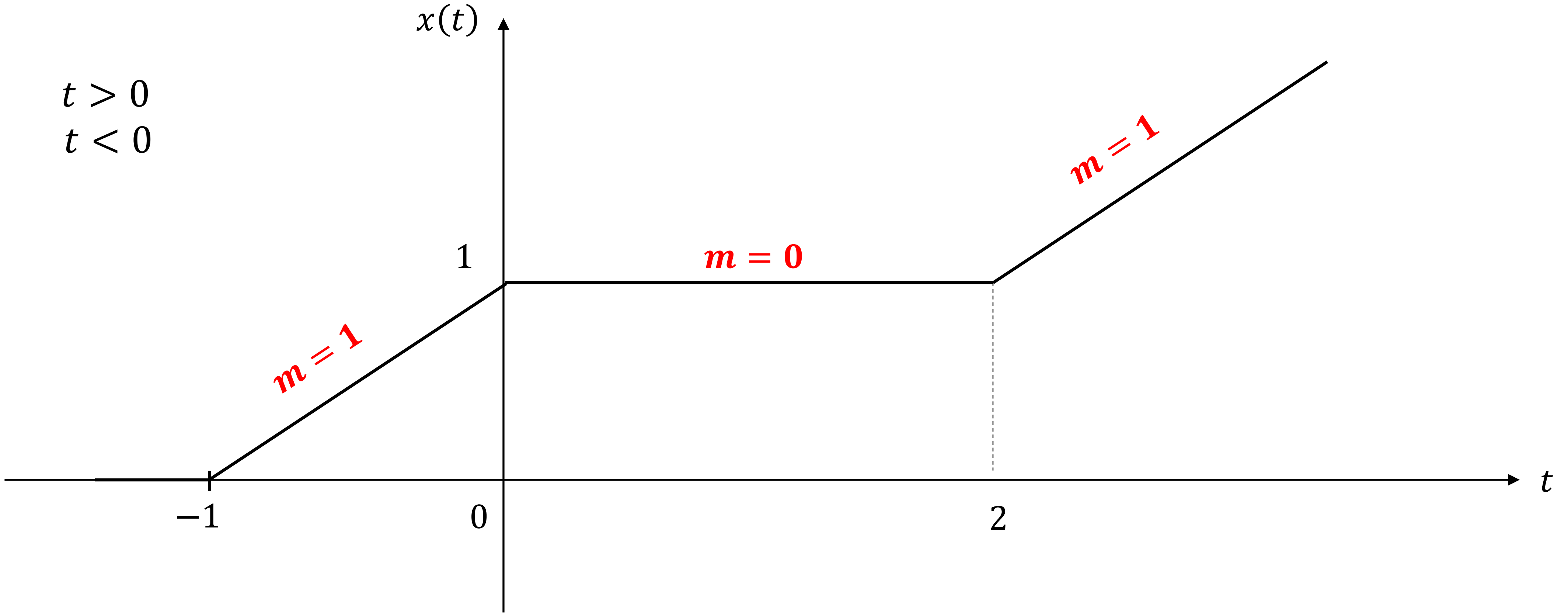
IAT 1 Solutions

Signals & Systems

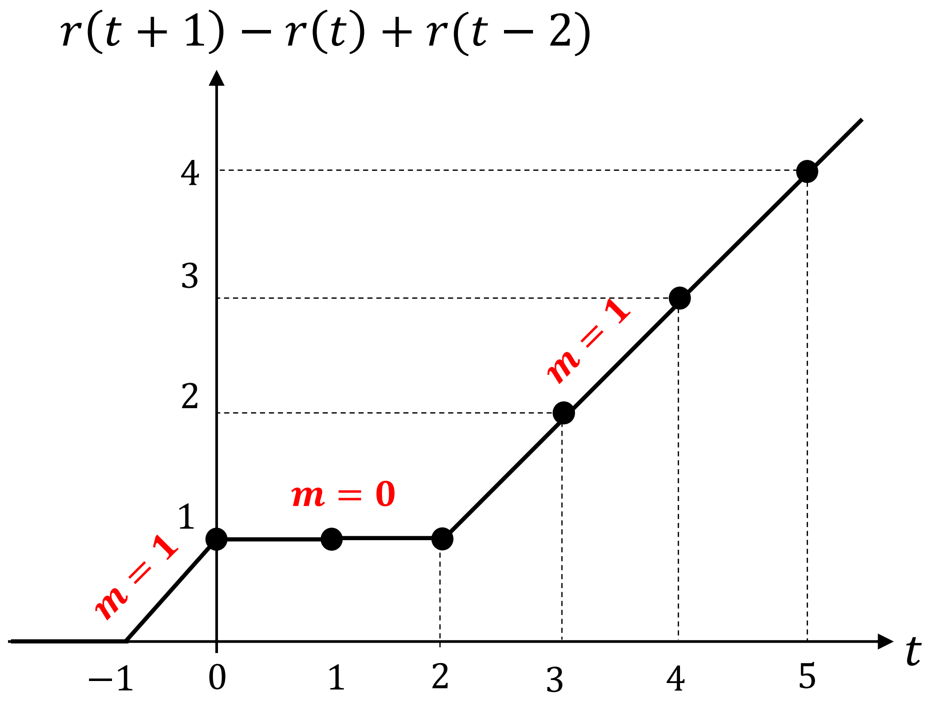
1 A. Sketch the following:

$$i) \quad y(t) = r(t+1) - r(t) + r(t-2)$$

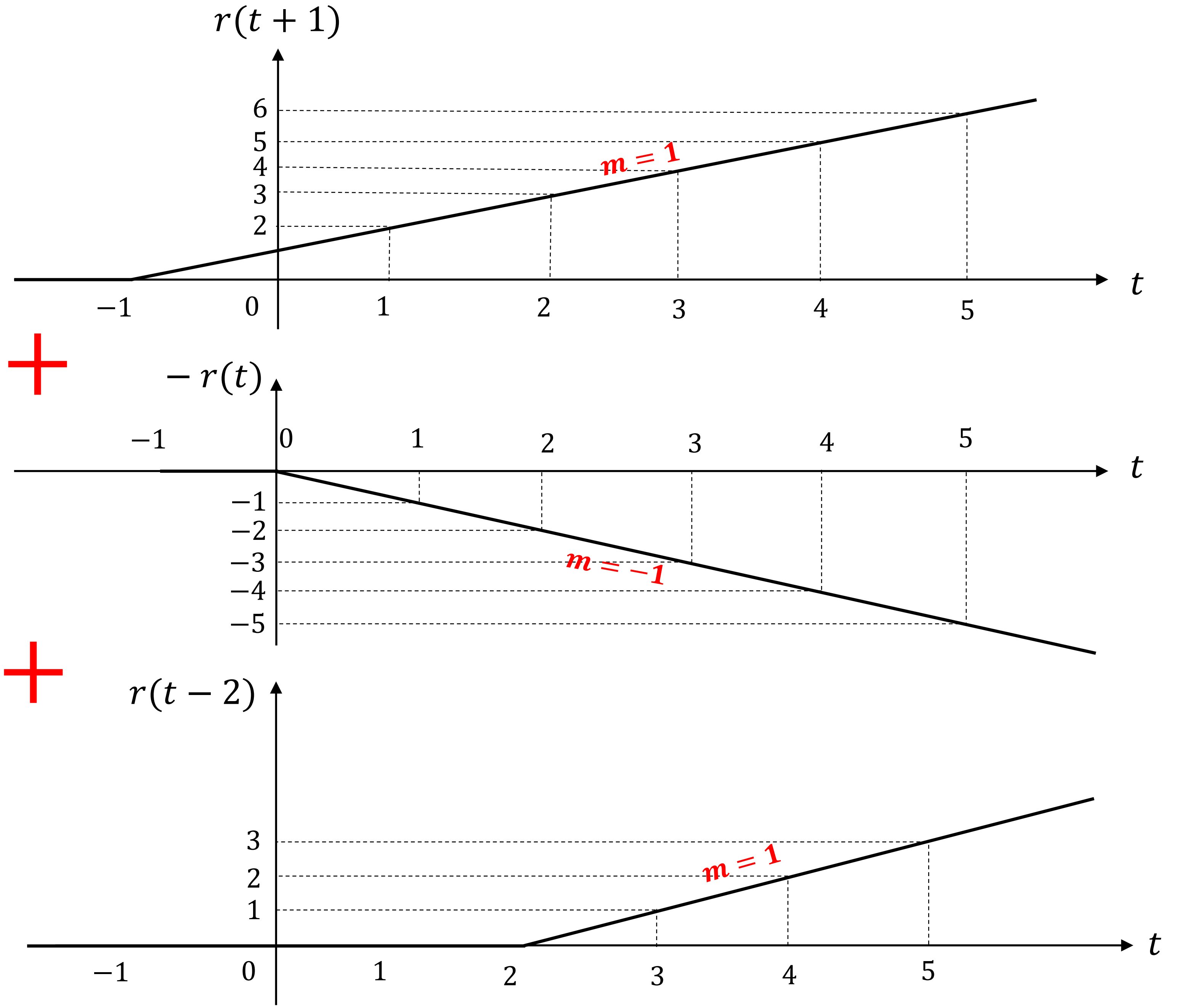
$$r(t) = \begin{cases} t, & t > 0 \\ 0, & t < 0 \end{cases}$$



$$-r(t) = -\begin{cases} t, & t > 0 \\ 0, & t < 0 \end{cases} \qquad r(t+1) = \begin{cases} t+1, & t+1 > 0 \\ 0, & t+1 < 0 \end{cases} \qquad r(t-2) = \begin{cases} t-2, & t-2 > 0 \\ 0, & t-2 < 0 \end{cases}$$

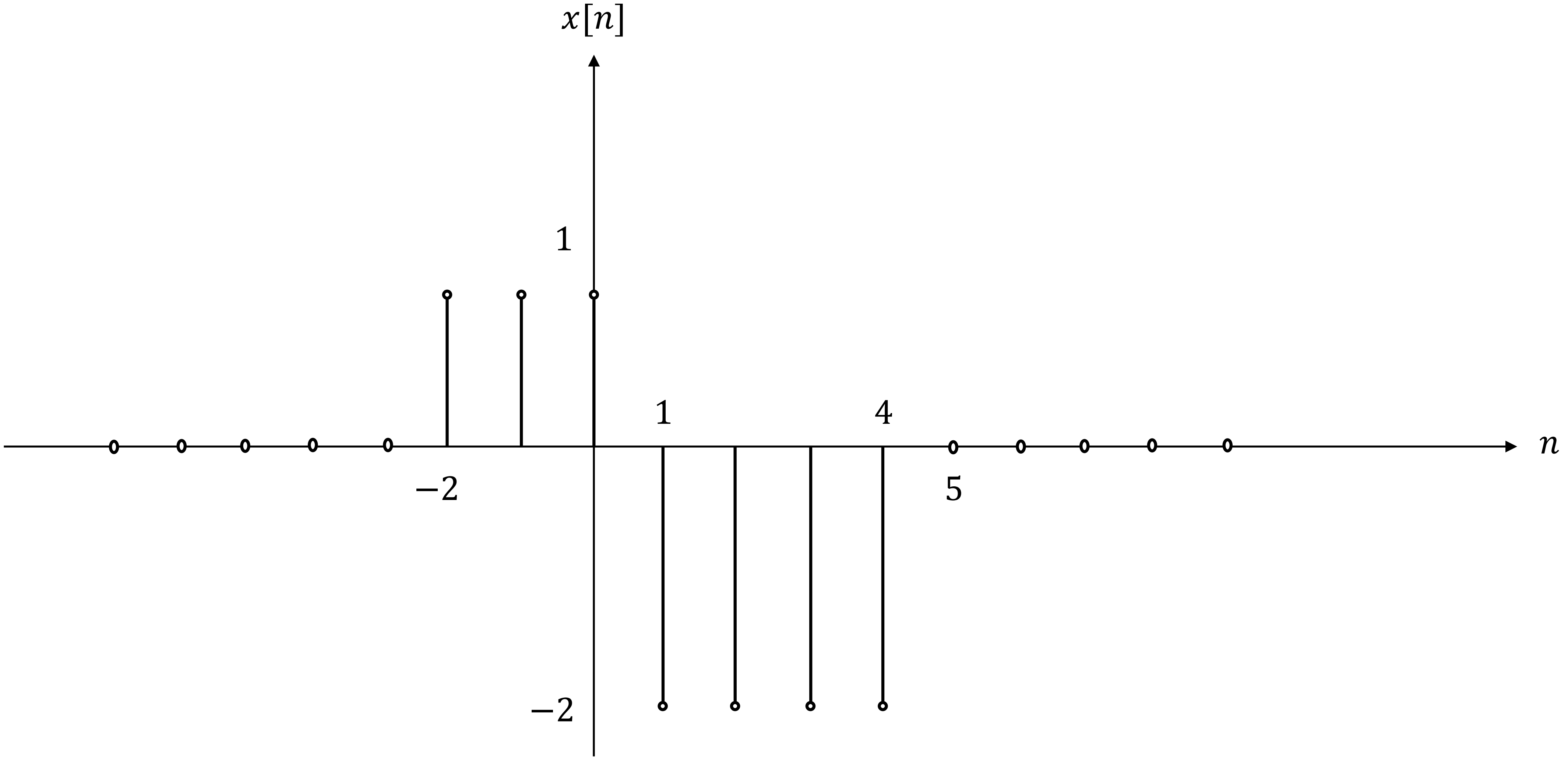


r(t + 1) − r(t) + r(t − 2)
4
3
2
1
m = 1
m = 0
m = 1
−1
0
1
2
3
4
5
t



r(t + 1)
6
5
4
3
2
m = 1
−1
0
1
2
3
4
5
t
+
− r(t)
−1
0
1
2
3
4
5
t
−1
−2
−3
−4
−5
m = −1
+
r(t − 2)
3
2
1
m = 1
−1
0
1
2
3
4
5
t

$ii)\quad x[n] = u[n+2] - 3u[n-1] + 2u[n-5]$

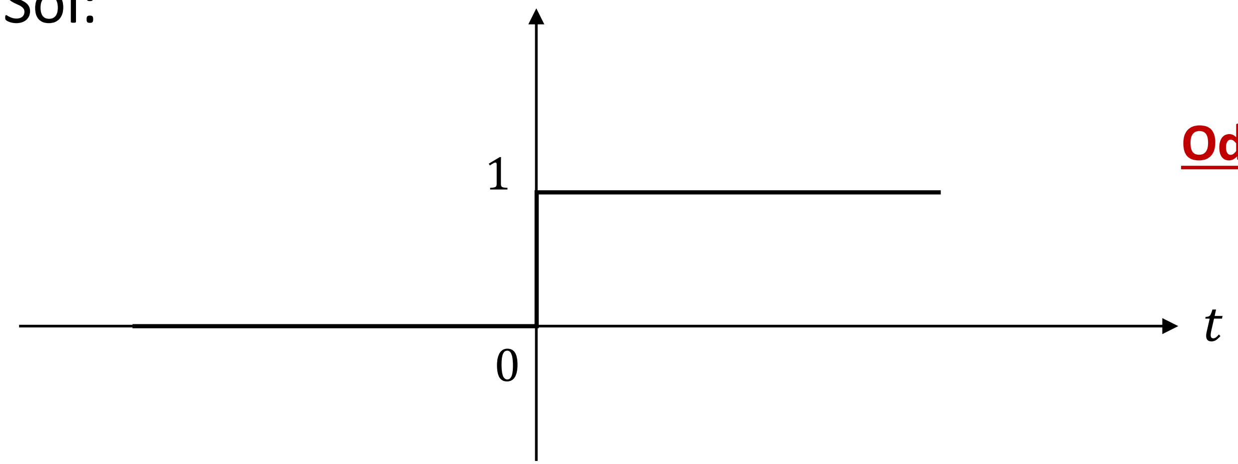


B. Find the even and odd parts of the following signals:

$i)\ y(t) = u(t)$

Sol:

