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CMR Institute of Technology, Bangalore
DEPARTMENT OF INFORMATION SCIENCE AND ENGINEERING
I - INTERNAL ASSESSMENT

Semester: 4-CBCS 2018
Subject: COMPLEX ANALYSIS, PROBABILITY AND STATISTICAL METHODS (18MAT41)
Faculty: Ms Girisha . A

Date: 19 May 2021
Time: 09:00 AM - 10:30 AM
Max Marks: 50

Instructions to Students :

1. Part A is compulsory
2. Answer any 6 questions from Part B

PART A

Answer All Questions

Q.No		Marks	CO	PO	BT/CL
1	. State and prove Cauchy –Riemann equations in Cartesian form.	8	CO1	PO1	L2

PART B

Answer any6 question(s)

Q.No		Marks	CO	PO	BT/CL
2	. The pressure and volume of a gas are related by the equation $pv^{\gamma} = k$, where γ and k are constants. Fit the equation to the following set observations: (table below)	7	CO4	PO2	L3
3	. The following table gives the results of the measurements of train resistance; V is the velocity in miles per hour; R is the resistance in pounds per ton: (table below) If R is related to V by the relation $R = a + bV + cV^2$, find a, b and c.	7	CO4	PO2	L3
4	. Find the Coefficient of correlation between industrial production and export using the following data and comment on the result (table below)	7	CO4	PO2	L3
5	. The regression equations of two variables x and y are $x = 0.7y + 5.2$, $y = 0.3x + 2.8$. Find the means of variables and and the coefficient of correlation between them.	7	CO4	PO2	L3
6	. Calculate the rank correlation coefficient from the following data showing ranks of 10 students in two subjects: (table below)	7	CO4	PO2	L3
7	. Show that the given function is analytic and hence find the derivative of $f(z) = z^n$	7	CO1	PO1	L2
8	. Determine the analytic function whose real part is $x\, sinx\, coshy - y\, cosx\, sinhy$	7	CO1	PO1	L2

Table for Q2:

$p(kg/cm^2)$	0.5	1.0	1.5	2.0	2.5	3.0
$v(liters)$	1.62	1.00	0.75	0.62	0.52	0.46

Table for Q3:

V	20	40	60	80	100	120
R	5.5	9.1	14.9	22.8	33.3	46.0

Table for Q4:

Production(in crore tons)	55	56	58	59	60	60	62
Exports(in crore tons)	35	38	38	39	44	43	45

Table for Q6:

Maths	3	8	9	2	7	10	4	6	1	5
Physics	5	9	10	1	8	7	3	4	2	6

Q1 Derive CR equations in cartesian form; $u_x = v_y$ & $u_y = -v_x$ statement $\rightarrow$ ①

Proof:- Let $f(z) = u + iv$ be analytic at any point $z = x + iy$

$\because$ $f(z) = u(x,y) + iv(x,y)$ is analytic it is differentiable

$$\Rightarrow f'(z) = \lim_{\delta z \to 0} \frac{f(z+\delta z) - f(z)}{\delta z} \text{ exists and is unique} \rightarrow ①$$

But we know $f(z) = u(x,y) + iv(x,y)$ and $\delta z = \delta x + i\delta y$

& $f(z+\delta z) = u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y)$

$$\Rightarrow f'(z) = \lim_{\delta z \to 0} \frac{\{u(x+\delta x, y+\delta y) + iv(x+\delta x, y+\delta y)\} - \{u(x,y) + iv(x,y)\}}{\delta z}$$

$$\Rightarrow f'(z) = \lim_{\delta z \to 0}\left(\frac{u(x+\delta x, y+\delta y) - u(x,y)}{\delta z}\right) + i\lim_{\delta z \to 0} \frac{v(x+\delta x, y+\delta y) - v(x,y)}{\delta z} \rightarrow ①$$

$\rightarrow$ ②

$\because$ $\delta z \to 0$ we can have the following possibilities

Case i :- Let $\delta y = 0$ and $\delta x \to 0$ then $\delta z \to 0$ implies $\delta x \to 0$

$$\therefore ① \Rightarrow f'(z) = \lim_{\delta x \to 0} \frac{u(x+\delta x, y) - u(x,y)}{\delta x} + i\lim_{\delta x \to 0} \frac{v(x+\delta x, y) - v(x,y)}{\delta x}$$

$$\Rightarrow f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = u_x + iv_x \rightarrow ②$$

$\rightarrow$ ②

Case ii :- Let $\delta x = 0$ and $\delta y \to 0$, then $\delta z \to 0$ implies $i\delta y \to 0$ or $\delta y \to 0$

$$\text{then } ① \Rightarrow f'(z) = \lim_{\delta y \to 0} \frac{u(x, y+\delta y) - u(x,y)}{i\delta y} + i\lim_{\delta y \to 0} \frac{v(x, y+\delta y) - v(x,y)}{i\delta y}$$

$$\Rightarrow f'(z) = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} = \frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y} = v_y - iu_y \quad \left(\because \frac{1}{i} = \frac{-i^2}{i} = -i\right)$$

$\rightarrow$ ③

$\rightarrow$ ①

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from ② & ③ $f'(z) = u_x + iv_x = v_y - iu_y$

equating real and imaginary parts we get

$$u_x = v_y \text{ and } v_x = -u_y \rightarrow ①$$

These are CR eqns in Cartesian form.

