UNIVERSITY SOLUTION– JUNE 2018

Sub:	Elements of Civil Engineering and Mechanics	Sub Code:	17CIV23	Branch:	ALL

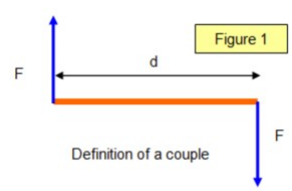
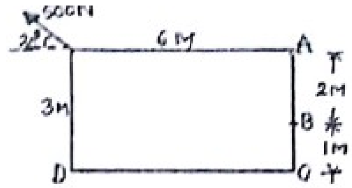
MODULE 1		
1a	Briefly explain the role of civil engineer in infrastructural development of the country	[08]
	Civil engineer has a very important role in the development of the following infrastructures Town planning Build suitable structures for the rural and urban areas for various utilities • Build tank, dams to exploit water resources • Purify the water and supply water to needy areas like houses, schools, offices and agriculture field. • Provide good drainage sysem and purification plants • Provide good drainage system and purification plants • provide and maintain communication systems like roads, railways , harbours and airports. • Monitor land, water and air pollution and take measures to control them.	
1b	Explain couple and explain its characteristics.	[06]
	Two parallel forces equal in magnitude and opposite in direction and seperated by a definite distance are said form a couple.fig.1 shows the representation  **<u>Characteristics</u>** • The sum of forces forming a couple in any direction is zero, which means the translator effect of the couple is zero. The rotational effect of couple on the body is zero. • The rotational effect of a couple about any point is a constant and it is equal to the product of the magnitude of the forces and perpendicular distance between the two forces. • The effect of couple is unchanged if couple is rotated, shifted and replaced by another pair of forces whose rotational effects are the same.	
1 c	Find the moment of force at A B C D as shown in the figure.	[06]

Fig.Q1(c)



M_A = 500sin30 X 6 = 1500Nm
M_B = 500sin30 X 6 + 500cos30 X 2 = 2366.02Nm
M_C = 500sin30 X 6 + 500cos30 X 3 =2799.03Nm
M_D = - 500cos30 X 3 = 1299.03Nm

OR

2 a	State basic idealization in mechanics	[08]

Particle
An object that has no size but has a mass concentrated at a point, is called a particle. In mathematical sense a particle is a body whose dimensions approach zero so that it may be analyzed as a point mass.

Rigid Body
A body is said to be rigid when the relative movements between its parts are negligible. Actually, every body must deform to a certain degree under the action of forces, but in many cases the deformation is negligible and may not be considered in the analysis. This rigid body concept leads to simplified computations. Refer Fig. 1.2 (a).

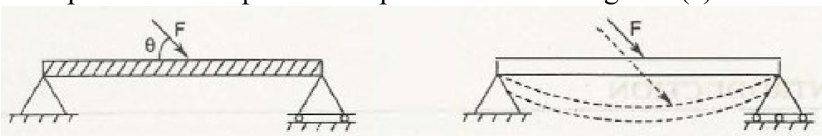
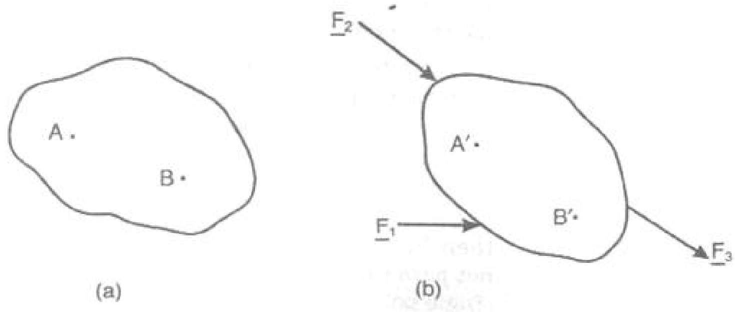


Fig. 1.2 *(a) Rigid body* **Fig. 1.2** *(b) Deformable body*

Continuum
A body consists of several particles. It is a well known fact that each particle can be sub-divided into molecules, atoms and electrons. It is not feasible to solve any engineering problem by treating a body as a conglomeration of such discrete particles. The body is assumed to consist of a continuous distribution of matter which will not separate even when various forces considered are acting simultaneously. In other words, we say the body is treated as a continuum.

