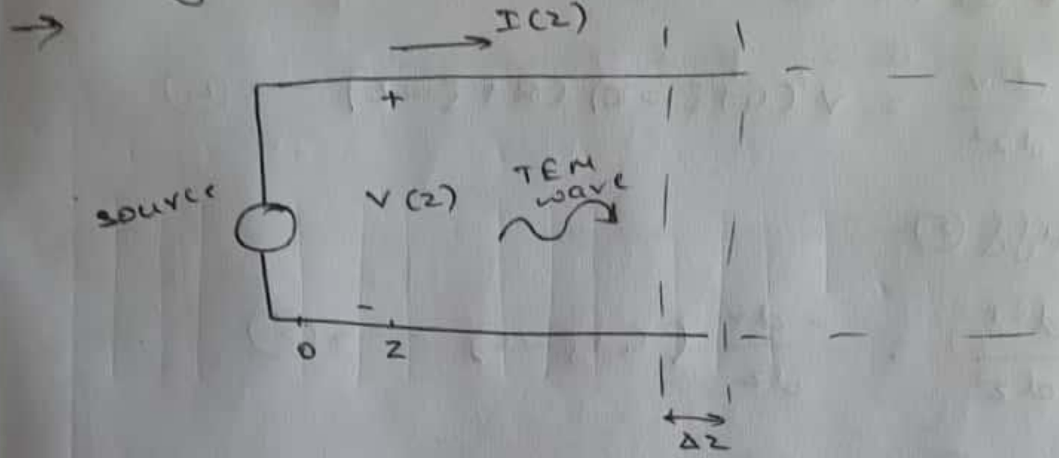
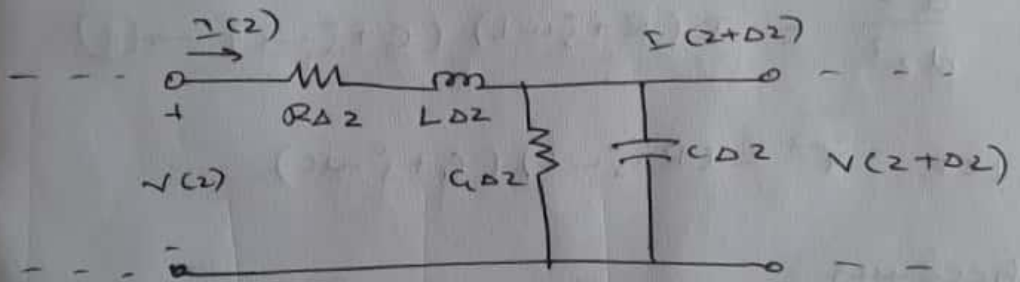
IAT-1 MWA



Δz can be written as



$$V(z) - I(z)\left[R\Delta z + j\omega L\Delta z\right] - (V+\Delta V)(z) = 0$$

$$-I(z)(R+j\omega L) = \frac{(V+\Delta V)(z) - V(z)}{\Delta z}$$

$$\underset{\Delta z\to 0}{lt} -I(z)(R+j\omega L) = \underset{\Delta z\to 0}{lt} \frac{\Delta V(z)}{dz}$$

$$-I(R+j\omega L) = \frac{dV}{dz} \rightarrow (1)$$

$$I(z) - (V+\Delta V)(G\Delta z + j\omega C\Delta z) = I(z) + \Delta I(z)$$

$$-(V+\Delta V)\Delta z(G+j\omega C) = I(z) + \Delta I(z) - I(z)$$

$$\underset{\Delta z\to 0}{lt} -(V+\Delta V)(G+j\omega C) = \underset{\Delta z\to 0}{lt} \frac{\Delta I(z)}{\Delta z}$$

$$-V(G+j\omega C) = \frac{dI}{dz} \rightarrow (2)$$

diff (1)

$$\frac{d^2V}{dz^2} = -\frac{dI}{dz}(R+j\omega L) \rightarrow (3)$$

sub (2) in (3)

$$\frac{d^2V}{dz^2} = V(G+j\omega C)(R+j\omega L) \rightarrow (4)$$

diff (2)

$$\frac{d^2I}{dz^2} = -\frac{dV}{dz}(G+j\omega C) \rightarrow (5)$$

sub (1) in (5)