$y(t) = u(t)$



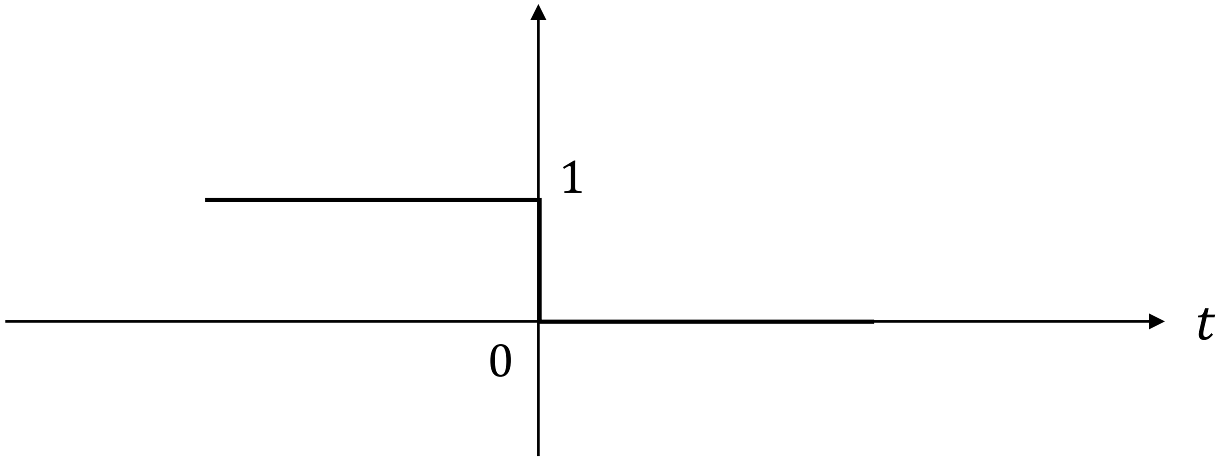
<u>Even component:</u>

$$y_e(t) = \frac{1}{2}[y(t) + y(-t)]$$

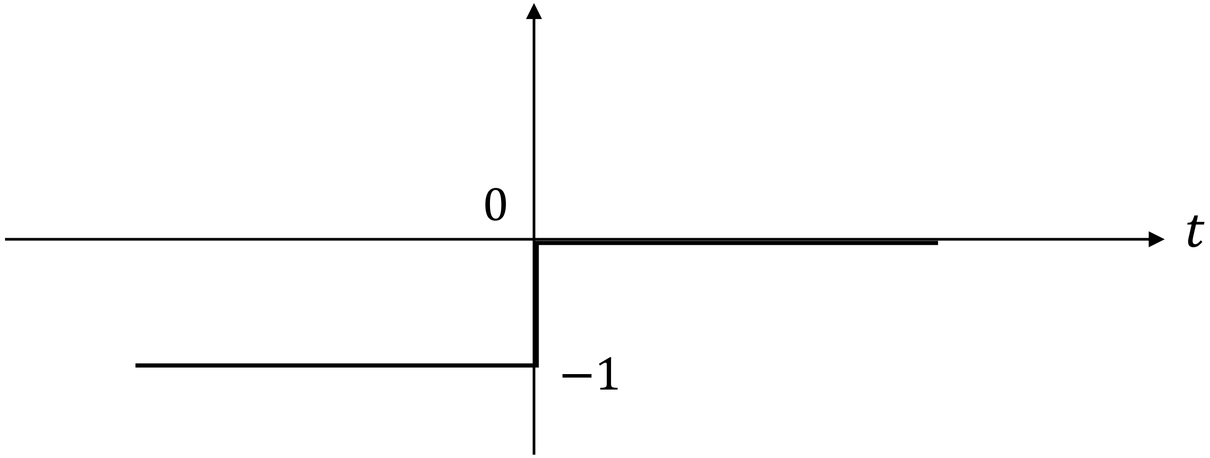
<u>Odd component:</u>

$$y_o(t) = \frac{1}{2}[y(t) - y(-t)]$$

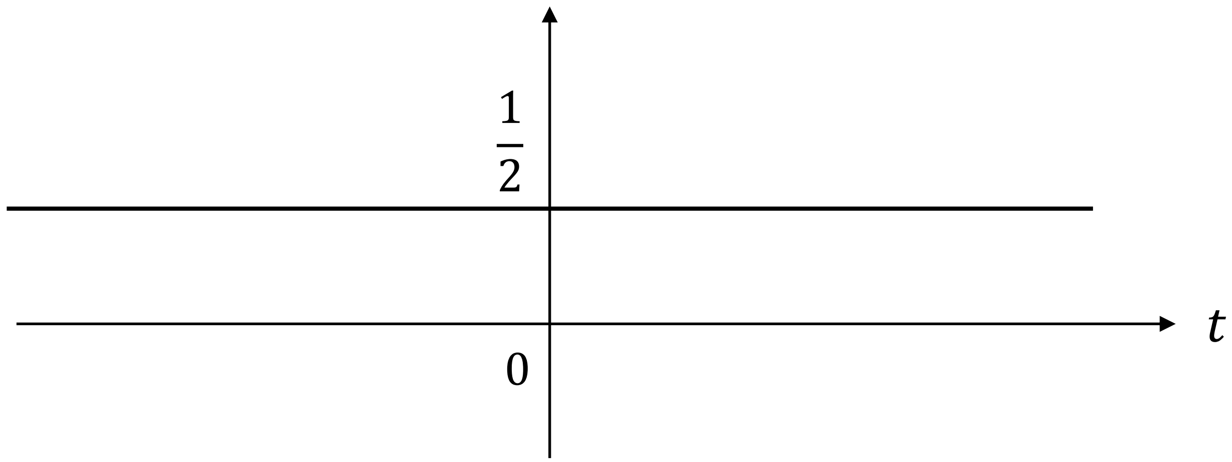
$y(-t) = u(-t)$



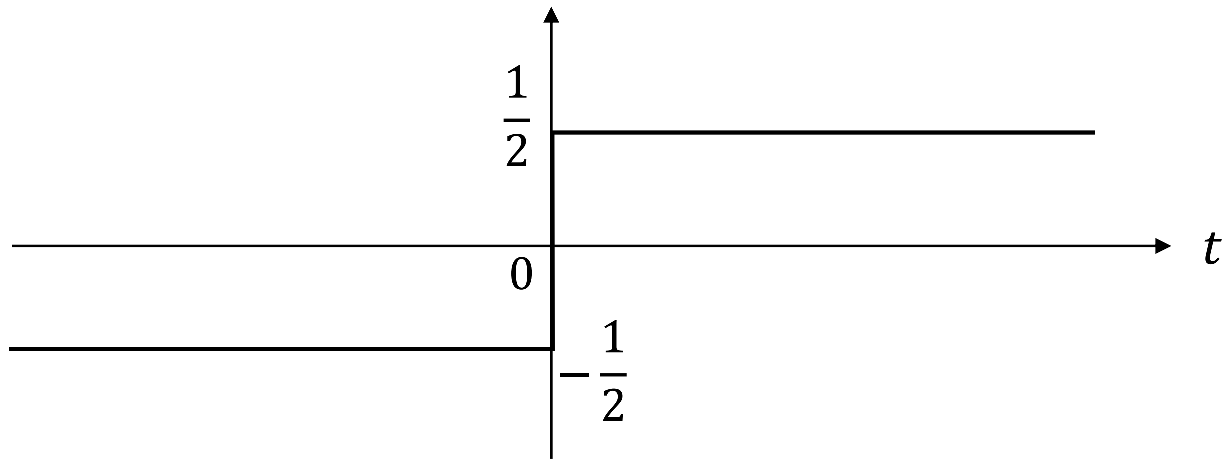
$-y(-t) = -u(-t)$



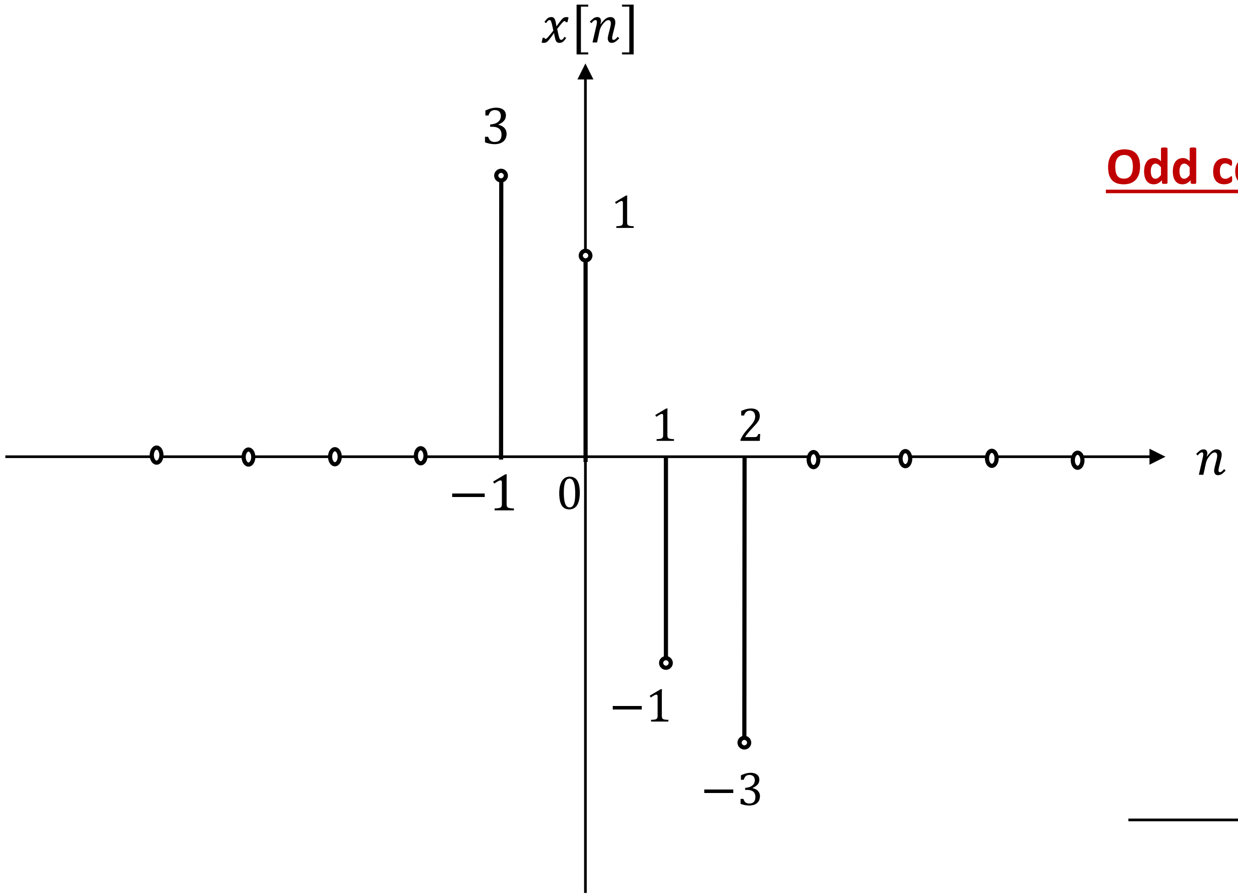
$$y_e(t) = \frac{1}{2}[y(t) + y(-t)]$$



