural strength is defined as the stress that is obtained from the yield just before the flexure test. It represents the highest stress that is experienced within the material at the moment of its yield. σ is used as the symbolic representation of flexural strength.

Bending equation derivation

Following are the assumptions made before the derivation of the bending equation:

- The beam used is straight with a constant cross-section.
- The beam used is of homogeneous material with a symmetrical longitudinal plane.
- The plane of symmetry has all the resultant of applied loads.
- The primary cause of failure is buckling.
- E remains same for tension and compression.
- Cross-section remains the same before and after bending.

Consider an unstressed beam, which is subjected to a constant bending moment such that the beam bends up to radius R. The top fibres are subjected to tension whereas the bottom fibres are subjected to compression. The locus of points with zero stress is known as neutral axis.



With the help of the above figure, the following are the steps involved in the derivation of the bending equation:

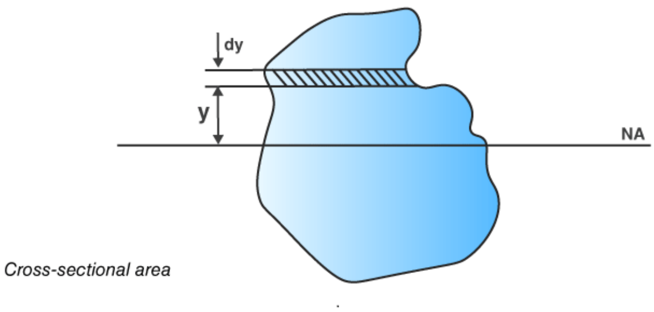
Strain in fibre AB is the ratio of change in length to original length.

Strain in fibre AB=(A′B′−AB)/AB ∴strain=(A′B′−C′D)/′C′D′ (as AB = CD and CD = C'D') CD and C'D' are on the neutral axis and stress is assumed to be zero, therefore strain is also zero on the neutral axis.

=[(R+y)Θ−RΘ]/RΘ =[RΘ+yΘ−RΘ]/RΘ =y/R* σ/E = y/R where E is young's Modulus of Elasticity

Or

σ/y=E/R



σ=E/R * y (eq.1)

F=σδA=E/R * yδA (force acting on the strip with area dA)

Fy=E/R * y^2δA (momentum about neutral axis)

M=∑E/R * y^2 (total momentum for entire cross-sectional area)

δA=E/R∑y^2δA ∑y^2δA is known as second moment of area and is represented as I.

∴M=E/R I (eq.2)

From eq.1 and eq.2 we get,

σ/y = M/I = E/R

Therefore, the above is the bending theory equation.

8.b. A solid circular shaft has to transmit power of 1000KW at 120 rpm. Find the diameter of the shaft if the shear stress of the material is not to exceed 80 N/mm^2. The maximum torque is 1.25 times the mean torque.

what percentage saving in material could be obtained if the shaft is replaced by a hollow one whose internal diameter is 0.6 times the external diameter? The length of the shaft, material and maximum shear stress being same. (12 Marks)

Solution: Given Data

P = 1000KW

N = 120rpm

τ_{max} = 80 N/mm^2

i) **Average torque to be transmitted**

$P = 2\pi N T_{ave}/60$

$T_{ave} = 1000000*60/2\pi 120 = 79577.47$ N-m

$T_{max} = 1.25T_{ave} = 1.25*79577.47 = 99471.84$ N-m

ii) **Polar Moment of Inertia for circular shaft**

$I_p = \pi D^4/32$

iii) **diameter of solid shaft:**

$$T_{max} = \frac{\sigma s}{R} * Ip$$

$$T_{max} = \frac{\sigma s}{R} * \frac{\pi D^4}{32}$$

$$99471.84 \times 10^3 = \frac{80}{D/2} * \frac{\pi D^4}{32}$$

D = 185mm

iv) Percentage saving in material could be obtained if the shaft is replaced by a hollow one whose internal diameter is 0.6 times the external diameter.