Rigid Body
As already stated, in Civil Engineering, we treat a body as rigid, when the relative position of any two particles in the body do not change even after the application of a system of forces. For examples, let the body shown in figure (a) move to a position as shown in figure (b) when the system of forces F_2 and F_3 are applied. If the body is treated as a rigid body, the relative position of A to B is the same as A' and B', i.e.,
AB = A'B'

<table>
<tr><td></td><td>

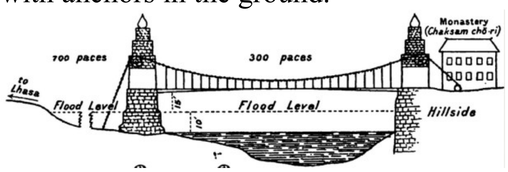


</td><td></td></tr>
<tr><td>2b</td><td>

Explain the following with neat sketch

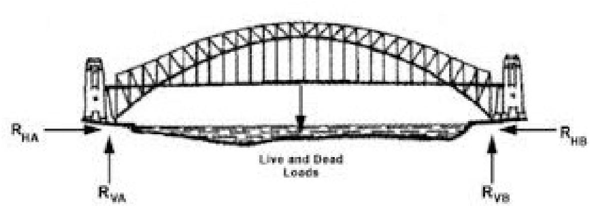
i) Suspension bridge

A suspension bridge is a type of bridge in which the deck (the load-bearing portion) is hung below suspension cables on vertical suspenders. This type of bridge has cables suspended between towers, plus vertical suspender cables that carry the weight of the deck below, upon which traffic crosses. The suspension cables must be anchored at each end of the bridge, since any load applied to the bridge is transformed into a tension in these main cables. The main cables continue beyond the pillars to deck-level supports, and further continue to connections with anchors in the ground.



ii) Arch bridge

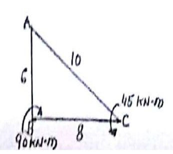
An arch bridge is a bridge with abutments at each end shaped as a curved arch. Arch bridges work by transferring the weight of the bridge and its loads partially into a horizontal thrust restrained by the abutments at either side. A viaduct (a long bridge) may be made from a series of arches, although other more economical structures are typically used today.



</td><td>[06]</td></tr>
<tr><td>2c</td><td>

In the triangle ABC, a force at 'A' produces a clockwise moment of 90kN-m at B and an anticlockwise moment of 45kN-m at C. Find the magnitude and direction of the force as shown in fig.Q2(c). (06 Marks)

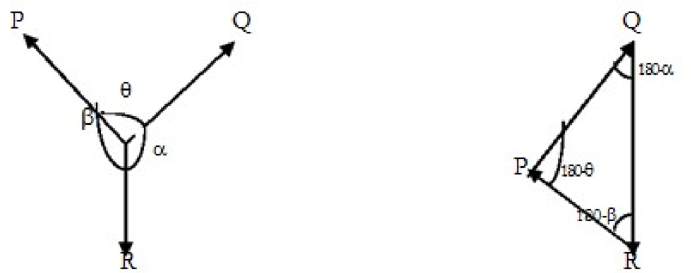
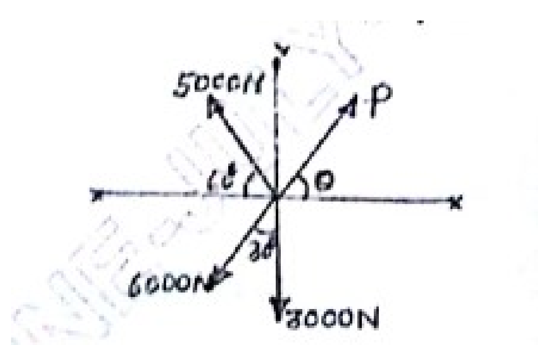
Fig.Q2(c)