$$y_o(t) = \frac{1}{2}[y(t) - y(-t)]$$



$ii)\ x[n] = 1 - 2n\,,\ \ -2 < n \leq 2$

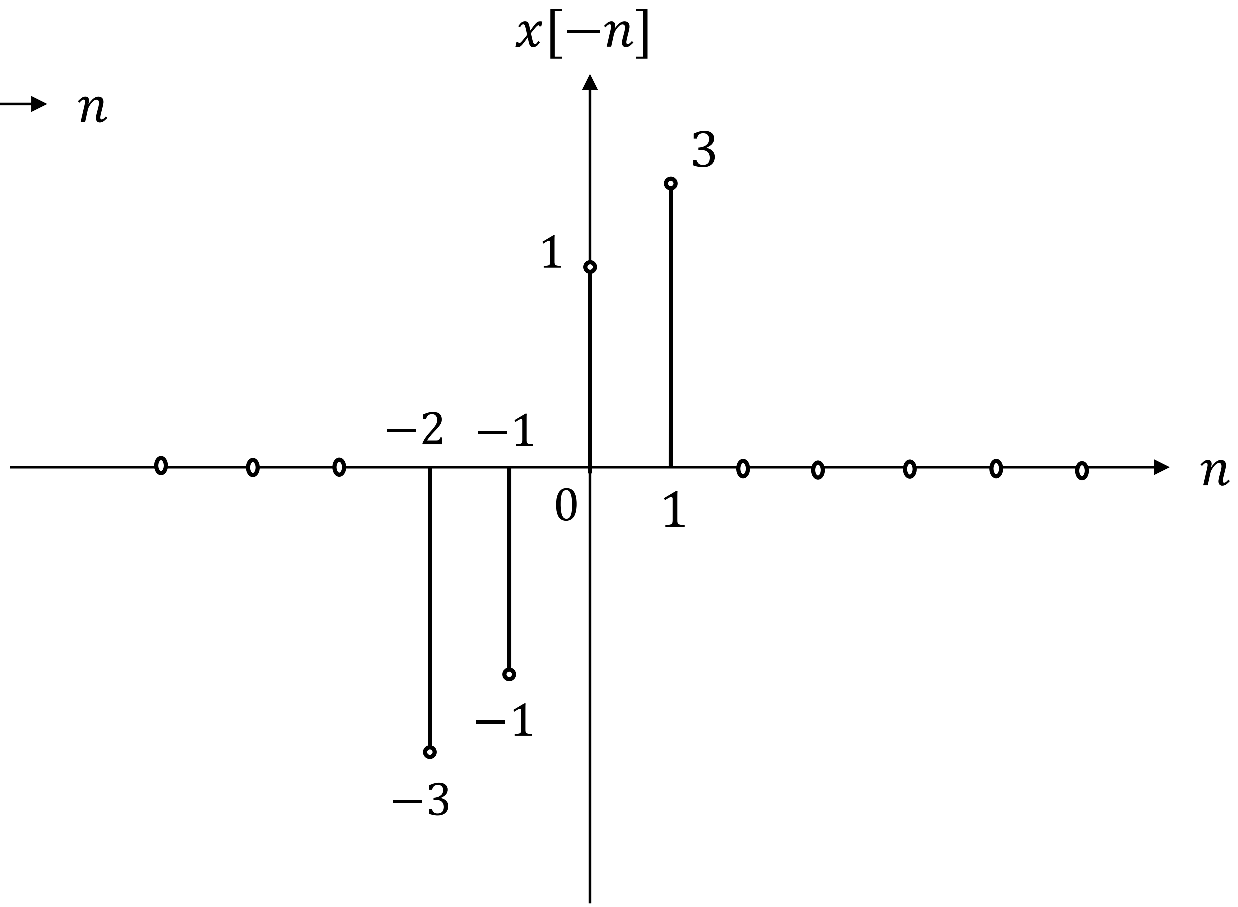


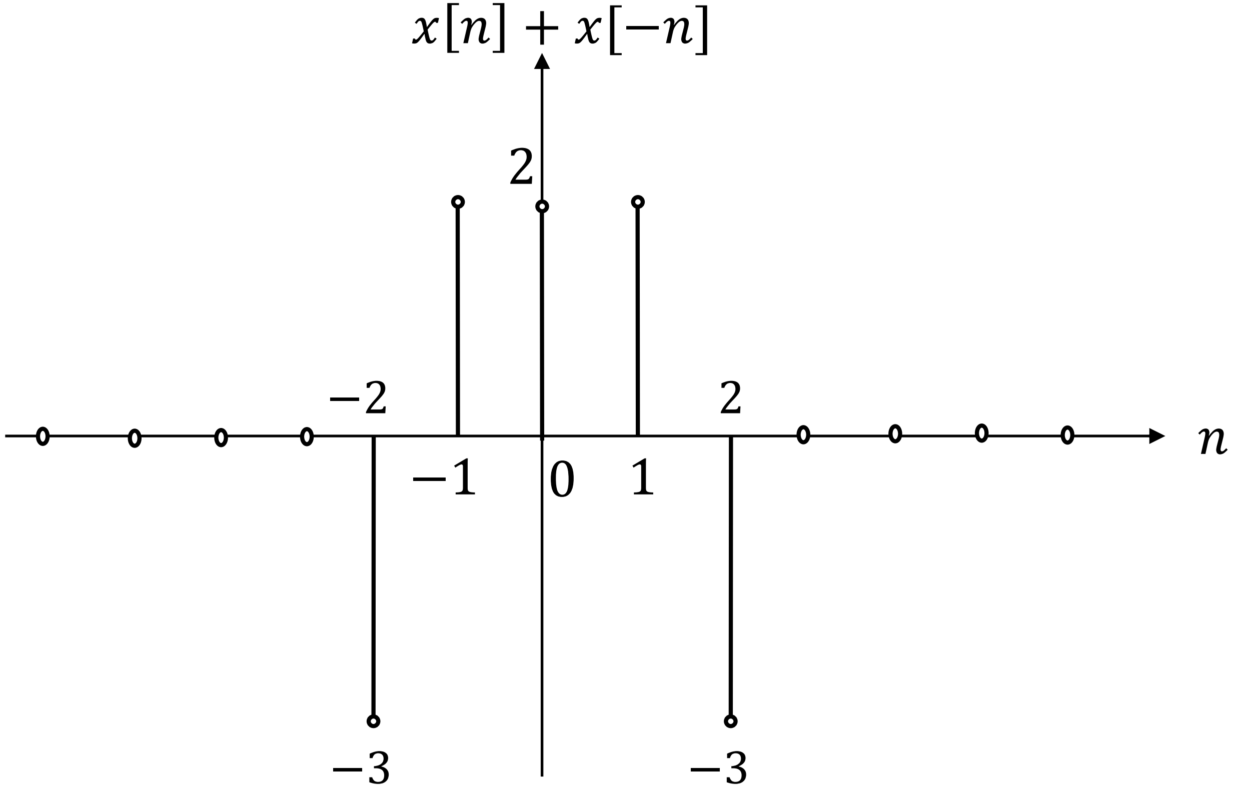
Even component:

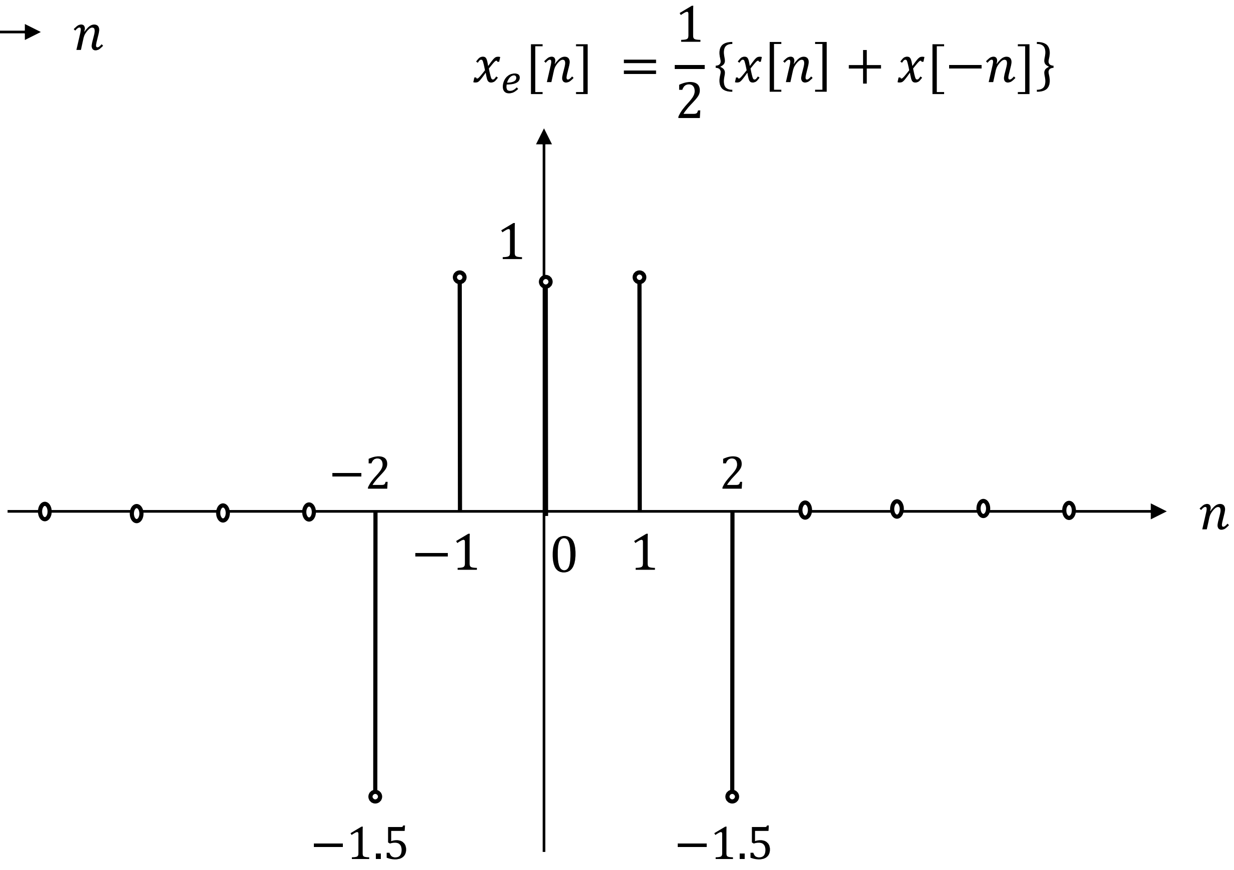
$$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$$

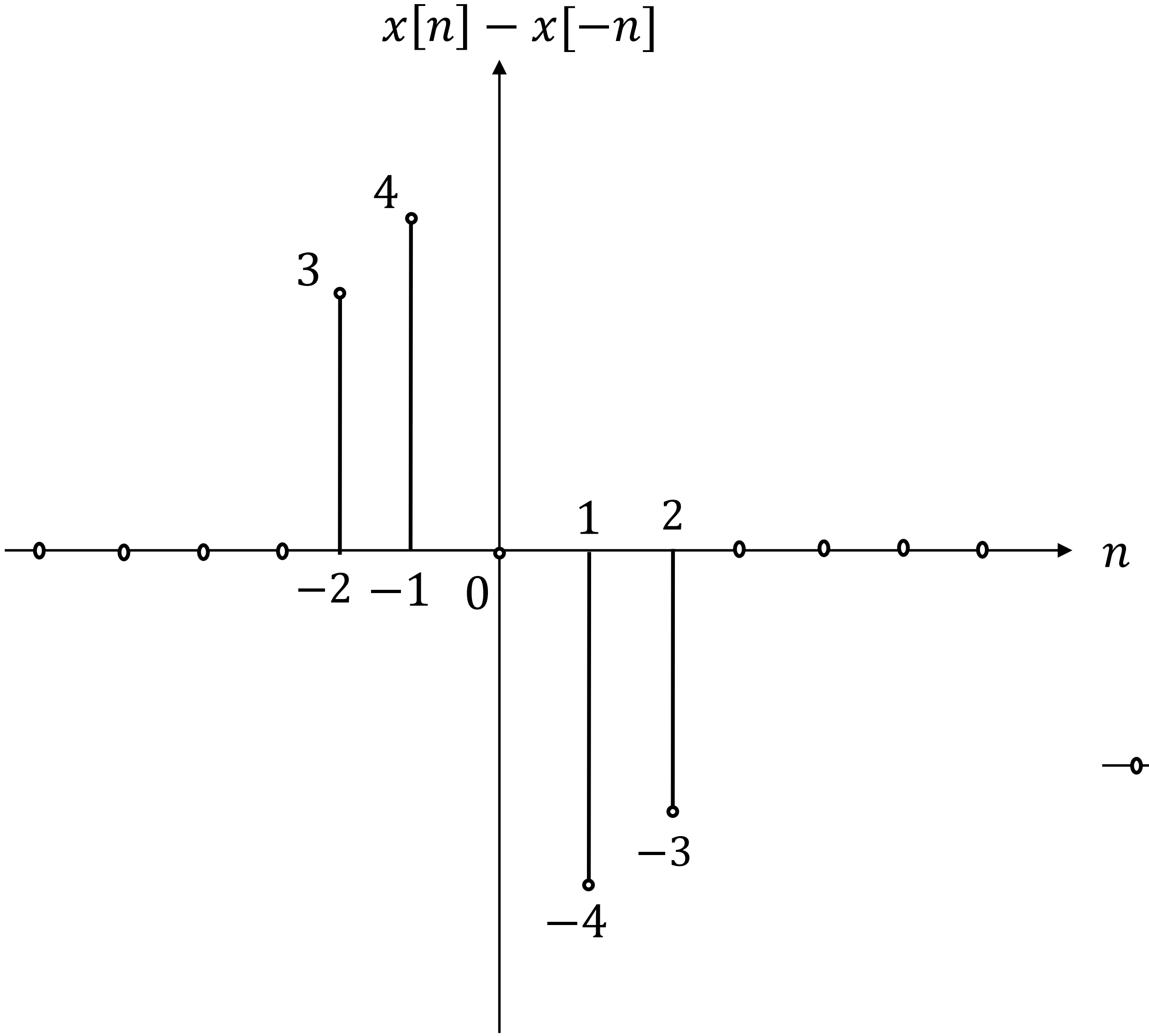
Odd component:

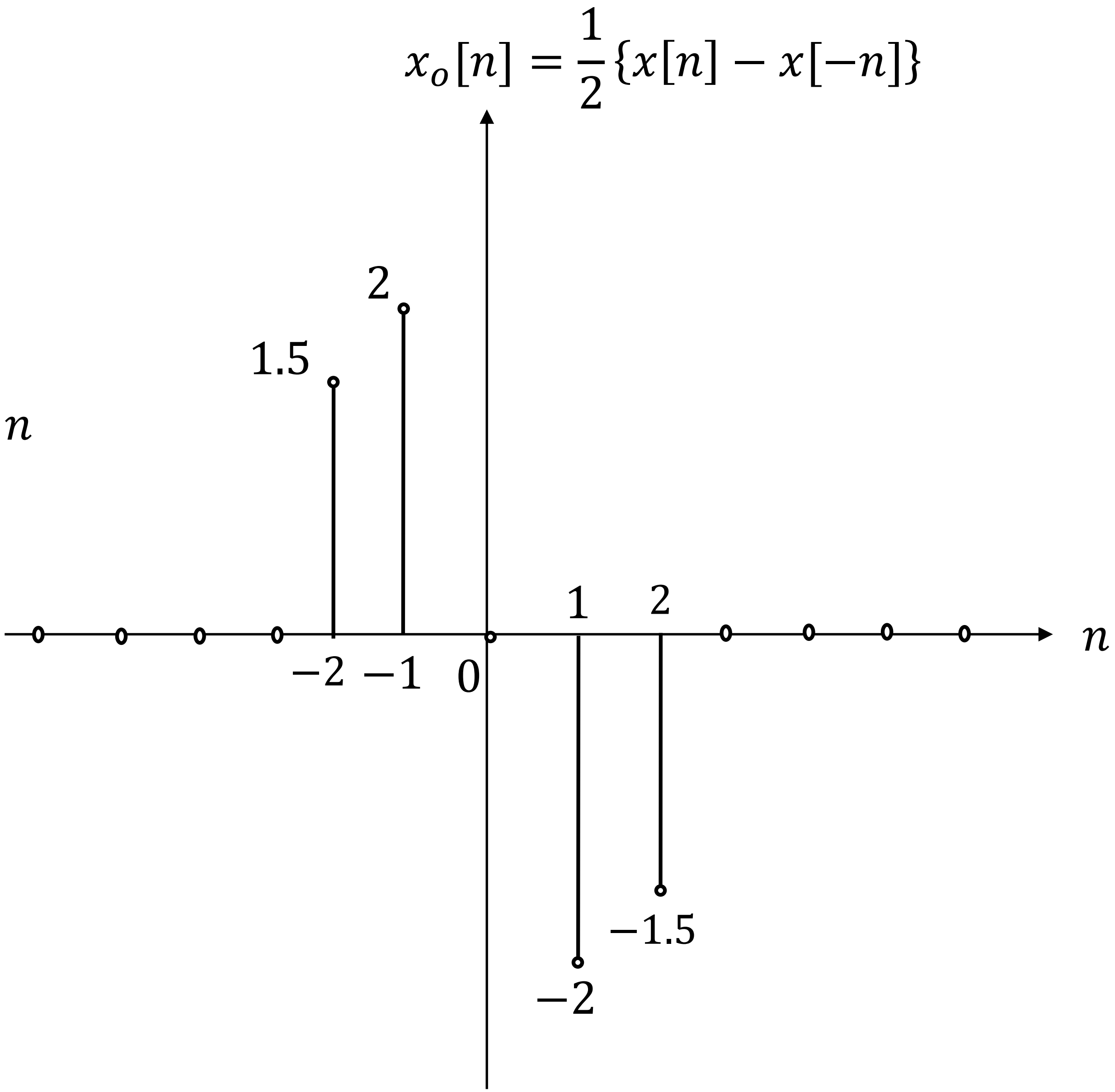
$$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$$