Q2) Given eqn is $pv^\gamma = k$

$\Rightarrow \log pv^\gamma = \log k \Rightarrow \log p + \log v^\gamma = \log k \Rightarrow \log p + \gamma \log v = \log k$

put $X = \log p$; $Y = \log v$; $a = \gamma$ $b = \log k \rightarrow ①$

$\Rightarrow X + aY = b$ The Normal eqns are $\sum X + a\sum Y = 6b$
$\sum X^2 + a\sum XY = b\sum X \rightarrow ①$

Table :-

p	v	$X = \log p$	$Y = \log v$	X^2	XY
0.5	1.62	-0.6931	0.4824	0.4804	-0.3344
1.0	1.00	0	0	0	0
1.5	0.75	0.4055	-0.2877	0.1644	-0.1167
2.0	0.62	0.6931	-0.4780	0.4804	-0.3313
2.5	0.52	0.9163	-0.6539	0.8396	-0.5992
3.0	0.46	1.0986	-0.7765	1.2069	-0.8531
		2.4204	-1.7134	3.1717	-2.2347

$\rightarrow ③$

So the Normal eqns are $2.4204 + a(-1.7134) = 6b$
$3.1717 + a(-2.2347) = 2.4204b \rightarrow ①$

Solving these eqns for a and b and writing γ for a and $k = e^b$ we get $\gamma = \cancel{1.2776}\ 1.4225$ & $k = \cancel{1.1039}\ 0.9970$ $\therefore$ $\boxed{pv^{\cancel{1.2776}\ 1.4225} = \cancel{1.1039}\ 0.997}$ $\rightarrow ①$

Q3. The eqn is $R = a + bV + cV^2 \Rightarrow$ Normal eqns are
$$\Sigma R = 6a + b\Sigma V + c\Sigma V^2$$
$$\Sigma RV = a\Sigma V + b\Sigma V^2 + c\Sigma V^3$$
$$\Sigma RV^2 = a\Sigma V^2 + b\Sigma V^3 + c\Sigma V^4$$
→ (2)

Table

V	R	VR	V^2	V^2R	V^3	V^4
20	5.5	110	400	2200	8000	160000
40	9.1	364	1600	14560	64000	2560000
60	14.9	894	3600	53640	216000	12960000
80	22.8	1824	6400	145920	512000	40960000
100	33.3	3330	10000	333000	1000000	100000000
120	46.0	5520	14400	662400	1728000	207360000
$\Sigma V = 420$	$\Sigma R = 131.6$	$\Sigma VR = 12042$	$\Sigma V^2 = 36400$	$\Sigma V^2R = 12,11,720$	$\Sigma V^3 = 35,28,000$	ΣV^4 364000000 ~~361552000~~

→ (3)

The Normal eqns are
$$131.6 = 6a + 420b + 36400c$$
$$12042 = 420a + 36400b + 3528000c$$
$$1211720 = 36400a + 3528000b + 364000000c \quad \cancel{361552000c}$$
→ (1)

Solving these 3 linear eqns in a, b & c we get.

$a = 4.35$ ~~3.118~~; $b = +0.00241$; $c = 0.0029$ → (1)

Q4. The Correlation Coefficient r is given by $r = \dfrac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}}$

where $X = x - \bar{x}$ $Y = y - \bar{y}$

For the given data $\bar{x} = 58.57$ → (1)

$\bar{y} = 40.28$ → (1)

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
55	35	−3.57	−5.28	12.7449	27.8784	18.8496
56	38	−2.57	−2.28	6.6049	5.1984	5.8596
58	38	−0.57	−2.28	0.3249	5.1984	1.2996
59	39	0.43	−1.28	0.1849	1.6384	−0.5504
60	44	1.43	3.72	2.0449	13.8384	5.3196
60	43	1.43	2.72	2.0449	7.3984	3.8896
62	45	3.43	4.72	11.7649	22.2784	16.1896
				35.7143	83.4288	50.8572 $\to$ 3

$\bar{x} = \dfrac{410}{7} = 58.57$ $\quad \Sigma y = 282$ $\quad \bar{y} = \dfrac{282}{7} = 40.28$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2}\sqrt{\Sigma Y^2}} = \frac{50.8572}{\sqrt{35.7143}\sqrt{83.4288}} \to 1$$

$$= \frac{50.8572}{5.9761 \times 9.1339} = \cancel{\frac{50.8572}{5.9761 \times 9.1339}} = \frac{50.8572}{54.5851} = 0.9317 \to 1$$

Q5 Reg line of y on x is $y = 0.3x + 2.8$; Reg line of x on y is $x = 0.7y + 5.2$