Let D_s = diameter of solid shaft

D_o = outer diameter of hollow shaft

D_i = inner diameter of hollow shaft

Given that $D_i = 0.60 D_o$

As the length of the shaft, material and maximum shear stress being same, hence the torsion strength of both the shafts are equal

$$\left\{\frac{Ip}{R}\right\}_{solid} = \left\{\frac{Ip}{R}\right\}_{hollow}$$

$$\left\{\frac{\pi Ds^4/32}{Ds/2}\right\}_{solid} = \left\{\frac{\pi(Do^4 - Di^4)/32}{Do/2}\right\}_{hollow} \text{----------------------(1)}$$

Substituting Ds = 185mm, Do = 0.60Di in (i)

We get Do = 193.76mm, Di=116.256mm

Percentage saving in material = { $[D_s^2 - (D_o^2 - D_i^2)]/D_s^2$ }*100 = **24.46%**

9. a. Define slope, deflection and elastic curve. Explain Macaulay's method of determining slope and deflection. (10 Marks)

Elastic curve of the beam: The deformation of a beam is usually expressed in terms of its deflection from its original unloaded position. The deflection is measured from the original neutral surface of the beam to the neutral surface of the deformed beam. The configuration assumed by the deformed neutral surface is known as the **elastic curve of the beam**.

Slope of Beam: The slope of a beam is the angle between deflected beam to the actual beam at the same point.

Deflection of Beam: Deflection is defined as the vertical displacement of a point on a loaded beam

9.b. compare the crippling loads given by Euler's and Rankine's formula for a tubular steel column 2.5m long having outer diameter as 40mm and 30mm respectively. The column is loaded through pin joints at the ends. Take permissible compressive stress as 320N/mm², Rankin's constant as 1/7500 and E=210GPa. For what length of the column of their cross section, does the Euler's formula cease to apply.

Solution: given data

Length of column = 2.5m

Outer dia. = 40mm

Inner dia. = 30mm

End condition = hinged on both sides

Effective length = 2.5m = 2500mm

σ_c = 320N/mm²

Rankin's constant = a = 1/7500

E = 210GPa = $2x10^5$ N/mm²

(i) Area of cross section = $\pi/4(Do^2 – Di^2) = \pi/4(40^2 – 30^2)$ = 549.77 mm²
(ii) MI = $\pi/64(Do^4 – Di^4) = \pi/64(40^4 – 30^4)$ = 85902.92 mm⁴

Euler's crippling load = $\Pi^2 E I / L^2$

$= \pi^2\ 2x10^5 *85902.92/2500^2$

= 27130.4N

Rankin's crippling load = $\frac{\sigma c * A}{1+a\left(\frac{le}{kmin}\right)^2}$ **where** $k_{min} = \sqrt{(I_{min}/A)} = \sqrt{(85902.92 /549.77)}$ = 12.5

Rankin's crippling load = $\frac{320*549.77}{1+1/7500\left(\frac{2500}{12.5}\right)^2}$ = **27777.85N**

10.a. differentiate between short and long column and what are the limitations of Euler's theory.

S.No.	Short Column	Long Column
1.	If the ratio of effective length to its least lateral dimension is less than or equal to 12 then it is called short column	If the ratio of the effective length of the column to its least lateral dimensions is greater than 12 then it is called a long column
2.	The effective length to least radius of gyration ratio is less than or equal to 40.	The effective length to least radius of gyration ratio is greater than 40.
3.	Buckling tendency is very low	Long and cylinder columns buckle easily
4.	The crushing tendency is very high	It has a very low crushing tendency.
5.	The load-carrying capacity is high as compared to long columns of the same cross-sectional area.	The load-carrying capacity of a long column is less as compared to a short column of the same cross-sectional area.
6.	The failure of the short columns is due to their crushing.	All the long columns fail due to their buckling.
7.	It has a high load-carrying capacity because of its low height.	It has less load carrying capacity because of its more height.
8.	It has a high load-carrying capacity because of its low height.	It has less load carrying capacity because of its more height.
9.	They are subjected to compressive stresses.	They are subjected to buckling stresses.
10.	Its slenderness ratio is less than 45	Its slenderness ratio is more than 45
11.	Short columns have a large lateral dimension as compared to its height	Long columns have a small lateral dimension as compared to its height
12.	The short column is stronger than a long column and it is highly preferable	Long column is weaker than a short column and generally, it is not preferred.