$x[n] + x[-n]$
2
-2
2
n
-1
0
1
-3
-3


$x_e[n] = \frac{1}{2}\{x[n] + x[-n]\}$
1
-2
2
n
-1
0
1
-1.5
-1.5


$x[n] - x[-n]$
3
4
1
2
n
-2
-1
0
-3
-4


$x_o[n] = \frac{1}{2}\{x[n] - x[-n]\}$
1.5
2
1
2
n
-2
-1
0
-1.5
-2

2. Check whether the following signals are periodic or not. If they are periodic find their period.

$i)\ \ x[n] = \cos\left(\frac{n\pi}{5}\right)\sin\left(\frac{n\pi}{3}\right)$

Sol: Using the identity $\cos(A)\sin(B) = \frac{1}{2}\{\sin(A+B) - \sin(A-B)\}$

$$x[n] = \frac{1}{2}\left\{\sin\left(\frac{n\pi}{5} + \frac{n\pi}{3}\right) - \sin\left(\frac{n\pi}{5} - \frac{n\pi}{3}\right)\right\} = \frac{1}{2}\left\{\sin\left(\frac{3n\pi + 5n\pi}{15}\right) - \sin\left(\frac{3n\pi - 5n\pi}{15}\right)\right\}$$

$$x[n] = \frac{1}{2}\left\{\sin\left(\frac{8n\pi}{15}\right) + \sin\left(\frac{2n\pi}{15}\right)\right\}$$

Ω_1 Ω_2

$$\Omega_1 = \frac{2\pi}{N_1}m$$

$$N_1 = \frac{2\pi}{\Omega_1}m = \frac{2\pi}{\frac{8\pi}{15}}m$$

$$N_1 = \frac{15}{4}m$$

Choose $m = 4, 8, 12, \ldots\ldots$

This results in $N_1 = 15, 30, 45, \ldots\ldots.$

$\boxed{N_1 = 15}$

$$\Omega_2 = \frac{2\pi}{N_2} m$$

$$N_2 = \frac{2\pi}{\Omega_2} m = \frac{2\pi}{\frac{2\pi}{15}} m$$

$$N = 15m$$

Choose $m = 1, 2, 3, \ldots\ldots$

This results in $N_1 = 15, 30, 45, \ldots\ldots.$

$$\boxed{N_2 = 15}$$

$$\frac{N_2}{N_1} = \frac{15}{15}$$

$$\frac{N_2}{N_1} = \frac{1}{1}$$ **(Rational number)**

$$N = N_2 = N_1$$

$$\boxed{N = 15}$$

Hence the fundamental period of the given sequence is $N = 15$.

$ii)\ \ x[n] = \cos(3n) + \sin(2\pi n)$

$$\underbrace{\cos(3n)}_{\Omega_1} \qquad \underbrace{\sin(2\pi n)}_{\Omega_2}$$

Sol:

$$\Omega_1 = \frac{2\pi}{N_1} m$$

For any integer m

$$N_1 = \frac{2\pi}{\Omega_1} m = \frac{2\pi}{3} m$$

This does not result in integer value of N_1

$$\boxed{N_1 = \frac{2\pi}{3} m}$$

$$\Omega_2 = \frac{2\pi}{N_2} m$$

$$N_2 = \frac{2\pi}{\Omega_2} m = \frac{2\pi}{2\pi} m$$

For integer $m = 1, 2, \ldots.$

$$N_2 = m$$

$$\boxed{N_2 = 1}$$

$\frac{N_2}{N_1}$ is irrational

Hence the given signal $x[n]$ is nonperiodic.

$$iii)\ x(t) = \sin(2\pi t) + \sin(3\pi t)$$

ω_1 ω_2

Sol:

$$\omega_1 = 2\pi$$

$$\frac{2\pi}{T_1} = 2\pi$$

$$\frac{1}{T_1} = 1$$

$$\boxed{T_1 = 1}$$

$$\omega_2 = 3\pi$$

$$\frac{2\pi}{T_2} = 3\pi$$

$$\frac{2}{T_2} = 3$$

$$\boxed{T_2 = \frac{2}{3}}$$

$$\frac{T_2}{T_1} = \frac{\frac{2}{3}}{1}$$

$$\frac{T_2}{T_1} = \frac{2}{3}$$

$$T = 3T_2 = 2T_1$$

rational number

$$T = 3\left(\frac{2}{3}\right) = 2(1)$$

$$\boxed{T = 2}$$

Hence the given signal $x(t)$ is periodic signal.

$iv)\ \ x[n] = (-1)^n, \qquad \forall n$

Sol: $x[n] = (-1)^n$

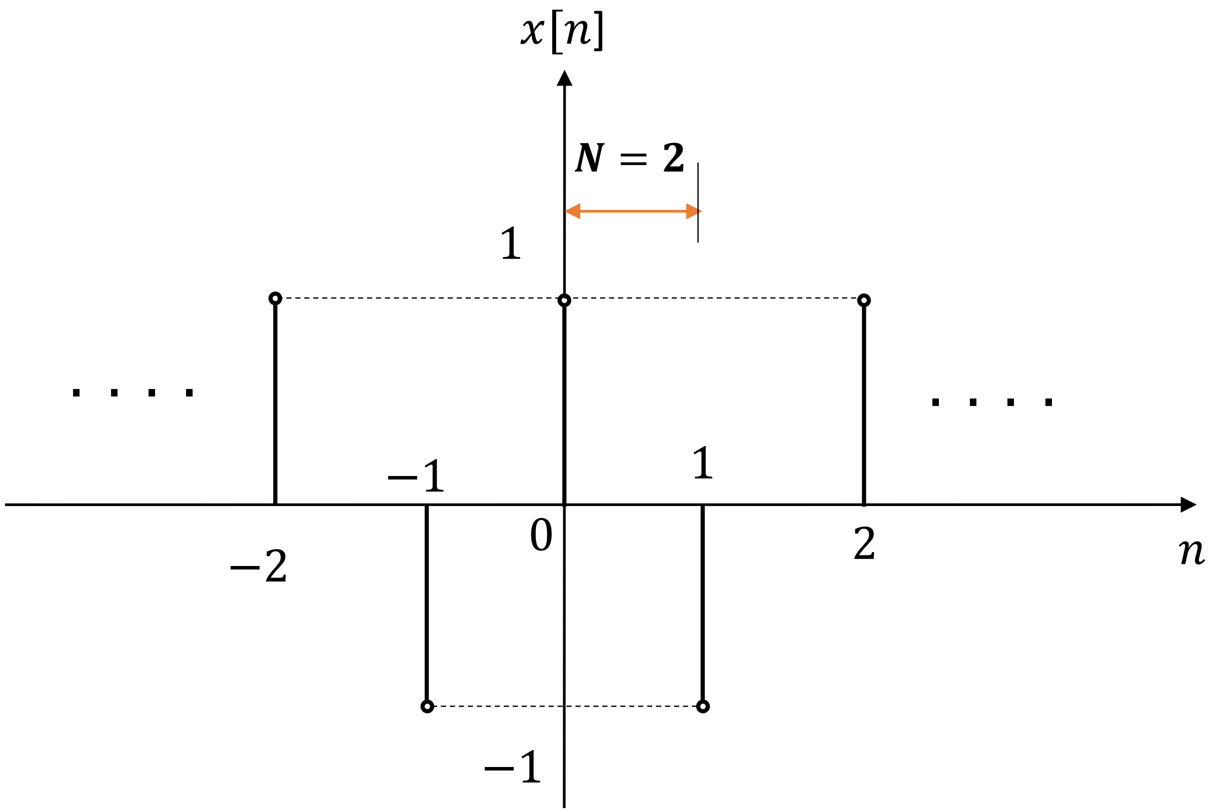
$n = -2, \quad x[-2] = (-1)^{-2} = 1$

$n = -1, \quad x[-1] = (-1)^{-1} = -1$

$n = 0, \quad x[0] = (-1)^{0} = 1$

$n = 1, \quad x[1] = (-1)^{-1} = -1$

$n = 2, \quad x[2] = (-1)^{2} = 1$



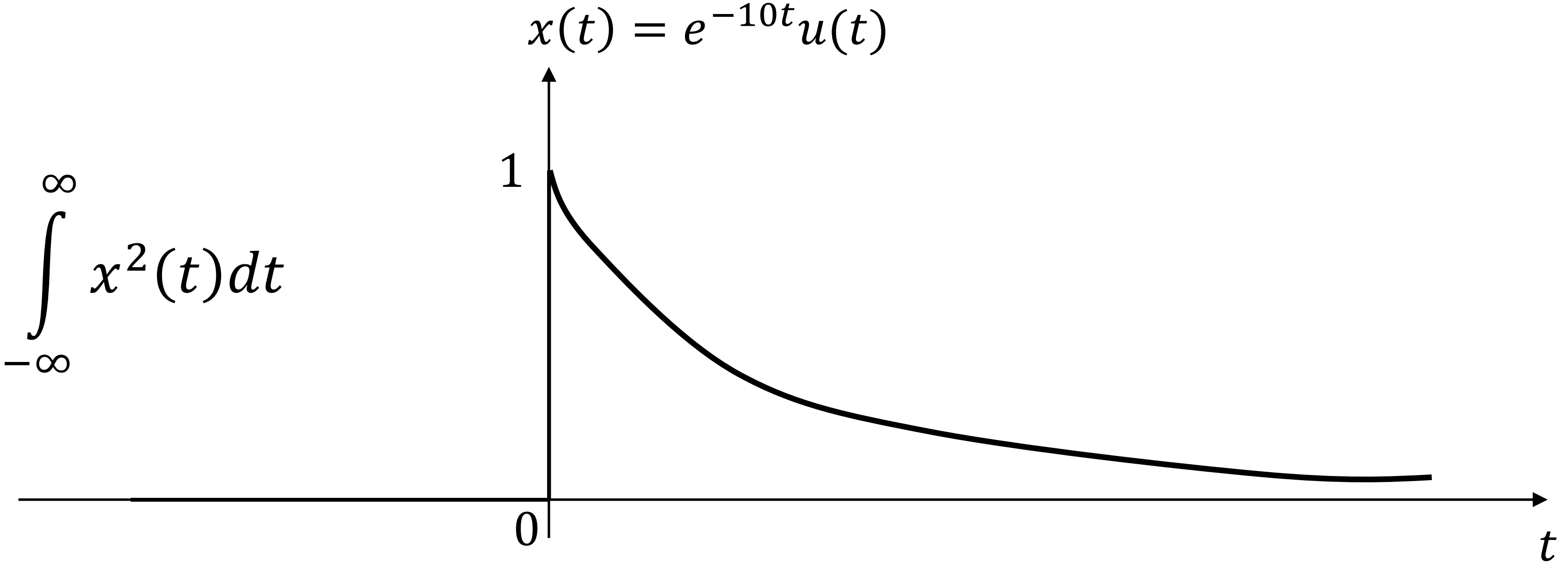
Hence the given signal $x[n]$ is periodic signal.

3. Determine if the following signals are energy or power signals. Obtain the energy and power values.

$i)\ x(t) = e^{-10t}u(t)$

Sol:

$$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt$$



$$E = \int_{0}^{\infty} x^2(t)dt$$

$$= \int_{0}^{\infty} (e^{-10t})^2 dt = \int_{0}^{\infty} e^{-20t} dt = \left[\frac{e^{-20t}}{-20}\right]_0^{\infty}$$

$$= -\frac{1}{20}[0-1]$$

$$\boxed{E = \frac{1}{20} \text{ Joules}}$$

Hence the given signal is an Energy signal.

$ii) \quad x[n] = \left(\frac{1}{2}\right)^n u[n]$

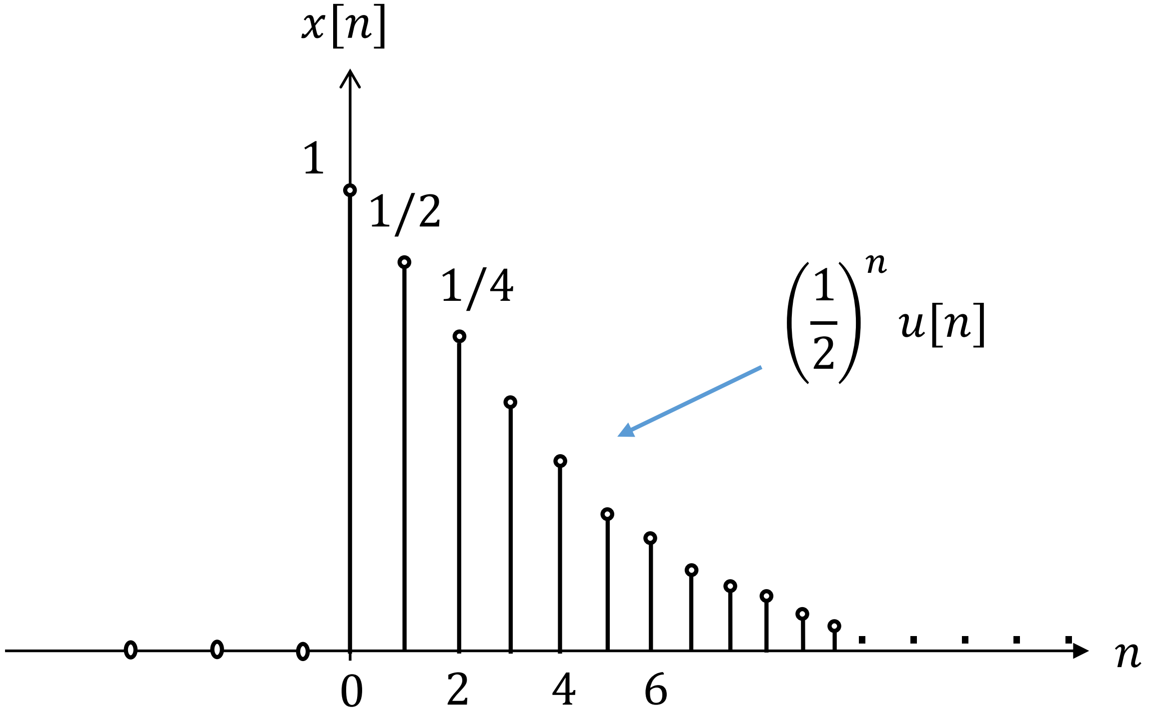
Sol:

$$E = \sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=0}^{\infty} x^2[n]$$

$$= \sum_{n=0}^{\infty} \left(\left(\frac{1}{2}\right)^n\right)^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1-\frac{1}{4}}$$

$$\boxed{E = \frac{4}{3}\,\mathrm{J}} < \infty$$



Hence the given signal $x[n]$ is an energy signal.