$\Rightarrow b_{yx} = 0.3$ & $b_{xy} = 0.7$ $\to 1$ $\Rightarrow r = \sqrt{b_{yx} \times b_{xy}} \to 1$ $= \sqrt{0.3 \times 0.7} = +0.458 \approx 0.46 \to 1$

$\bar{x}$ & $\bar{y}$ are obtained by solving
$$\left.\begin{aligned} \bar{y} - 0.3\bar{x} &= 2.8 \\ 0.7\bar{y} - \bar{x} &= -5.2 \end{aligned}\right\} \to 2$$

Solving these two equations for $\bar{x}$ & $\bar{y}$ we get

$$\left.\begin{aligned} \bar{x} &= 9.06 \\ \bar{y} &= 5.52 \end{aligned}\right\} \to 2$$

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Q6 rank correlation Coefficient $\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$ $\rightarrow$ 2

Here $n=10$; $\sum d^2 = (3-5)^2 + (8-9)^2 + \cdots (5-6)^2 = 24$ $\rightarrow$ 3

$\therefore \rho = 1 - \frac{6(24)}{10(10^2-1)} = 1 - \frac{144}{990} = 1 - 0.14545$

$= 0.8545$ $\rightarrow$ 2

Q7 Given $f(z) = z^n$; To prove f is analytic & to find $\frac{d}{dz}(f(z))$

Let $w = f(z) = (re^{i\theta})^n = r^n e^{in\theta} = r^n(\cos n\theta + i\sin n\theta)$ $\rightarrow$ 2

$\Rightarrow u(r,\theta) = r^n \cos n\theta$ & $v(r,\theta) = r^n \sin n\theta$.

$\Rightarrow u_r = nr^{n-1}\cos n\theta$ | $v_r = nr^{n-1}\sin n\theta$

$u_\theta = r^n(-\sin n\theta)n$ | $v_\theta = r^n \cdot \cos n\theta \cdot n$ $\rightarrow$ 2

$\therefore ru_r = r n r^{n-1}\cos n\theta = nr^n \cos n\theta = v_\theta$

$rv_r = r n r^{n-1}\sin n\theta = nr^n \sin n\theta = -u_\theta$

$\Rightarrow$ CR eqns are satisfied $\Rightarrow f(z) = z^n$ is analytic $\rightarrow$ 1

Now $\frac{dw}{dz} = f'(z) = e^{-i\theta}(u_r + iv_r) = e^{-i\theta}(nr^{n-1}\cos n\theta + i\, nr^{n-1}\sin n\theta)$ $\rightarrow$ 1

$= nr^{n-1}\left[e^{-i\theta}(\cos n\theta + i\sin n\theta)\right]$

$= nr^{n-1}\left[e^{-i\theta}(e^{in\theta})\right]$

$= nr^{n-1}e^{i(n-1)\theta} = n(re^{i\theta})^{n-1} = nz^{n-1}$ $\rightarrow$ 1

$\Rightarrow f'(z) = nz^{n-1}$

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Q8 To find $f(z) = u+iv$ where $f(z)$ is analytic and

$$u(x,y) = x\sin x\cosh y - y\cos x\sinh y$$

$\because f(z) = u+iv$ is analytic u&v Satisfy CR eqns ie $u_x = v_y$, $u_y = -v_x$ $\rightarrow$ 1

$$\Rightarrow u_x = \cosh y\,(x\cdot\cos x + \sin x) - y\sinh y(-\sin x)$$

$$\Rightarrow u_x = \cosh y\,(x\cos x + \sin x) + y\sinh y\sin x \rightarrow 1$$

and $u_y = x\sin x\cdot\sinh y - \cos x(y\cdot\cosh y + \sinh y)$

But we know $u_y = -v_x$

$$\Rightarrow v_x = \cos x\,(y\cosh y + \sinh y) - x\sin x\sinh y \rightarrow 1$$

Also $f'(z) = u_x + iv_x \rightarrow 1$

$$= \cosh y(x\cos x + \sin x) + y\sinh y\sin x + i\left(\cos x\,(y\cosh y + \sinh y) - x\sin x\sinh y\right)$$

By Milne-Thompson method put $x = z$ & $y = 0 \rightarrow 1$

$$\Rightarrow f'(z) = \cosh 0\,(z\cos z + \sin z) + 0\cdot\sinh 0\sin z + i\left(\cos z\,(0\cdot\cosh 0 + \sinh 0) - z\sin z\sinh 0\right)$$

$$= \{1\,(z\cos z) + \sin z\} + i\{\cos z(0) - 0\}$$

$$f'(z) = z\cos z + \sin z \rightarrow 1$$

$$\Rightarrow f(z) = \int(z\cos z + \sin z)\,dz = \int z\cos z\,dz + \int\sin z\,dz$$

$$= z(\sin z) - (1)(-\cos z) + (-\cos z) + c$$

$$\Rightarrow f(z) = z\sin z + c. \rightarrow 1$$