Limitations of Euler's theory

The general expression of bucking load for the long column as per Euler's theory is given as,

$P = \Pi^2 E I / L^2$

$\sigma = \Pi^2 E / (Le / k)^2$

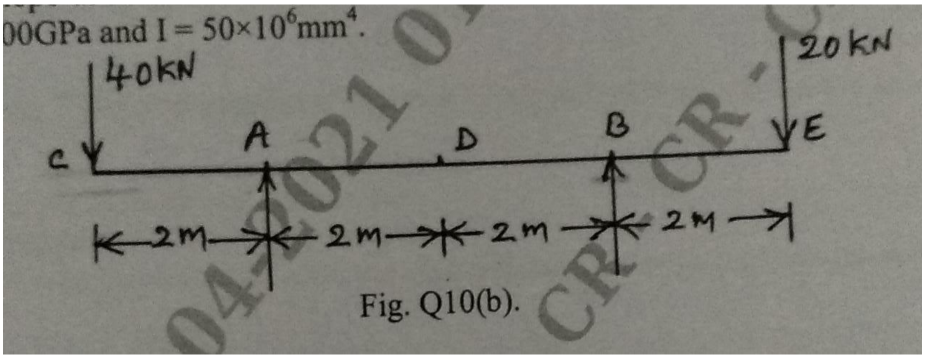
We know that, Le / k = slenderness ratio

Limitation 1: The above formula is applied only for long columns

Limitation 2: As the slenderness ratio decreases the crippling stress increases. Consequently if the slenderness ratio reaches to zero, then the crippling stress reaches infinity, practically which is not possible.

Limitation 3: if the slenderness ratio is less than certain limit, then crippling stress is greater than crushing stress ,which is not possible practically. Therefore, up to limiting extent Euler's formula is applicable with crippling stress equal to crushing stress.

b. Calculate slope at A and deflection at D for the overhanging beam shown in fig. Q10(b). Take E=200GPa and I=50x10^6 mm^4.



Solution:

Given that

E=200GPa = 2*10^5 N/mm^2

I=50x10^6 mm^4

(i) Determination of reactions:

$R_A + R_B = 40+20 = 60KN$

$-40x2-R_Bx4+20x6 = 0 \Rightarrow R_B = 10KN$ and $R_A=50KN$

(ii) Differential equation for curve

EI $\frac{d^2y}{dx^2}$ = -40x+50(x-2) + 10(x-6) -----------i

EI $\frac{dy}{dx}$ = -40x^2/2+50(x-2)^2/2 + 10(x-6)^2/2 ----------------ii

EI y = -40x^3/6+50(x-2)^3/6 + 10(x-6)^3/6 ----------------iii

(iii) Slope at A

EI $\frac{dy}{dx}$ = $-40x^2/2+50(x-2)^2/2 + 10(x-6)^2/2$ ----------------ii

At x = 2,

$2*10^5 * 50x10^6 \frac{dy}{dx} = -40*2^2/2$

$\frac{dy}{dx}$ = 0.008 radians

(iv) Deflection at D

EI $y = -40x^3/6+50(x-2)^3/6 + 10(x-6)^3/6$ ----------------iii

X= 4m

$2*10^5 * 50x10^6\ y = -40*4^3/6+50(4-2)^3/6$

Y = 36mm downwards