$iii)\ x(t) = \sin(4\pi t)$

$$\omega = 4\pi$$

$$\frac{2\pi}{T} = 4\pi$$

$$\frac{1}{T} = 2$$

$$\boxed{T = \frac{1}{2}\text{ sec}}$$

Sol:

$$E = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} (\sin(4\pi t))^2 dt = \infty$$

The given signal $x(t)$ is **not a energy signal**

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \frac{1}{T}\int_0^T x^2(t)dt$$

$$= \frac{1}{T}\int_0^T (\sin(4\pi t))^2 dt = \frac{1}{T}\int_0^T \sin^2(4\pi t)\,dt = \frac{1}{T}\int_0^T \frac{1}{2}[1-\cos(8\pi t)]dt$$

$$= \frac{1}{2T}\left\{\int_0^T dt - \int_0^T \cos(8\pi t)\,dt\right\} = \frac{1}{2T}\left\{[t]_{t=0}^{T} - \left[\frac{\sin(8\pi t)}{8\pi}\right]_{t=0}^{T}\right\}$$

$$P = \frac{1}{2T}\left\{T - \frac{1}{8\pi}[\sin(8\pi T) - 0]\right\}$$

For $T = \frac{1}{2}$ sec, $P = \left\{\frac{1}{2} - \frac{1}{8\pi}[\sin(4\pi) - 0]\right\}$

$$\boxed{P = \frac{1}{2}\mathrm{W}}$$

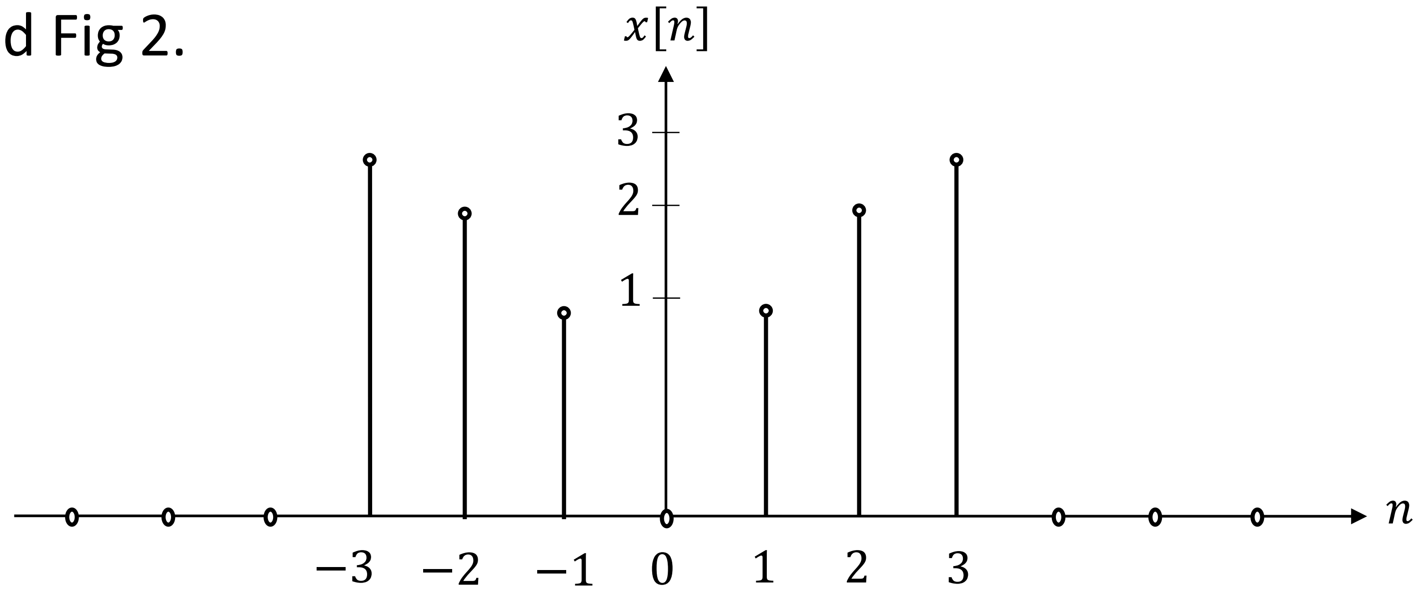
Hence the given signal is a power signal.

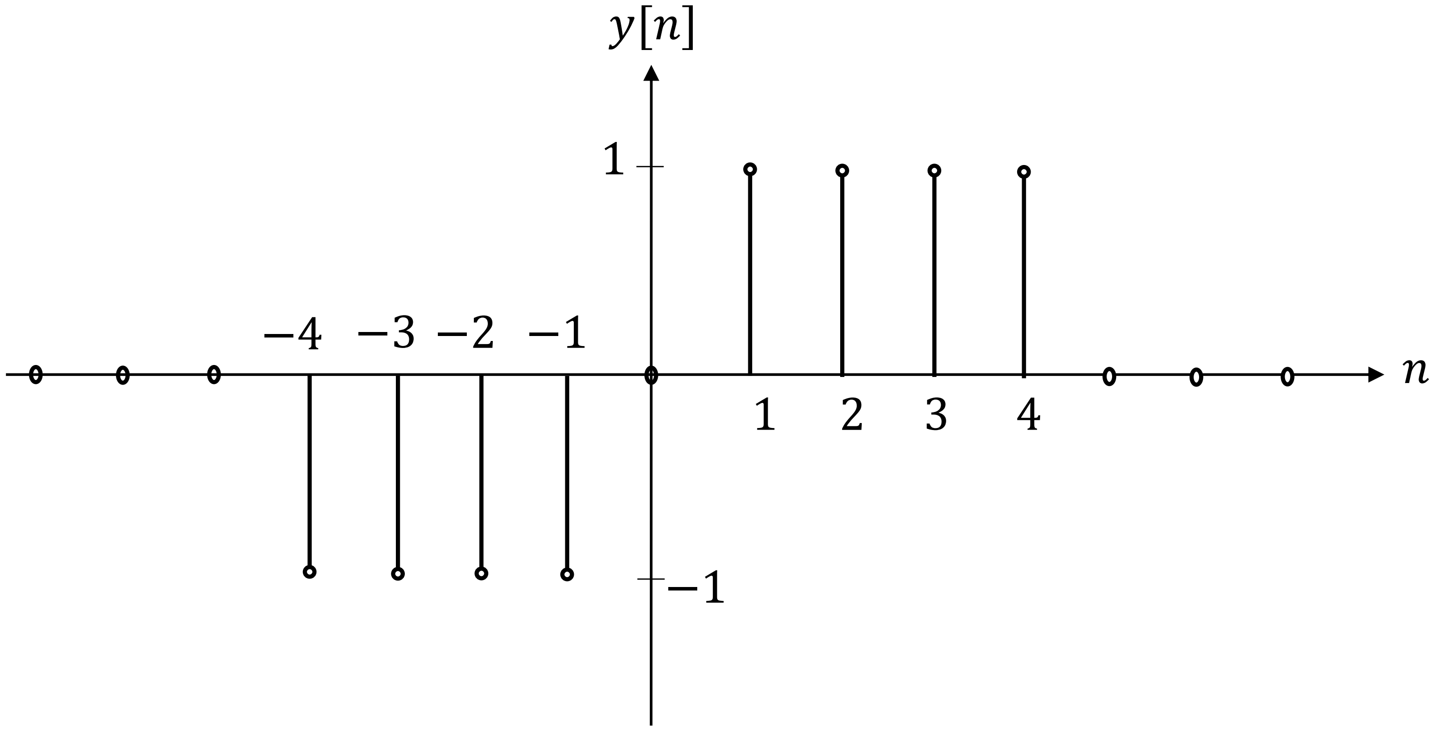
4. Let $x[n]$ and $y[n]$ be given in Fig 1 and Fig 2.
Sketch the following signals.