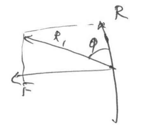
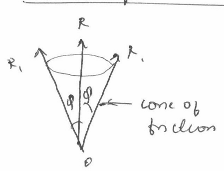
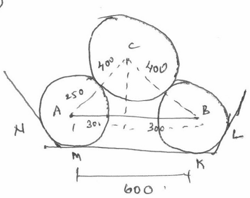
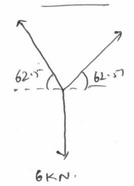
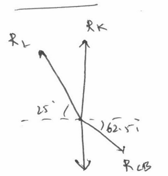
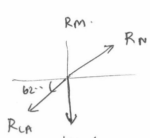


Moment at B =90kNm

$F\cos\theta \times 6 = 90$

</td><td>[06]</td></tr>
</table>

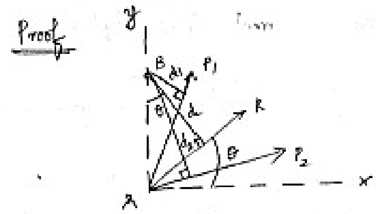
	Moment at O = 45kNm $F\sin\theta \times 8 = 45$, $\theta = 33.69$, $F = 18.02$kN	
	MODULE 2	
2 a	State and prove lamis theorem. Also specify the significance.	[10]
	Lami's theorem states that, if three concurrent forces act on a body keeping it in Equilibrium, then each force is proportional to the sine of the angle between the other two forces. Let P, Q, R be the 3 concurrent forces in equilibrium as shown in fig. (a) Since the forces are vectors, we can move them to form a triangle as shown in fig. (b)  Fig (a) Applying sine rule we get $P/(\sin(180-\alpha))=Q/(\sin(180-\beta))=R/(\sin(180-\theta))P/(\sin\alpha)=Q/(\sin\beta)=R/(\sin\theta)$ **Significance of Lamis theorem** Lami's theorem is used to describe and equilibrium of three forces and works in three dimensions as well as two dimensions. Lami's theorem is applied in static analysis of mechanical and structural systems.	
2b	Four forces are acting on a gusset plate of a bridge truss as shown in the figure. Determine the force P and angle θ to maintain equilibrium of joint.  $\sum F_X = -5000\cos 60 - 6000\sin 30 + P\cos\theta = 0$ $P\cos\theta = 5500$ $\sum F_Y = 5000\sin 60 - 3000 - 6000\cos 30 + P\sin\theta = 0$ $P\sin\theta = 3866.02$	[10]

	$\theta = 34.99^{o}$ $P = 6713.44kN$	
4a	4) b) <u>Angle of friction.</u> The angle which the Resultant Reaction R_1 due to Normal reaction R & friction F make with the normal to the surface is called Angle of friction. $\boxed{\tan\varphi = \frac{F}{R}}$  <u>co-efficient of friction</u> It is the ratio of limiting friction 'F' to the normal reaction R b/w two surfaces. $\boxed{\mu = \frac{F}{R}}$	
4b	<u>cone of friction.</u>  The cone having the point of contact as the vertex O, the normal OR at the point of contact as its axis & φ as the semi-vertex angle is called cone of friction	
4c	c)  $\cos\theta = \frac{300}{850} \Rightarrow \theta = 62.51^{\circ}$ <u>Sphere C</u>  $\frac{6}{\sin 54.98} = \frac{R_{AC}}{\sin 152.51} = \frac{R_{BC}}{\sin 152.51}$ $R_{AC} = R_{BC} = 3.38\ KN$ <u>Sphere B</u>  $\Sigma F_x = 0,$ $3.38\cos 62.51 - R_L\cos 25 = 0$ $\boxed{R_L = 1.72\ KN.}$ $\Sigma F_y = 0, \quad R_K - 4 + 1.72\sin 25 - 3.38\sin 62.50 = 0$ $R_K = 6.27\ KN$  $\Sigma F_x = 0,$ $R_N\cos 15 - 3.38\cos 62.51 = 0$, $\Sigma F_y = 0$ $R_N = 1.51KN$ $R_m - 4 + 1.61\sin 15 - 3.38\sin 62.5$ $R_m = 6.58\ KN.$	