$$\frac{d^2I}{dz^2} = I(R+j\omega L)(G+j\omega C) \rightarrow (6)$$

Let $\gamma^2 = (R+j\omega L)(G+j\omega C)$

(4) becomes

$$\frac{d^2V}{dz^2} = V(z)\gamma^2$$

$$\boxed{V(z) = V_i e^{-\gamma z} + V_r e^{\gamma z}}$$

(6) becomes

$$\frac{d^2I}{dz^2} = I(z)\gamma^2$$

$$\boxed{I(z) = I_i e^{-\gamma z} - I_r e^{\gamma z}}$$

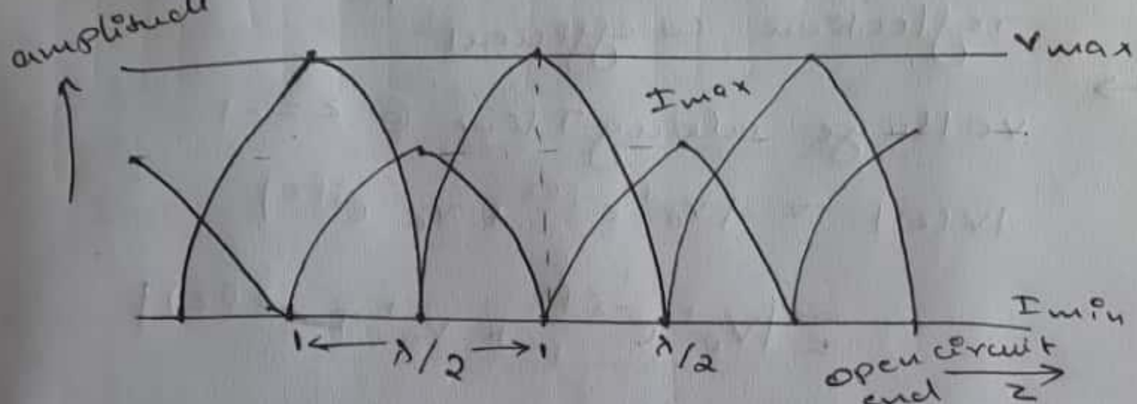
where, i - forward travelling wave

r - reverse travelling wave.

3) a) what are standing waves? Draw the standing wave pattern for
i) open circuit termination
ii) short circuit termination
iii) Matched load

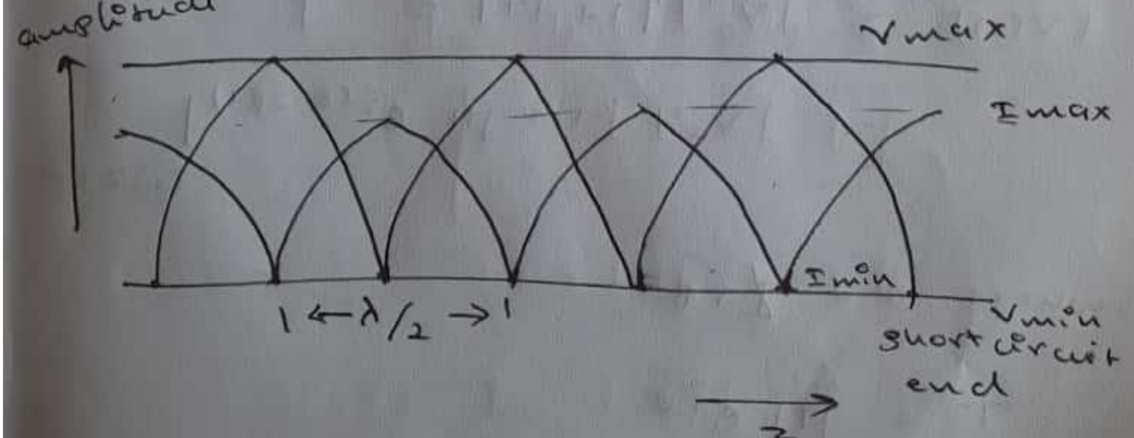
→ standing wave is two travelling waves components add in phase at same points and substract at other points