$i)\ x[2n] + y[n-4]$

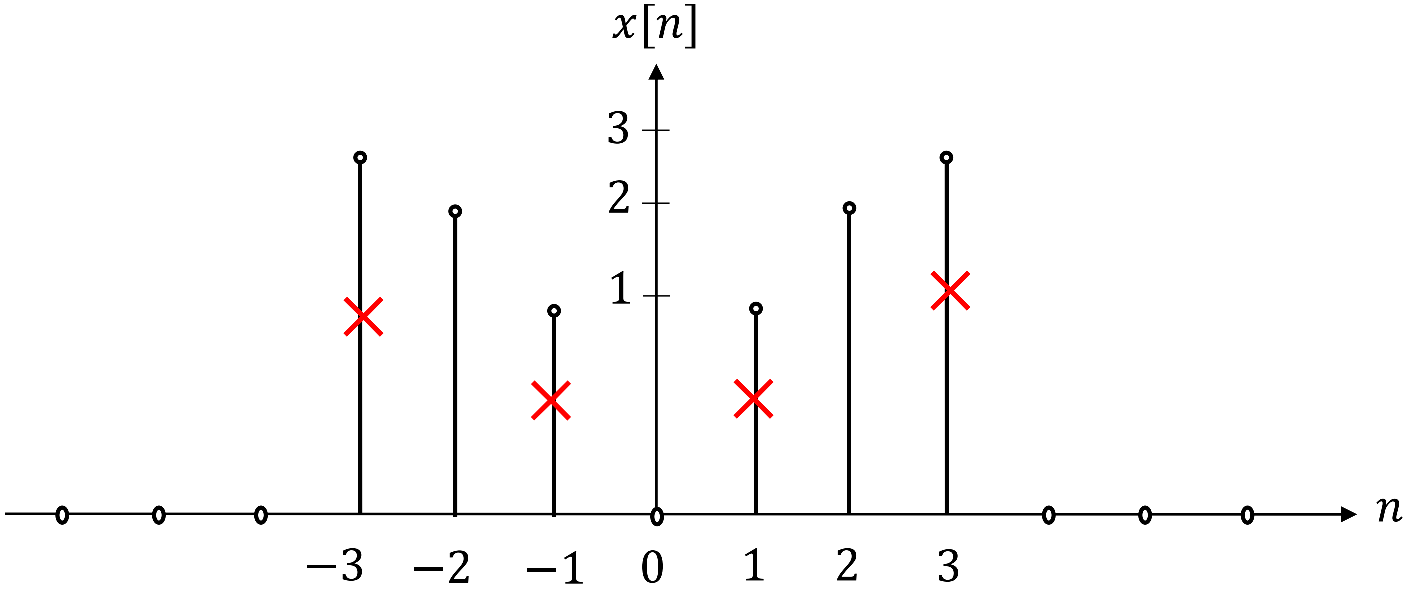
$ii)\ x[-n]y[-n]$

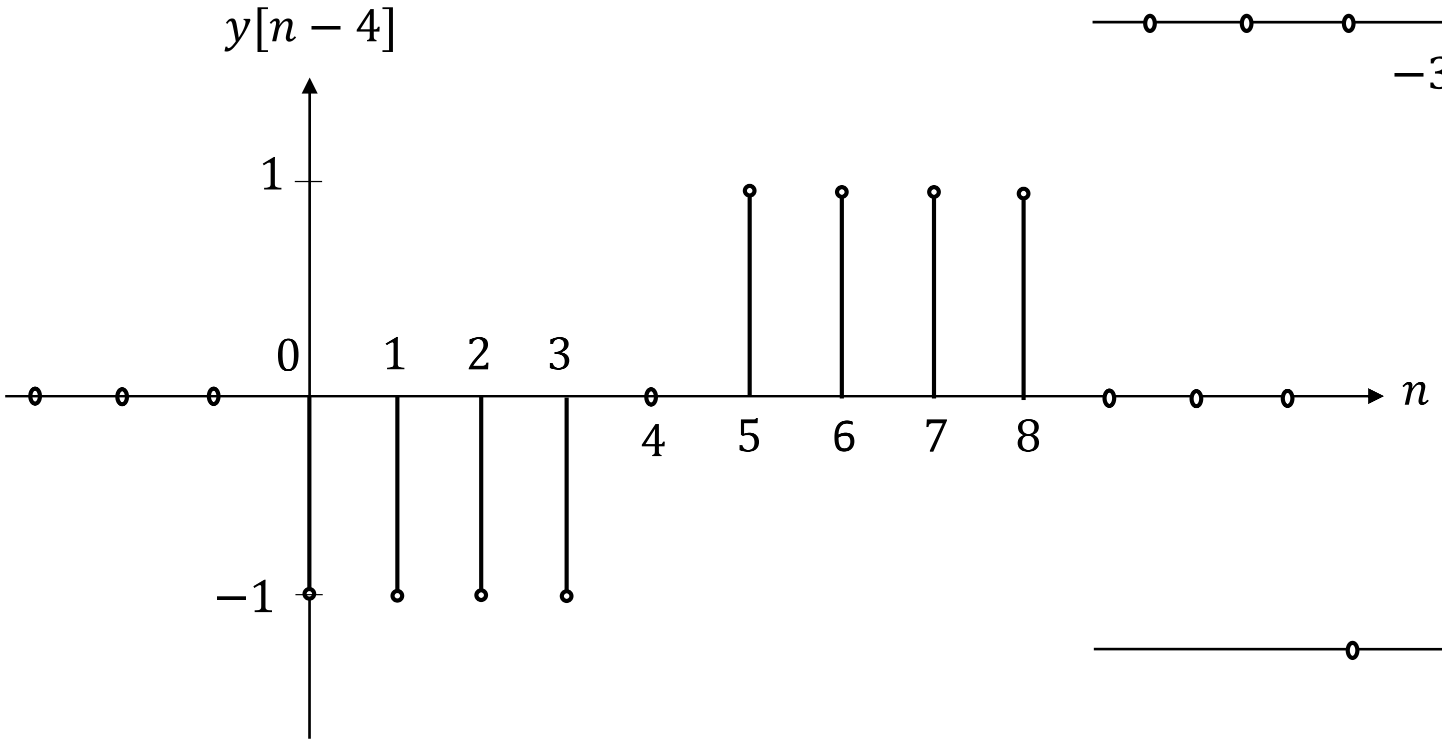
$iii)\ x[n+2]\, y[6-n]$

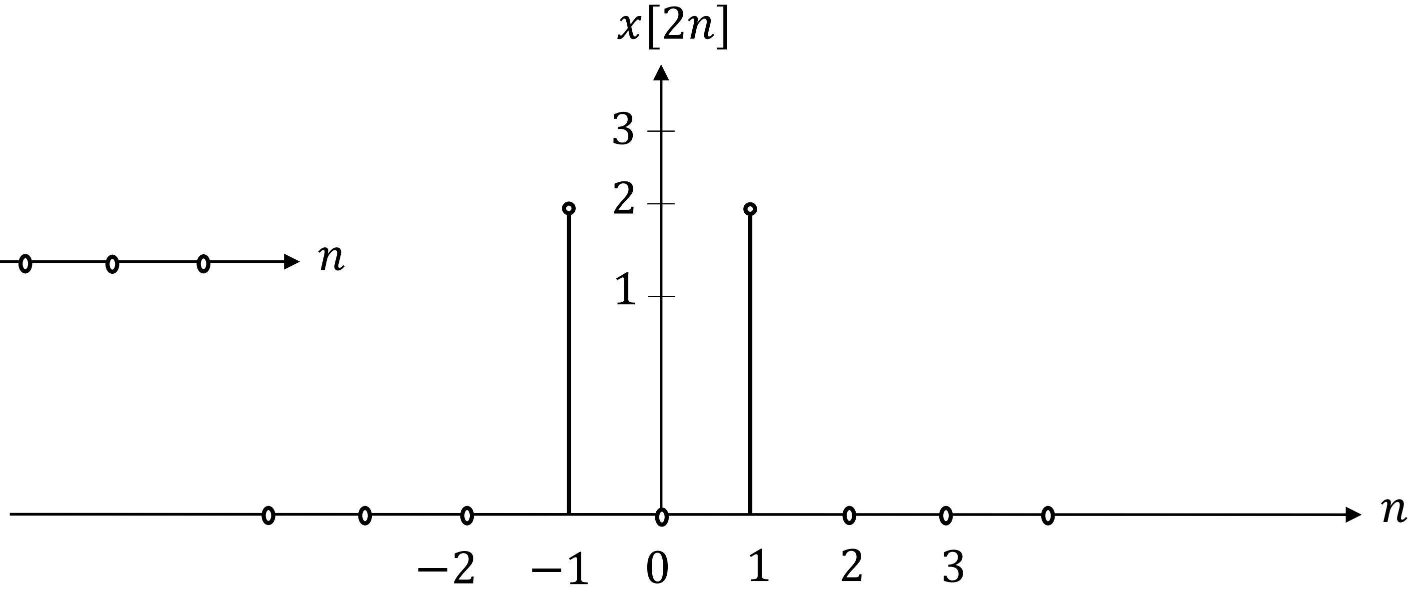


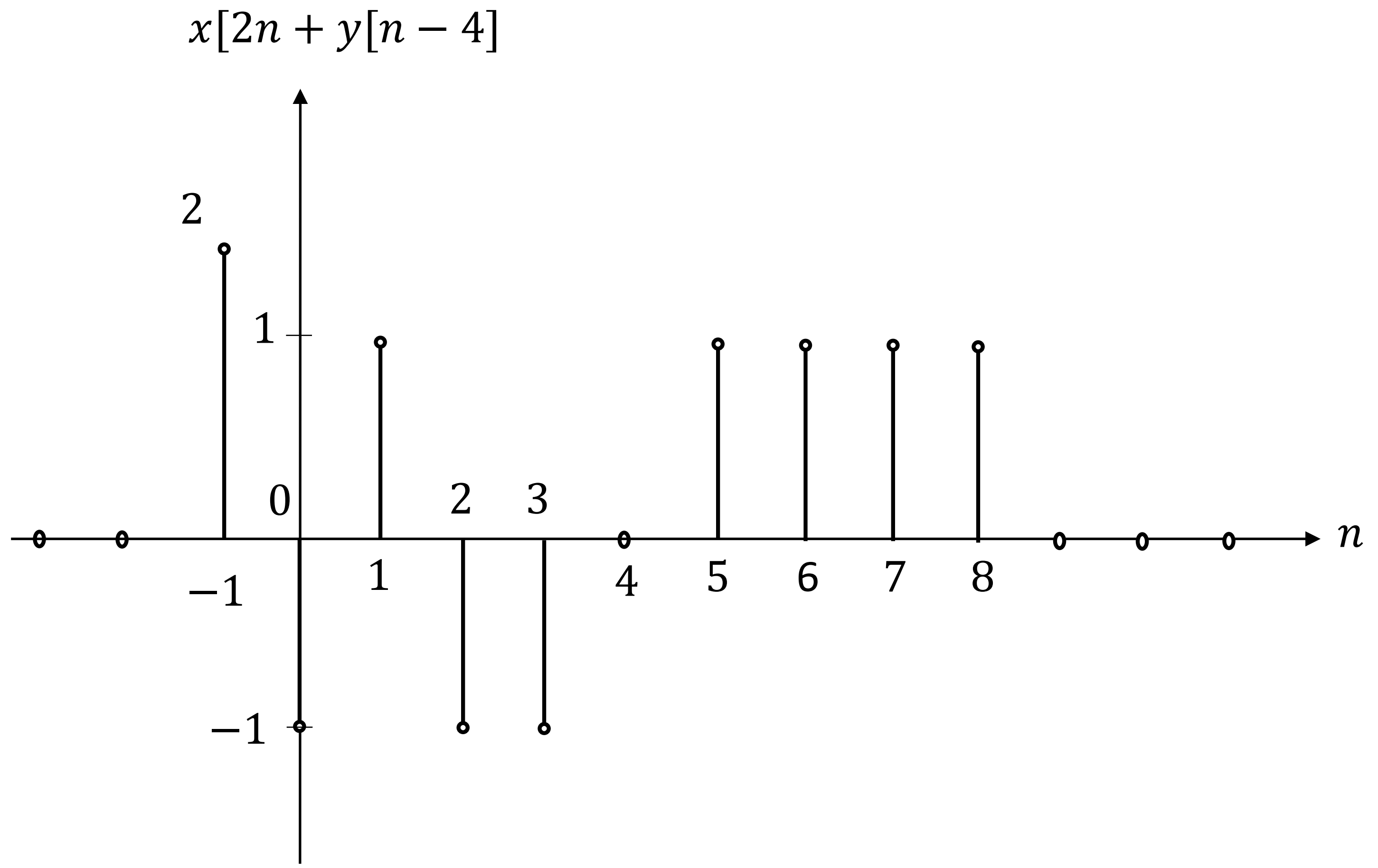


$i)\ \ x[2n] + y[n-4]$



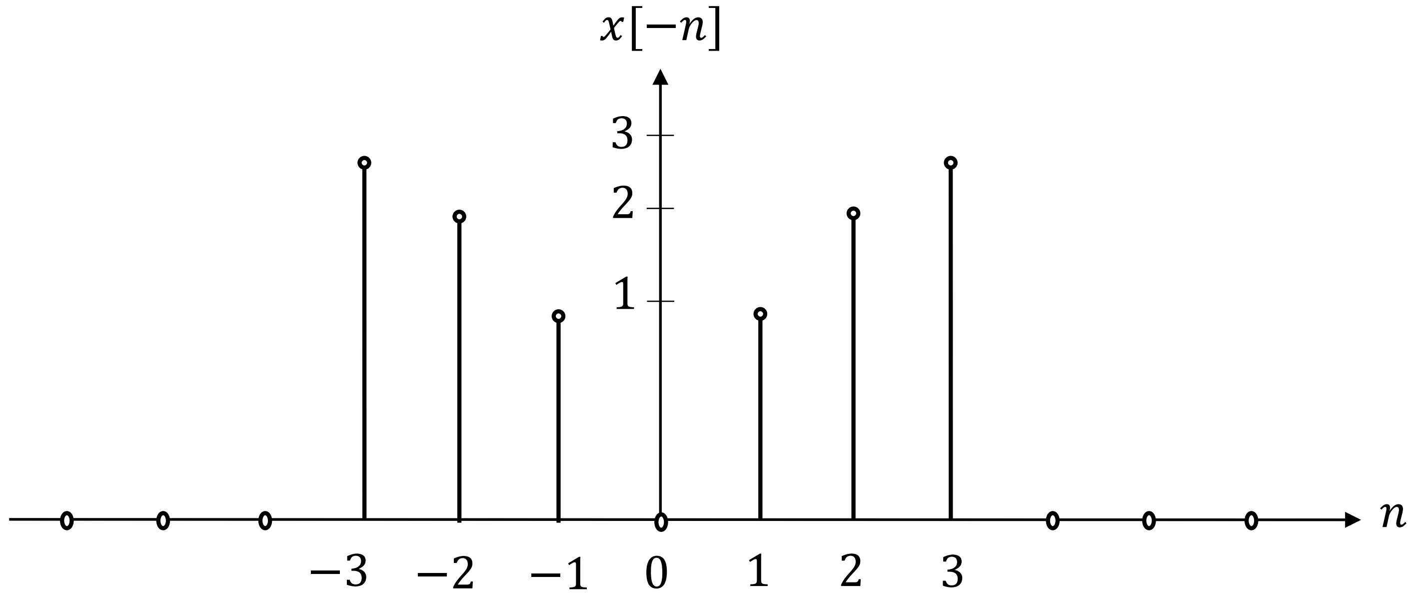


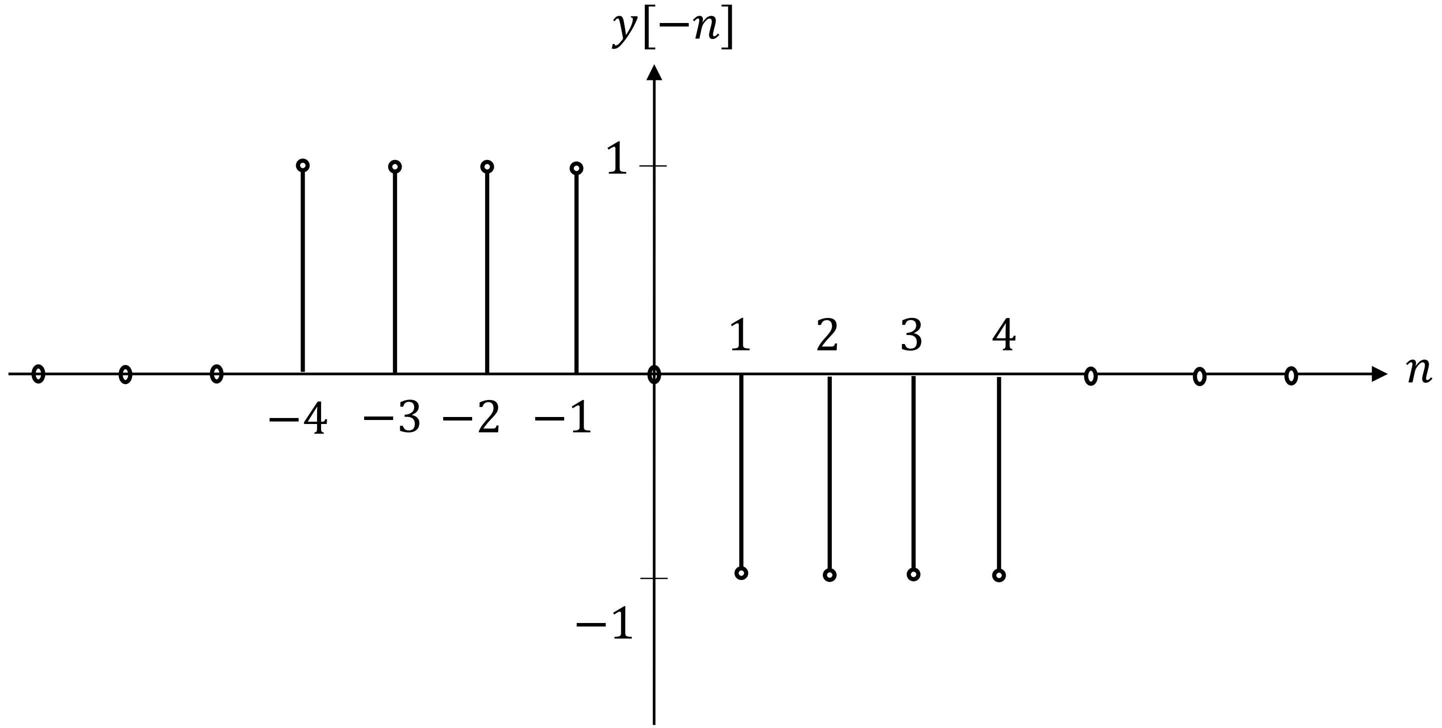


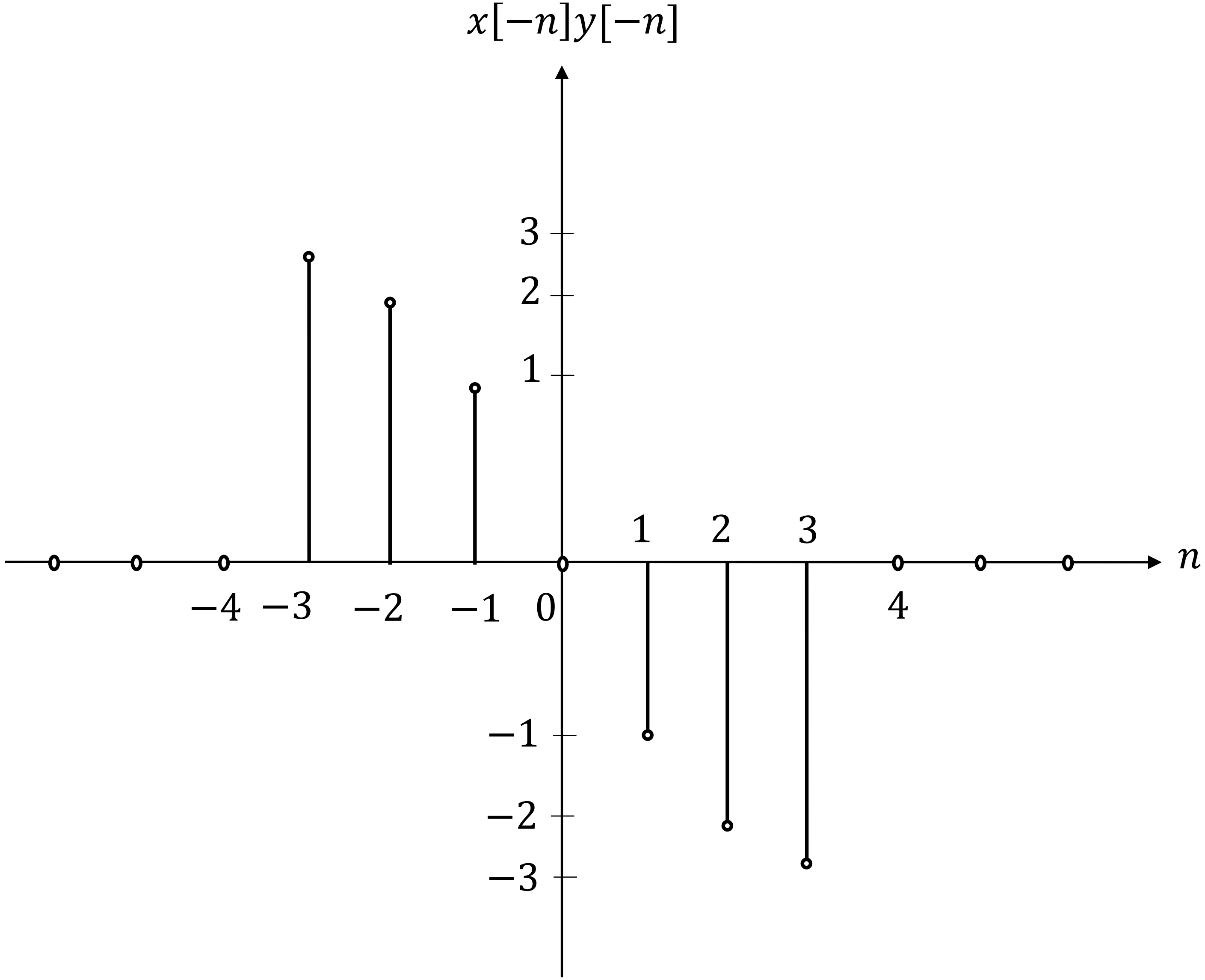


$x[2n + y[n-4]$
2
1
0
2
3
n
-1
1
4
5
6
7
8
-1

$ii)\ x[-n]y[-n]$





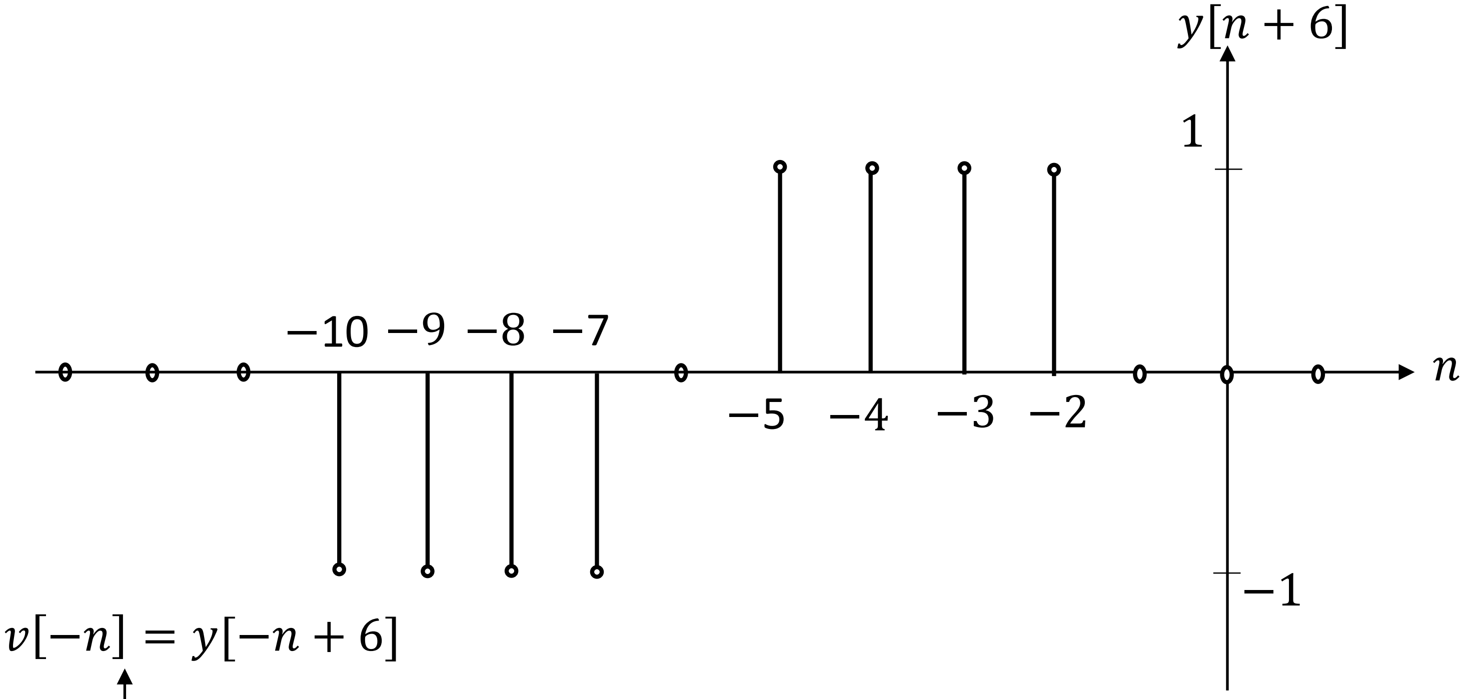

$x[-n]y[-n]$
3
2
1
-1
-2
-3
n
-4 -3 -2 -1 0 1 2 3 4

$iii)\ x[n+2]\, y[6-n]$

$y[6-n] = y[-n+6]$

Step 1: Time-shifting operation is performed first on $x[n]$