MODULE 3

5 a.	State and prove Varignon's theorem of moment.	[10]

Proof



$R \times d_2 = R\sin\theta \times 0 + R\cos\theta \times AB$

$R \times d_2 = R_x \cdot AB$

$P_1 \cdot d_1 = P_{1x}\ AB$

$P_2\ d_2 = P_{2x}\ AB$

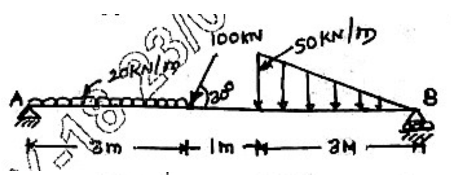
$P_1 \cdot d_1 + P_2\ d_2 = AB\,(\underbrace{P_{1x} + P_{2x}})$

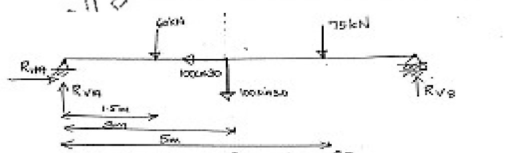
$= AB\,(R_x)$

$= Rd$

$\Rightarrow P_1 d_1 + P_2 d_2 = Rd$

5 b.	Determine the support reactions.	[10]





$\Sigma F_x = 0 \Rightarrow R_{HA} = 100\cos 30 = \underline{86.6\ kN}$

$\Sigma F_y = 0 \Rightarrow R_{VA} - 60 - 100\sin 30 - 75 + R_{VB} = 0$

$R_{VA} + R_{VB} = 185\ kN$

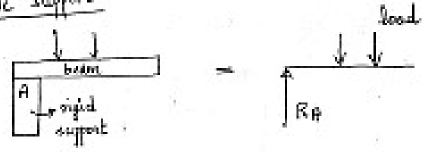
$\Sigma M_A = 0 \Rightarrow (60 \times 1.5) + (100\sin 30 \times 3) + (75 \times 5)$
$(-R_{VB} \times 7) = 0$

$\Rightarrow R_{VB} = 87.85\ kN$

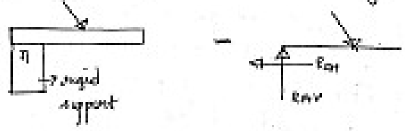
$R_{VA} = 185 - R_{VB}$
$= 185 - 87.85$
$= \underline{97.14\ kN}$

6 a.	Explain with neat sketches: (i) Types of loads (ii) Types of support (iii) Types of beams	[10]

1. Simple support



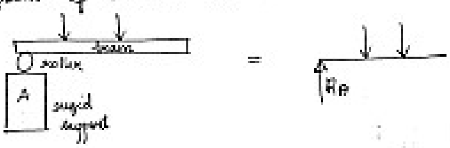
Here the beam is simply placed on a rigid support. Here the reaction will be perpendicular to the rigid support opposite to the loading direction.

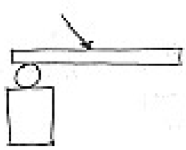


This R_{AH} is actually due to friction at point of contact b/w beam and the rigid support. Since this point of contact is very small if inclined loads come on the beam we will opt for other supports (hinged support).

