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Internal Assessment Test I–June 2021

Sub:	Advanced Calculus and Numerical Methods					Sub Code:	18MAT21	
Date:	**22/06/2021**	Duration:	**90 mins**	Max Marks:	**50**	Sem / Sec:	**Common for all Branches**	OBE

		MARKS	CO	RBT
1.	Solve $y'' - 4y' + 4y = e^{2x} + cosx + 4.$	[08]	CO2	L3
2.	Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$ by Method of variation of parameters.	[07]	CO2	L3
3.	Solve $(2x+3)^2\frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$.	[07]	CO2	L3
4.	Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10(x + \frac{1}{x})$	[07]	CO2	L3
5.	The current i and the charge q in a series having inductance L, capacitance C and emf E satisfy $L\frac{d^2q}{dt^2} + \frac{q}{C} = E$. Solve for '$q$' and '$i$' in terms of t and also at t=0.	[07]	CO2	L3
6.	Form the partial differential equation by elimination arbitrary function $f\left(z^2 - xy, \frac{x}{z}\right) = 0$	[07]	CO3	L3
7.	Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$ subject to the condition that $z = 1\ and\ \frac{\partial z}{\partial x} = y\ \ when\ x = 0.$	[07]	CO3	L3

1) Here,

we have,

$$\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 4y = e^{2x} + \cos x + 4$$

$$D^2y - 4Dy + 4y = e^{2x} + \cos x + 4$$

$$[D^2y - 4Dy + 4y] = e^{2x} + \cos x + 4$$

$$[D^2 - 4D + 4]y = e^{2x} + \cos x + 4$$

The auxillary eqn is.

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

so. $y_2 = (c_1 + c_2x)e^{2x}$

Now,

$$y_c = \frac{e^{2x} + \cos x + 4}{D^2 - 4D + 4}$$

$$y_c = \frac{e^{2x}}{D^2-4D+4} + \frac{\cos x}{D^2-4D+4} + \frac{4e}{D^2-4D+4}$$

$$= \frac{xe^{2x}}{2D-4} + \frac{\cos x}{3-4D} + \frac{4}{4}$$

$$= \frac{xe^{2x}}{2} + \frac{\cos x\,(3+4D)}{9-16D^2} + 1$$

$$= \frac{x^2e^{2x}}{2} + \frac{3\cos x - 4\sin x}{9+16} + 1$$

$$\Rightarrow \frac{x^2e^{2x}}{2} + \frac{3\cos x - 4\sin x}{25} + 1$$

So, solution, here is,

$$y = y_p + y_c$$

$$= c_1e^{2x} + c_2xe^{2x} + \frac{x^2e^{2x}}{2} + \frac{3\cos x - 4\sin x}{25} + 1$$

2. we have $[D^2-6D+9]y = \frac{e^{3x}}{x^2}$

A.E,
$$m^2-6m+9=0$$

or
$$m=3,3$$

$$\therefore \quad y_c = (c_1+c_2x)e^{3x}$$

Let $y = Ae^{3x}+Bxe^{3x}$ be the complete solution of the given equation.

we have
$$y_1 = e^{3x} \qquad y_2 = xe^{3x}$$
$$y_1' = 3e^{3x} \qquad y_2' = 3xe^{3x}+e^{3x}$$

$$W = y_1y_2' - y_2y_1' = e^{6x} \qquad \text{Also } \phi(x) = \frac{e^{3x}}{x^2}$$

$$A' = \frac{-y_2\phi(x)}{W} \qquad B' = \frac{y_1\phi(x)}{W}$$

$$A' = \frac{-xe^{3x}.e^{3x}/x^2}{e^{6x}} \qquad B' = \frac{e^{3x}.e^{3x}/x^2}{e^{6x}}$$

$$A' = -1/x \qquad B' = 1/x^2$$

$$A = -\int \frac{1}{x}dx + k_1 \qquad B = \int \frac{1}{x^2}dx + k_2$$

i.e $A = -\log x + K_1$ $\qquad B = -\frac{1}{x} + K_2$

substituting these in $y = Ae^{3x} + Bxe^{3x}$,

$$y = (-\log x + K_1)e^{3x} + \left(-\frac{1}{x} + K_2\right) x e^{3x}$$

$$y = (K_1 + K_2 x)e^{3x} - e^{3x}\log x - e^{-3x},$$

The term $-e^{3x}$ can be neglected.