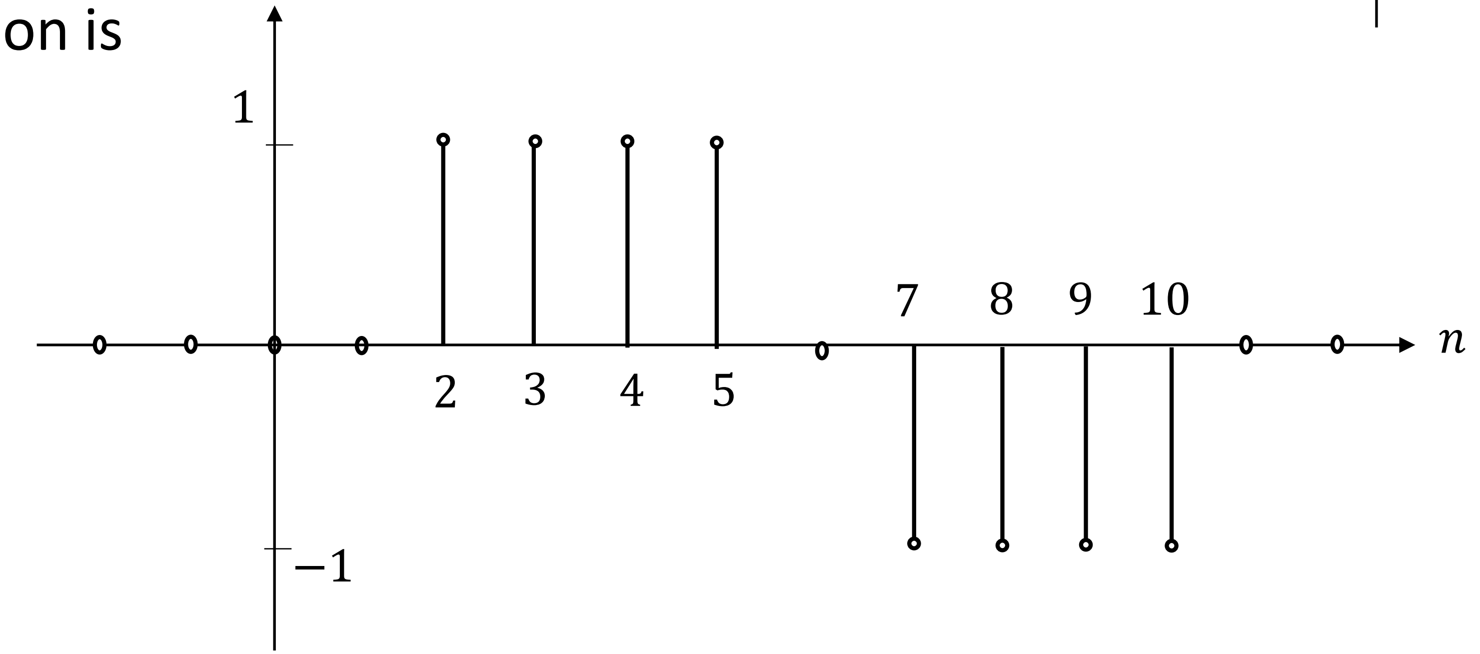
$$v[n] = y[n+6]$$

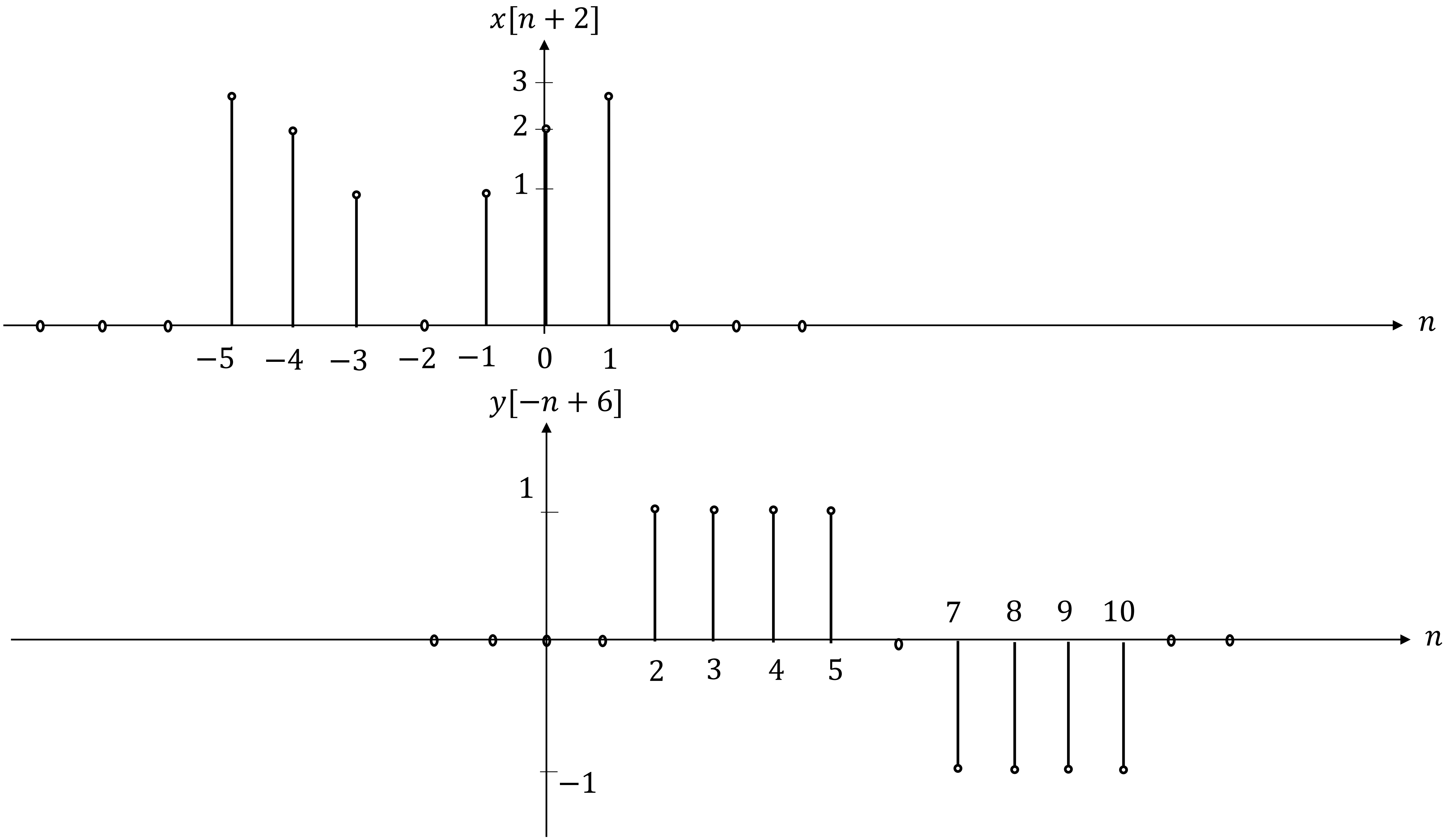


Step 2: Time-reversal operation is performed on $v[n]$

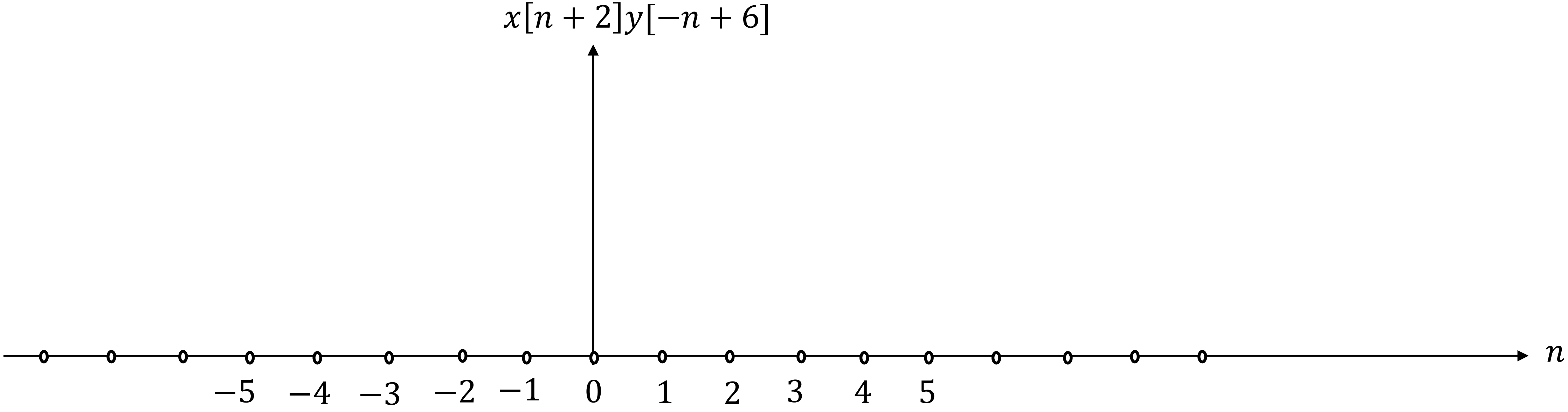
$$w[n] = v[-n]$$

$$= y[-n+6]$$





$x[n+2]$
3
2
1
n
−5
−4
−3
−2
−1
0
1
$y[-n+6]$
1
−1
n
2
3
4
5
7
8
9
10


$x[n+2]y[-n+6]$
n
-5 -4 -3 -2 -1 0 1 2 3 4 5

5. Figure shows a staircase like signal $x(t)$ that may be viewed as the superposition of rectangular pulses. Starting with the compressed version of rectangular pulse $g(t)$ shown in Fig 2, construct the waveform of Fig 1 and express $x(t)$ in terms of $g(t)$.