2. Roller Support

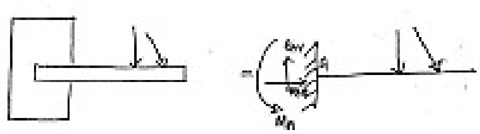
The beam is placed on a rigid support with a roller in between to remove the friction at point of contact between the beam and the support





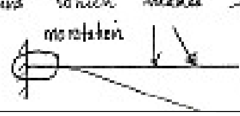
– unstable
as no horizontal
reaction can be developed at A
→ the beam slips of the support

Fixed Support

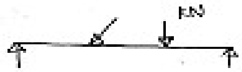


This support restricts the beam from translation in vertical and horizontal direction as well as from rotation.

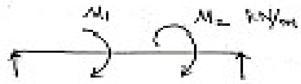
eg: When reinforcements in beam are taken into columns which makes the support fixed.



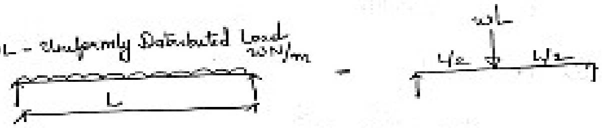
1) Point loads or Concentrated loads.



) Couple moments

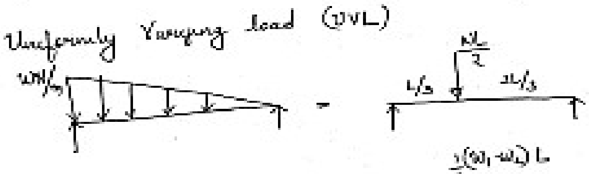


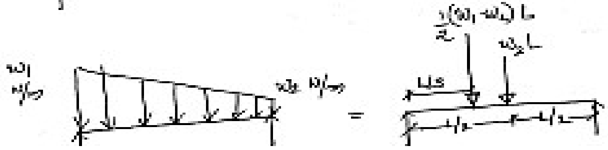
) UDL - Uniformly Distributed Load



eg: weight of beam always acts like a UDL.

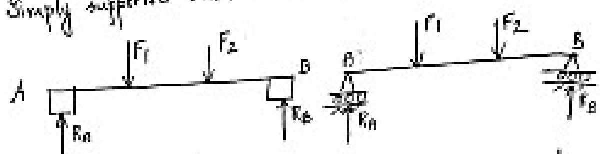
Uniformly Varying load (UVL)





Types of Beams

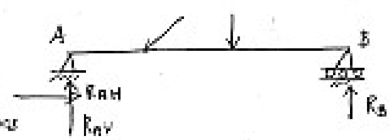
① Simply supported beams and Beams on roller support



2 eqns 2 unknowns The support reactions R_A and R_B must be found by $\boxed{\Sigma F_y = 0 \text{ and } \Sigma M = 0}$. If inclined loads come on the beam then the beam will become unstable.

② Beams with one end hinged and the other roller.

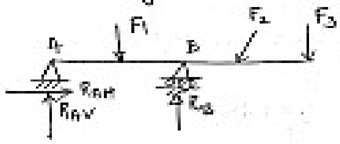
3 eqns 3 unknowns



The reactions R_{AH}, R_{AV}, R_B can be found by $\boxed{\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M = 0}$. The problem will simplify to the first case if inclined loads are not present.

<table>
<tr><td></td><td>

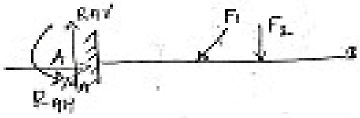
(3) Overhanging beams



Cannot be stable if both supports are rollers.

R_{AH}, R_{AV}, R_B can be solved by $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$
3 equations and 3 unknowns.

(4) Cantilever Beams.