Thus $\boxed{y = (K_1 + K_2 x)e^{3x} - e^{3x}\log x}$

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3. Given $(2x+3)^2 y'' - (2x+3)y' - 12y = 6x$ —(1)

Let put $(2x+3) = e^t$, $\log(2x+3) = t$

$$\Rightarrow x = \frac{e^t - 3}{2}$$

we have $(2x+3)y' = 2Dy$ $\qquad D = \frac{d}{dt}$

$(2x+3)y'' = 4D(D-D)y$

By (1) $(4D^2 - 6D - 12)y = 6\left(\frac{e^t - 3}{2}\right)$

$\Rightarrow$ $[4D^2 - 6D + 12]y = 3(e^t - 3)$

A.E $\qquad 4m^2 - 6m + 12 = 0$

$$m = \frac{3 \pm \sqrt{57}}{4}$$

$$y_c = CF = c_1 e^{\frac{3+\sqrt{57}}{4}t} + c_2 e^{\frac{3-\sqrt{57}}{4}t}$$

$$y_p = \frac{1}{4D^2-6D-12} 3(e^t-3)$$

$$= \frac{3e^t}{4D^2-6D-12} - \frac{9e^{0.t}}{4D^2-6D-12}$$

$$y_p = \frac{3e^t}{4(1)-6.1-12} - \frac{9.e^{0t}}{0-0-12}$$

$$y_p = -\frac{3e^t}{14} + \frac{3}{4}$$

$\therefore$

$$y = y_c + y_p$$

$$y = c_1 e^{\frac{3+\sqrt{57}}{4}t} + c_2 e^{\frac{3-\sqrt{57}}{4}t} - \frac{3e^t}{14} + \frac{3}{4}$$

$$\Rightarrow y = c_1 (2x+3)^{\left(\frac{3+\sqrt{57}}{4}\right)} + c_2 (2x+3)^{\frac{3-\sqrt{57}}{4}} - \frac{3(2x+3)}{14} + \frac{3}{4}$$

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4. Given $x^3y''' + 2x^2y'' + 2y = 10\left(x+\frac{1}{x}\right)$

we put $x = e^t$ or $\log x = t$

put $xy' = Dy$

$x^2y'' = D(D-1)y$ $\qquad D = \frac{d}{dt}$

$x^3y''' = D(D-1)(D-2)y$

Then given eqn.

$$[D^3 - D^2 + 2]y = 10(e^t + e^{-t})$$

A.E $\quad m^3 - m^2 + 2 = 0$

$m = -1,\ 1 \pm i$

$$y_c = C_1 e^{-t} + e^t [C_2 \cos t + C_3 \sin t]$$

Now

$$y_p = \frac{1}{D^3 - D^2 + 2} 10(e^t + e^{-t})$$

$$y_p = 10\left[\frac{e^t}{D^3 - D^2 + 2} + \frac{e^{-t}}{D^3 - D^2 + 2}\right]$$

$$y_p = 10\left[\frac{e^t}{1 - 1 + 2} + \frac{e^{-t}}{-1 - 1 + 2}\right]$$

(Dr = 0)

$$= 10\left[\frac{e^t}{2} + \frac{t e^{-t}}{3D^2 - 2D}\right]$$

$$= 10\left[\frac{e^t}{2} + t\frac{e^{-t}}{3 + 2}\right]$$

$$y_p = 5e^t + 2te^{-t}$$

so complete Solution

$$y = \frac{C_1}{x} + x\left[C_2 \cos(\log x) + C_3 \sin(\log x)\right] + 5x + \frac{2\log x}{x}$$