* open circuit ~~transm.~~ termination



$Z_L = \infty$, $\Gamma = 1$

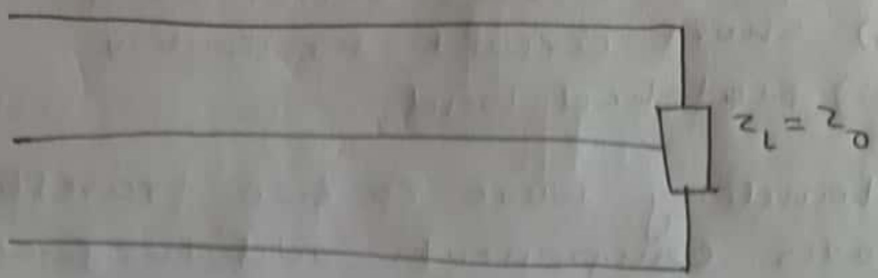
* Short circuit termination



$Z_L = 0$ $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$\Gamma = -1$

* Matched load termination



$Z_L = Z_0$, $\Gamma = 0$

No standing wave

3b) Derive the relation b/w SWR & reflection co-efficient

→

voltage along T line @ $z=-l$

$$|V(z)| = |V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}|$$

$$= |V_0^+ e^{-j\beta z} + V_0^+ \Gamma e^{j\beta z})|$$

$\because \frac{V_0^-}{V_0^+} = \Gamma$, $V_0^- = \Gamma V_0^+$

$$|V(z)| = |V_0^+| |1 + |\Gamma| e^{2j\beta z}|$$

$$= |V_0^+| |1 + |\Gamma| e^{j(\theta - 2\beta l)}|$$

@ $z=-l$

where $\theta - \angle\Gamma$

$$\Gamma = |\Gamma| e^{j\theta}$$

voltage fluctuates along the line

$V_{max} = |V_0^+| |1 + |\Gamma||$ if $\theta = 0$

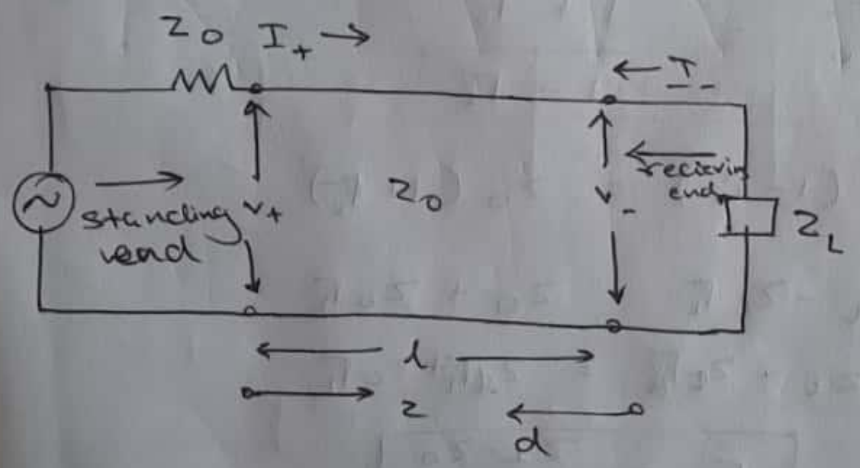
$V_{min} = |V_0^+| |1 - |\Gamma||$ $\theta = 180$

$$\frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\boxed{SWR = \frac{1+|\Gamma|}{1+|\Gamma|}}$$

4) Define reflection co-efficient (Γ). Derive the equation for Γ at the load end and also equation for Γ at a distance d from the load end

→ The reflection co-efficient is a parameter that discribes how much of an electromagnetic wave is reflect by an impedance discontinuity in the transmission medium.



$$V = V^+ e^{-\gamma z} + V^- e^{\gamma z} \rightarrow (1)$$

$$I = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z} \rightarrow (2)$$

$$V_L = V^+ e^{-\gamma l} + V^- e^{\gamma l} \rightarrow (3)$$

$$I_L = \frac{1}{Z_0}\left(V^+ e^{-\gamma l} - V^- e^{\gamma l}\right) \rightarrow (4)$$

$$Z_L = \frac{V_L}{I_L} = Z_0 \left[\frac{V_+ e^{\gamma l} + V^- e^{\gamma l}}{V^+ e^{-\gamma l} - V^- e^{\gamma l}}\right] \rightarrow (5)$$

$$\Gamma = \frac{V_{reflected}}{V_{incident}} = \frac{-I_r}{I_i}$$

$$\Gamma = \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}}$$

solving (5) for

$$\frac{Z_L}{Z_0} = \frac{V_+ e^{-\gamma l} + V^- e^{\gamma l}}{V_+ e^{-\gamma l} - V^- e^{\gamma l}}$$

$$= \frac{1 + \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}}}{1 - \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}}}$$

$$\frac{Z_L}{Z_0} = \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

$$Z_L (1 - \Gamma_L) = Z_0 (1 + \Gamma_L)$$

$$Z_L - Z_L \Gamma_L = Z_0 + Z_0 \Gamma_L$$

$$Z_L - Z_0 = Z_L \Gamma_L + Z_0 \Gamma_L$$

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\Gamma_L = |\Gamma_L| e^{j\theta_L}$$