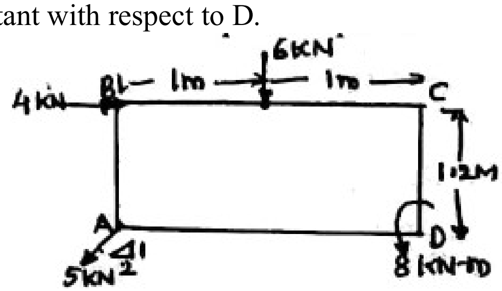


R_{AV}, R_{AH}, M_A can be solved by $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$
3 equations and 3 unknowns

In all the above case the number of unknown is equal to the number of equations. Hence all the above type of beams are <u>statically determinate</u>

</td><td></td></tr>
<tr><td>6 b</td><td>

Determine the resultant with respect to D.



$\Sigma F_x = 40 - 50\cos 26.56 = -4.72\text{ kN}$

$\Sigma F_y = -6 - 50\sin 26.56 = -28.36\text{ kN}$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= \sqrt{-4.72^2 + (28.36)^2}$

$= \underline{28.75\text{ kN}}$

$\theta = \tan^{-1}\dfrac{\Sigma F_y}{\Sigma F_x} = \tan^{-1}\dfrac{-28.36}{-4.72}$

$= \underline{80.55^\circ}$

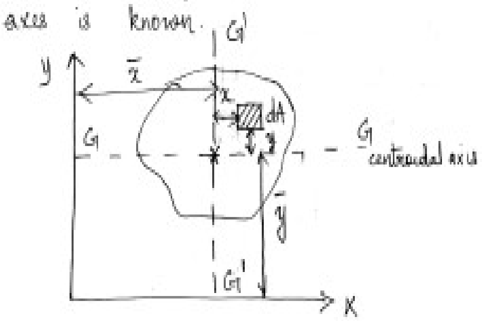
$\Sigma M_D = 40\times 1.2 - 6\times 1 + (50\times\cos 26.56\times 0)$
$- 50\times\sin 26.56\times 2 - 8 = -10.71\text{ kNm}$

$d = \dfrac{|\Sigma M_D|}{|R|} = \dfrac{10.71}{28.75} = \underline{0.37\text{ m}}$

</td><td>[10]</td></tr>
<tr><td>7 a.</td><td>State and Prove Parallel axis theorem.</td><td>[10]</td></tr>
</table>

PARALLEL AXIS THEOREM

This theorem is used to find the moment of inertia about an axis parallel to the centroidal axis if the area moment of inertia about centroidal axes is known.



$$I_{GG} = \int y^2\,dA$$

$$I_{xx} = \int (y+\bar{y})^2\,dA$$

$$= \int y^2\,dA + \bar{y}^2\int dA + \underbrace{2\bar{y}\int y\,dA}_{0}$$

$$\boxed{I_{xx} = I_{GG} + \bar{y}^2 A}$$

$\int y\,dA \rightarrow$ moment of area about centroidal axis $= 0$

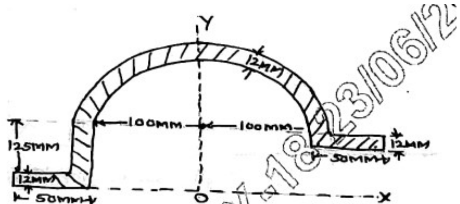
Similarly

$$\boxed{I_{yy} = I_{G'G'} + \bar{x}^2 A}$$

Statement of Theorem:
Moment of Inertia of an area about an axis in the plane of area is equal to the moment of inertia about an axis passing through the centroid and parallel to the given axis plus the product of the area and square of the distance between the two parallel axes.

The second term is always positive because of the square term and hence I_{xx} is always greater than I_{GG}.
$\Rightarrow$ Moment of inertia about centroidal axis is the least.