5. Given $\frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{E}{L}$ Denoting, $\lambda^2 = \frac{1}{LC}$ and $\mu = \frac{E}{L}$

$\Rightarrow$ $(D^2+\lambda^2)q = \mu$

AE $m^2+\lambda^2=0$ $\therefore$ $m = \pm i\lambda$

CF $= q_c = c_1\cos\lambda t + c_2\sin\lambda t$

P.I $= q_p = \frac{\mu}{D^2+\lambda^2} = \frac{\mu e^{0t}}{D^2+\lambda^2} = \frac{\mu}{\lambda^2}$

$\Rightarrow$ $q(t) = c_1\cos\lambda t + c_2\sin\lambda t + \frac{\mu}{\lambda^2}$ —— (1)

Also $q'(t) = -\lambda c_1\sin\lambda t + \lambda c_2\cos\lambda t$ —— (2)

But $q(0)=0$ and $q'(0)=0$ by data

Hence (1) and (2),

$0 = c_1 + \frac{\mu}{\lambda^2}$ and $0 = \lambda c_2$ $\Rightarrow$ $\boxed{c_1 = -\frac{\mu}{\lambda^2}}$ and $\boxed{c_2 = 0}$

By (1)

$q(t) = -\frac{\mu}{\lambda^2}\cos\lambda t + \frac{\mu}{\lambda^2}$

or $q(t) = \frac{\mu}{\lambda^2}[1-\cos\lambda t]$ where $\mu/\lambda^2 = \frac{E/L}{1/LC} = EC$

Thus $\boxed{q(t) = EC\left[1-\cos\sqrt{1/LC}\, t\right]}$

& $\boxed{i(t) = q'(t) = E\sqrt{C/L}\,\sin\sqrt{1/LC}\, t}$

6

Given $f(z^2-xy, \frac{x}{z}) = 0$

We have

$$u = z^2 - xy \qquad v = \frac{x}{z}$$

so

$$\frac{\partial u}{\partial x} = 2zp - y \qquad \frac{\partial v}{\partial x} = \frac{z - xp}{z^2}$$

$$\frac{\partial u}{\partial y} = 2zq - x \qquad \frac{\partial v}{\partial y} = x\left(-\frac{q}{z^2}\right)$$

To eliminate f,

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2zp - y & \frac{z - xp}{z^2} \\ 2zq - x & \frac{-xq}{z^2} \end{vmatrix} = 0$$

$$\Rightarrow \quad xyq - 2z^2q + xz - x^2p = 0$$

The required PDE is $\boxed{q(xy - 2z^2) + xz = x^2p}$

7. Given $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} - 4z = 0$, $z = 0$ & $\frac{\partial z}{\partial y} = y$ when $x = 0$

The given PDE assumes the form of an ODE

$$(D^2 + 3D - 4)z = 0 \quad \text{where } D = \frac{d}{dx}$$

AE is $m^2 + 3m - 4 = 0$ or $(m-1)(m+4) = 0$

$$\therefore \quad m = 1, -4$$

$\Rightarrow$ The solution of the ODE

$$z = c_1 e^{x} + c_2 e^{-4x}$$

Solution of the PDE

$$z = f(y)e^{x} + g(y)e^{-4x} \quad \text{--- (1)}$$

dgn partially w.r.t x, we get

$$\frac{\partial z}{\partial x} = f(y)e^{x} - 4g(y)e^{-4x} \quad \text{--- (2)}$$

By data, $z = 1$ and $\frac{\partial z}{\partial x} = y$ when $x = 0$

Hence (1) & (2) becomes

$$1 = f(y) + g(y) \quad \text{and} \quad y = f(y) - 4g(y)$$

By solving $f(y) = \frac{1}{5}(4+y)$ & $g(y) = \frac{1}{5}(1-y)$

we substitute these in (1)

$$\boxed{z = \frac{1}{5}(4-y)e^{x} + \frac{1}{5}(1-y)e^{-4x}}$$ is the required solution.