Γ for distance d is

$$\Gamma_d = \frac{V_0^- e^{\gamma (l-d)}}{V^+ e^{-\gamma (l-d)}}$$

$$\Gamma_d = \frac{V^- e^{\gamma l}}{V^+ e^{-\gamma l}} e^{-2\gamma d}$$

$$= \Gamma_L e^{-2\gamma d}$$

$$= \Gamma_L e^{-2\alpha d} \cdot e^{-2j\beta d}$$

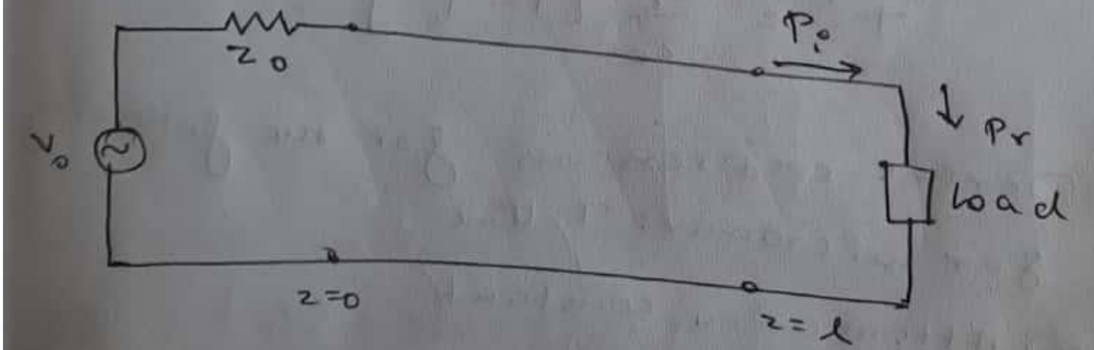
$$\boxed{\Gamma_d = |\Gamma_L| e^{-2\alpha d} e^{j(\theta_L - 2\beta d)}}$$

5) Define transmission co-efficient. Derive an expression for transmission co-efficient in the T L(?). Also derive the relationship between Γ and T.

→ The transmission co-efficient is a measure of how much of an electromagnetic wave (light) passes through a surface.

$$T = \frac{\text{transmitted voltage / current}}{\text{Incident voltage / current}}$$

$$= \frac{V_r}{V_i} = \frac{I_r}{I_i}$$



$$V_+ e^{-\gamma l} + V_- e^{\gamma l} = V_{tr} e^{-\gamma l} \rightarrow (1)$$

$$\frac{V_+}{Z_0} e^{-\gamma l} - \frac{V_-}{Z_0} e^{\gamma l} = \frac{V_{tr}}{Z_s} e^{-\gamma l} \rightarrow (2)$$

from ①

$$1 + \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{V_{tr} e^{-\gamma l}}{V_+ e^{-\gamma l}} = \frac{V_{tr}}{V_+}$$

$$1 + \Gamma_L = T$$

$$T = 1 + \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{2Z_L}{Z_L + Z_0}$$

In terms of power,

$$P_{transmitted} = P_{incident} - P_{reflected}$$

$$\frac{(V_{tr} e^{-\alpha l})^2}{2Z_L} = \frac{(V_+ e^{-\alpha l})^2}{2Z_0} - \frac{(V_- e^{\alpha l})^2}{2Z_0}$$

$$\frac{Z_0}{Z_L}\left[\frac{V_{tr} e^{-\alpha l}}{V_+ e^{-\alpha l}}\right]^2 = 1 - \left(\frac{V_- e^{\alpha l}}{V_+ e^{\alpha l}}\right)^2$$

$$\frac{Z_0}{Z_L} T^2 = 1 - (\Gamma_L)^2$$

$$T^2 = \frac{Z_L}{Z_0}\left[1 - \Gamma_L^2\right]$$

10) Derive expressions for the following for microwave T line.

a) Attenuation constant
b) phase constant
c) Characteristic impedence
d) velocity of propagation
e) wavelength.