7 b. Determine the centroid with respect to X and Y axis. [10]



Component	Area mm²	x (mm)	y (mm)
1	50x12 = 600	$-(100+\frac{50}{2})$ = -125	$\frac{12}{2}$ = 6
2	125x12 = 1500	$-(100+12/2)$ = -106	12 + 125/2 = 74.5
3	50x12 = 600	100 + 50/2 = 125	125 + 12/2 = 131
4	$-\frac{\pi\times(100)^2}{2}$ = 15707.9	0	$125+12+\frac{4\times100}{3\pi}$ = 179.44
5	$+\frac{\pi\times(112)^2}{2}$ = 19704.07	0	$125+12+\frac{4\times112}{3\pi}$ = 184.853

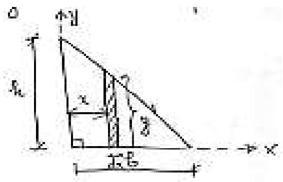
$$\bar{X} = \frac{(600\times-125)+(1500\times-106)+(600\times125)-(15707.9\times0)+(19704.07\times0)}{6696.17}$$

$$= -23.75\text{ mm}$$

$$\bar{Y} = \frac{(600\times6)+(1500\times74.5)+(600\times131)+-(15707.9\times179.44)+(19704.07\times184.853)}{6696.17}$$

$$= 151.03\text{ mm}$$

8a. Find the centroid of a triangle by method of integration.



similar triangles

$\frac{y'}{b-x} = \frac{h}{b}$

$y' = \frac{h}{b}(b-x)$

Area of the shaded element

$dA = y'dx$

$A = \int dA = \int_0^b y'dx$

$= \int_0^b \frac{h}{b}(b-x)\,dx$

$= \frac{h}{b}\left(bx - x^2/2\right)_0^b$

$= \frac{h}{b}\left(b^2 - b^2/2\right)$

$= \frac{h}{b} \times \frac{b^2}{2} = \frac{bh}{2}$

$\bar{X} = \frac{\int x\, y\, dx}{A}$

$= \frac{\int_0^b x \frac{h}{b}(b-x)\,dx}{\frac{1}{2}bh}$

$= \frac{\frac{h}{b}\int_0^b (bx - x^2)\,dx}{\frac{1}{2}bh}$

$= \frac{\frac{h}{b}\left(b\frac{x^2}{2} - \frac{x^3}{3}\right)_0^b}{\frac{1}{2}bh}$

$= \frac{h}{b} \times \frac{b^3}{2} - \frac{b^3}{3} \times \frac{2}{bh}$

$= \frac{h}{b} \times \frac{b^3}{6} \times \frac{2}{bh}$

$\bar{X} = \frac{b}{3}$

$\bar{y} = \frac{\int \bar{y}\, dA}{A}$ → distance of centroid of subdivision from X-axis

$= \frac{1}{A}\int_0^b \frac{h}{b}(b-x) \times \frac{h}{b}(b-x)\,dx \times \frac{1}{2}$ → centroidal distance $= y = y'/2$

$= \frac{1}{A}\frac{h^2}{b^2}\int_0^b \frac{(b^2 + x^2 - 2bx)}{2}\,dx$

$= \frac{1}{2A}\frac{h^2}{b^2}\left(b^2x + \frac{x^3}{3} - bx^2\right)_0^b = \frac{h^2}{2b^2A}\left(b^3 + \frac{b^3}{3} - b^3\right)$ =h/3

8b. Determine the moment of inertia with respect to its centroidal axes. [10]

$\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A}$

$= \frac{(120\times10)\times\frac{120}{2} + 100\times12\times\frac{12}{2} + 180\times10\times\frac{180}{2}}{(120\times10) + (100\times12) + (180\times10)}$

$= \frac{241200}{4200} = 57.43\,\text{mm}$

$\bar{Y} = \frac{120\times10\times\left[100+10+10/2\right] + 100\times12\times\left[\frac{100}{2}+10\right] + 180\times10\times\frac{10}{2}}{4200}$

$= 52.14\,\text{mm}$

$I_{cx} = \left[\frac{120\times10^3}{12} + (120\times10\times62.85^2)\right] + \left[\frac{100^3\times12}{12} + (100\times12\times7.86^2)\right] + \left[\frac{180\times10^3}{12} + (180\times10\times47.14^2)\right]$

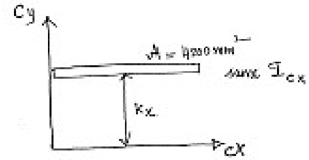
$I_{cx} = 9.84\times10^6\,\text{mm}^4$

$I_{cy} = \left[\frac{10\times120^3}{12} + (10\times120\times2.67^2)\right] + \left[\frac{12^3\times100}{12} + (12\times100\times51.43^2)\right] + \left[\frac{180^3\times10}{12} + (180\times10\times32.57^2)\right]$

$= 11.405\times10^6\,\text{mm}^4$

The least Moment of inertia is I_{cx}

$= 984 \times 10^6 \text{ mm}^4$



$A \cdot k_x^2 = I_{cx}$

$k_x = \sqrt{\frac{I_{cx}}{A}} = \sqrt{\frac{984 \times 10^6}{4200}} = \underline{\underline{48.40 \text{ mm}}}$

maximum radius of gyration

9a

9)

a) $x = 2y$, Horizontal componat $u_x = u\cos\theta = 18 \text{ m/s}$

$u_y = u\sin\theta = 24 \text{ m/s}$

$y = u\sin\theta \times t - 9t^2 \quad -(1)$

$y = 24t - 94.905t^2 \quad -(2)$

Solve (1) & (2) $\therefore x = 2y = u\cos\theta t = 18t$

$y = 9t$

$\therefore t = 3.058 \text{ s}$

$x = 55.04$

$y = 27.53$

9b

9)
b) For burglar car, $u = 0$, $a = 2 m/s^2$

t = time taken by police van to overtake burglar car.

Police vigilant team came after 5 s.

∴ burglars car will be in motion for $(t+5)$ sec

uniform velocity of police van = 20 m/s

when police car overtake burglar car, the distance travelled by both van will be same.

∴ Distance travelled by police van = uniform velocity × t

$S = 20t$. — (1)

The distance travelled by burglar car in $(t+5)$ sec

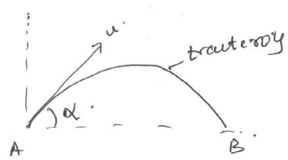
$S = ut + \frac{1}{2} at^2$

$S = 0 + \frac{1}{2} \times 2 (t+5)^2$ — (2)

10)

Projectile -

A particle projected upwards at a certain angle & moving under the combined effect of vertical & horizontal components of velocity is called projectile.



Angle of projection: It is the angle α with the horizontal at which the projectile is projected.

Range -

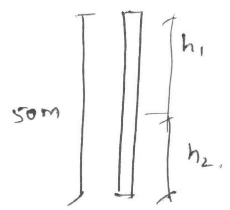
The distance b/w the point of projection & the point where the projectile strikes the ground is known as range.

Vertical height - The maximum ht reached by the particle from ground is called V.H.

Time of flight. Time take or total time taken from projection, to reach max ht & return back to ground is called time of flight

10)

b)



$h_1 + h_2 = 50m$.

For Stone 1,

$u = 0$, $h = h_1$, $a = 9.81\ m/s^2$.

$h = ut + \frac{1}{2}at^2$

$h_1 = 4.9\ t^2$ — (1)

For Stone -2

$u = 25\ m/s$, $h = h_2$, $a = -9.81\ m/s^2$.

$h_2 = 25t - \frac{1}{2} \times 9.81 \times t^2$

$h_2 = 25t - 4.9t^2$ — (2)

Solve (1) & (2) $\therefore$ $\boxed{t = 2\ s}$ — (2)

$h_1 = 19.62\ m$ from top

$h_2 = 30.38\ m$. from bottom.