→

a) attenuation constant

$$\Gamma = \sqrt{ZY}$$

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{j\omega L\left(\frac{R}{j\omega L}+1\right) j\omega C\left(\frac{G}{j\omega C}+1\right)}$$

$$= j\omega\sqrt{LC}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{1/2}$$

@ $R << \omega L$, $G << \omega C$

$$= j\omega\sqrt{LC}\left(1+\frac{R}{2j\omega L}\right)\left(+\frac{G}{2j\omega C}\right)$$

$$= j\omega\sqrt{LC}\left(1+\frac{G}{2j\omega C}+\frac{R}{2j\omega L}+\cancel{\frac{RG}{-j4\omega C L}}^{0}\right)$$

$$\Gamma = \frac{1}{2}\left(2j\omega\sqrt{LC}+\frac{\cancel{j\omega}\sqrt{LC}\,G}{\cancel{j\omega}C}+\frac{\cancel{j\omega}\sqrt{LC}\,R}{\cancel{j\omega}L}\right)$$

$$\alpha + j\beta = \frac{1}{2}\left(G\sqrt{\frac{L}{C}}+R\sqrt{\frac{C}{L}}\right)+j\omega\sqrt{LC}$$

$$\alpha = \left(\frac{R}{2}\sqrt{\frac{C}{L}}+\frac{G}{2}\sqrt{\frac{L}{C}}\right)$$

b) phase constant

$$\beta = \omega\sqrt{LC}$$

c) Charecteristic impedance

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$= \sqrt{\frac{j\omega L\left(\frac{R}{j\omega L}+1\right)}{j\omega C\left(\frac{G}{j\omega C}+1\right)}}$$

$$= \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{1/2}$$

$$= \sqrt{\frac{L}{C}}\left(1+\frac{1}{2}\frac{R}{j\omega L}\right)\left(1-\frac{1}{2}\frac{G}{j\omega C}\right)$$

$$= \sqrt{\frac{L}{C}}\left(1+\frac{R}{2j\omega L}-\frac{G}{2j\omega C}+\frac{RG}{4\omega^2 LC}\right)$$

@ $R << \omega L$, $G << \omega C$

$$\frac{1}{2}\left(\frac{R}{j\omega L}-\frac{G}{j\omega C}\right)+\frac{1}{4}\frac{RG}{\omega^2 LC} << 1$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

d) velocity of propagation (V_p)

$$V_p = \frac{\lambda}{t} = \lambda f = \frac{2\pi}{\beta} f$$

$$= \frac{2\pi f}{\omega\sqrt{LC}}$$

$$v_p = \frac{2\pi f}{2\pi f\sqrt{LC}}$$

$$\boxed{v_p = \frac{1}{\sqrt{LC}}}$$

e) wavelength (λ)

$$\lambda = \frac{2\pi}{\beta}$$

$$= \frac{2\pi}{\omega\sqrt{LC}}$$

$$= \frac{2\pi}{2\pi f\sqrt{LC}}$$

$$\boxed{\lambda = \frac{1}{f\sqrt{LC}}}$$

9) The characteristic impedance of a uniform transmission line is $2040\,\Omega$ @ $f = 800\,Hz$, $\gamma = 0.054\angle 87.9$

$R, L, G, C = ?$

→ $Z_0 = 2040\,\Omega$

$f = 800\,Hz$

$\gamma = 0.054\angle 87.9$

$\gamma = 1.97\times10^{-3} + 0.0539j$

$\alpha = 1.97\times10^{-3}$

$\beta = 0.0539$

~~$\alpha = 0.054$~~

~~$\beta = 87..$~~

$\omega = 2\pi f$

$= 2\times\pi\times800$

$= 5026.5\ Hz$

$G + j\omega C = 9.65 \times 10^{-7} + 2.64 \times 10^{-5} j$

$\boxed{G = 9.65 \times 10^{-7}}\ \Omega/m$

$\omega C = 2.64 \times 10^{-5}$

$C = \dfrac{2.64 \times 10^{-5}}{5026.5}$

$C = 5.25 \times 10^{-9}\ F/m$

$\boxed{C = 5.25\ nF/m}$

$\gamma Z_0 = R + j\omega L$

$R + j\omega L = (1.97 \times 10^{-3} + 0.0539j)(2040)$

$= 4.01 + 109.95j$

$\boxed{R = 4\ \Omega/m}$

$\omega L = 109.95$

$L = \dfrac{109.95}{\ldots}$