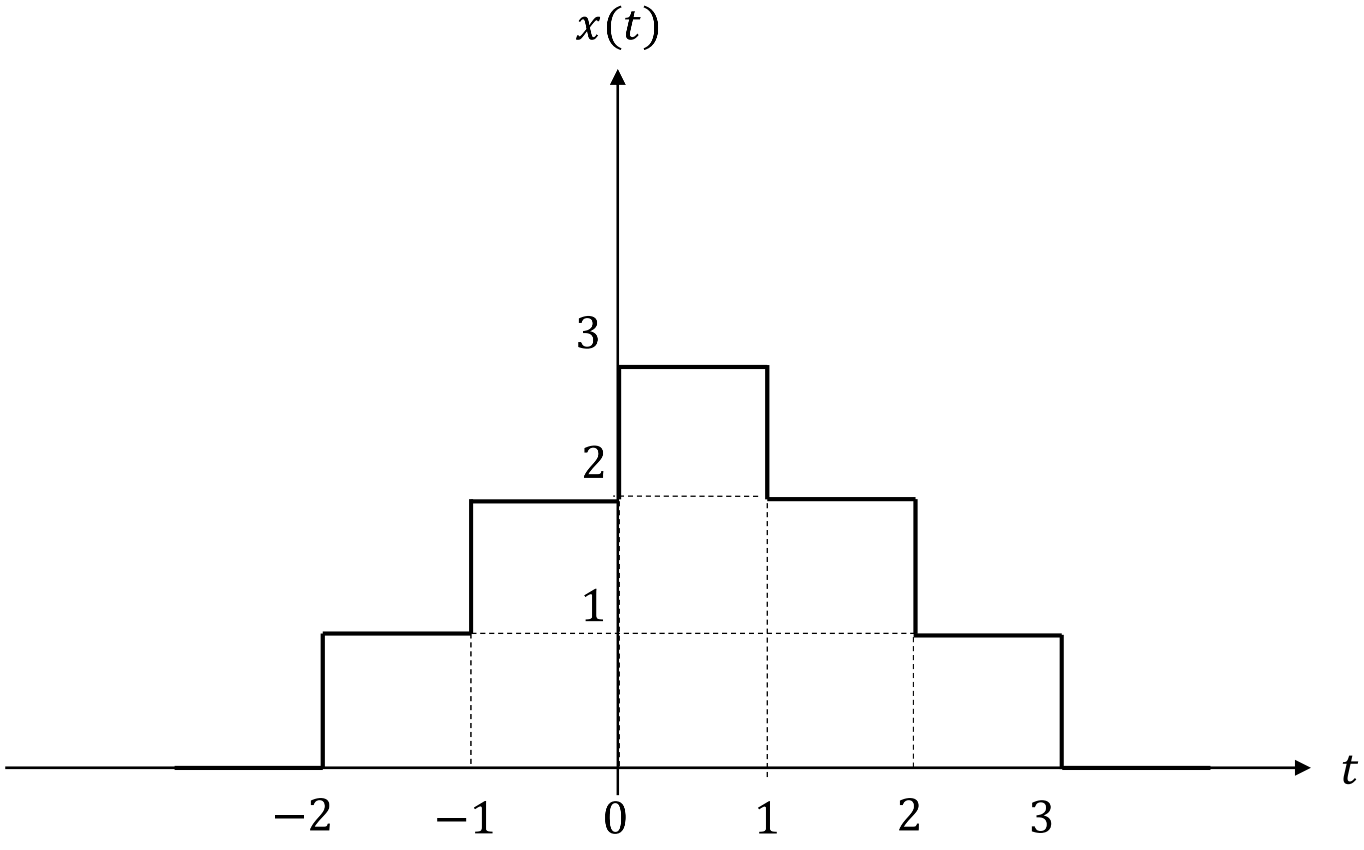


Fig 1

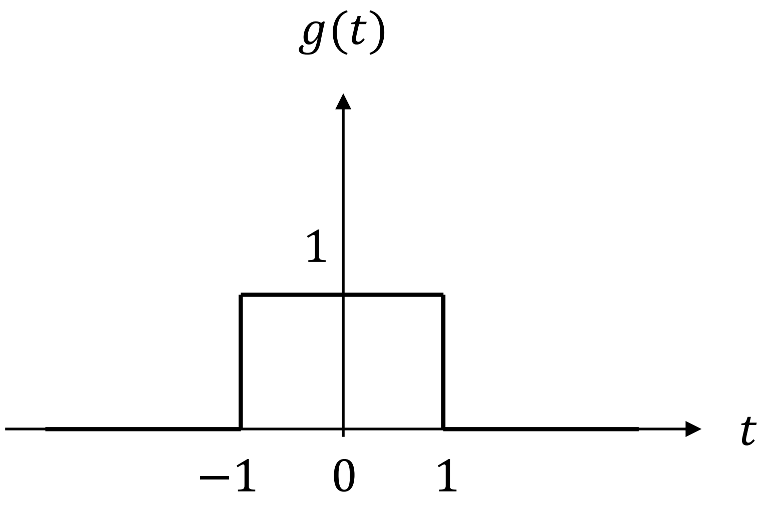
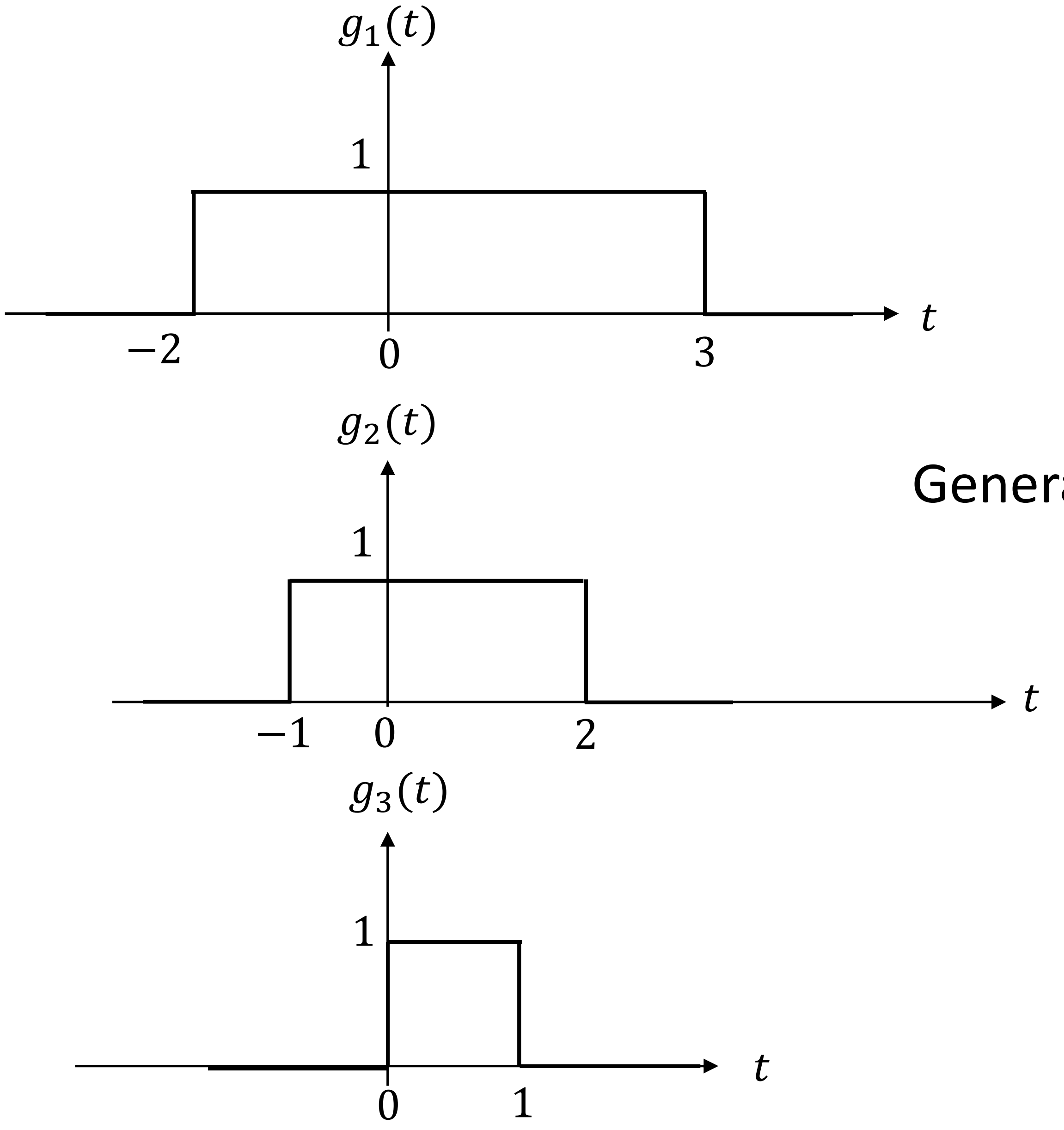


Fig 2



$$x(t) = g_1(t) + g_2(t) + g_3(t)$$

Generate each of these rectangular pulses from $g(t)$.

The width of given pulse $g(t)$ is 2

To generate $g_1(t)$, we let

$$g_1(t) = g(at - b)$$ where a and b are to be determined

Width of $g_1(t)$ is 5

This requires to choose $\boldsymbol{a = \frac{2}{5}}$

Expand $g(t)$ by a factor of $\frac{5}{2}$

Midpoint of $g_1(t)$ is at $t = 0.5$

Midpoint of $g(t)$ is at $t = 0$

Hence we must choose b to satisfy the condition

$$at - b = 0$$

At $t = 0.5$, $\quad a(0.5) - b = 0$

$$b = 0.5\mathrm{a}$$

$$b = 0.5\left(\frac{2}{5}\right)$$

$$\boldsymbol{b = \frac{1}{5}}$$

Hence

$$g_1(t) = g\left(\frac{2}{5}t - \frac{1}{5}\right)$$

To generate $g_2(t)$, we let

$$g_2(t) = g(at - b)$$ where a and b are to be determined

Width of $g_2(t)$ is 3

This requires to choose $\boldsymbol{a = \frac{2}{3}}$

Expand $g(t)$ by a factor of $\frac{3}{2}$

Midpoint of $g_2(t)$ is at $t = 0.5$

Midpoint of $g(t)$ is at $t = 0$

Hence we must choose b to satisfy the condition

$$at - b = 0$$

At $t = 0.5$, $a(0.5) - b = 0$

$$b = 0.5\mathrm{a}$$

$$b = 0.5\left(\frac{2}{3}\right)$$

$$\boldsymbol{b = \frac{1}{3}}$$

Hence

$$\boxed{g_2(t) = g\left(\frac{2}{3}t - \frac{1}{3}\right)}$$

To generate $g_3(t)$, we let

$$g_3(t) = g(at - b)$$ where a and b are to be determined

Width of $g_3(t)$ is 1

Compress $g(t)$ by a factor of 2

This requires to choose $\boldsymbol{a = 2}$

Midpoint of $g_3(t)$ is at $t = 0.5$

Midpoint of $g(t)$ is at $t = 0$

Hence we must choose b to satisfy the condition

$$at - b = 0$$

At $t = 0.5$, $a(0.5) - b = 0$

$$b = 0.5\mathrm{a}$$

$$\boldsymbol{b = 1}$$

Hence

$$\boxed{g_3(t) = g(2t - 1)}$$

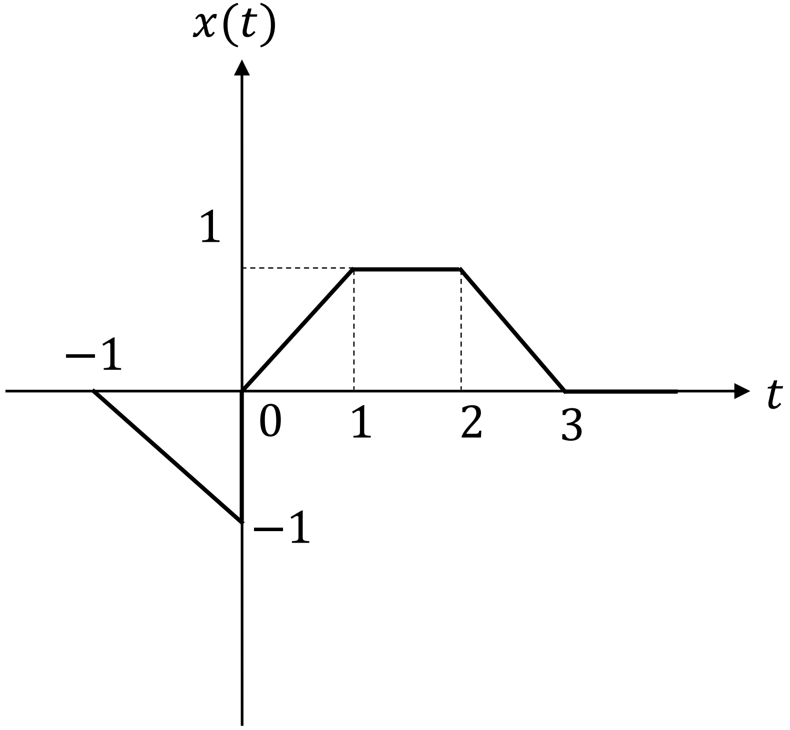
$$\boxed{x(t) = g\left(\frac{2}{5}t - \frac{1}{5}\right) + g\left(\frac{2}{3}t - \frac{1}{3}\right) + g(2t - 1)}$$

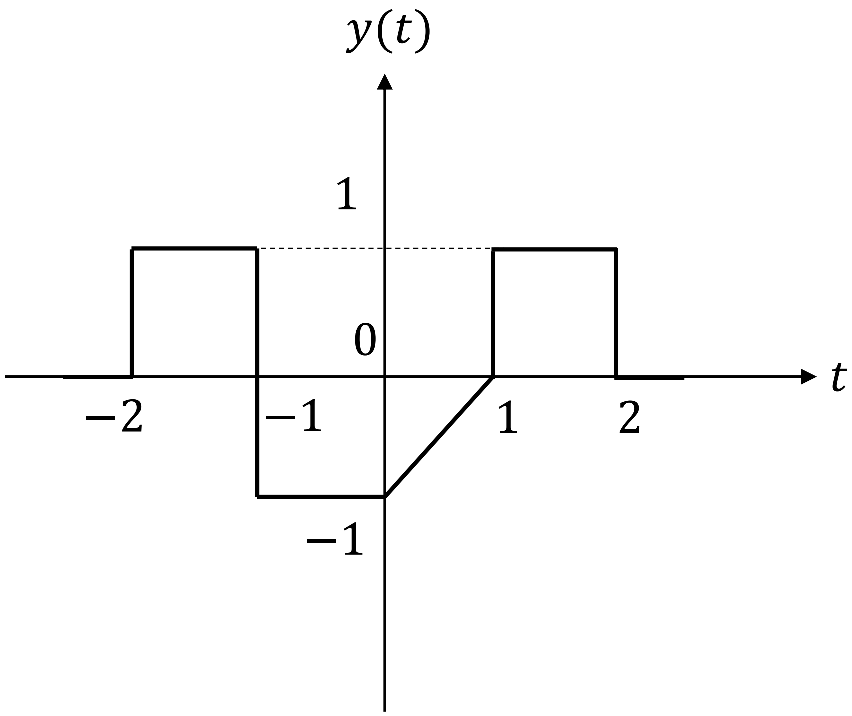
6. Let $x(t)$ and $y(t)$ be given in Figs (a) and (b) respectively. Sketch the following signals.

(a) $x(t)y(t-1)$

(b) $x(4-t)y(t)$

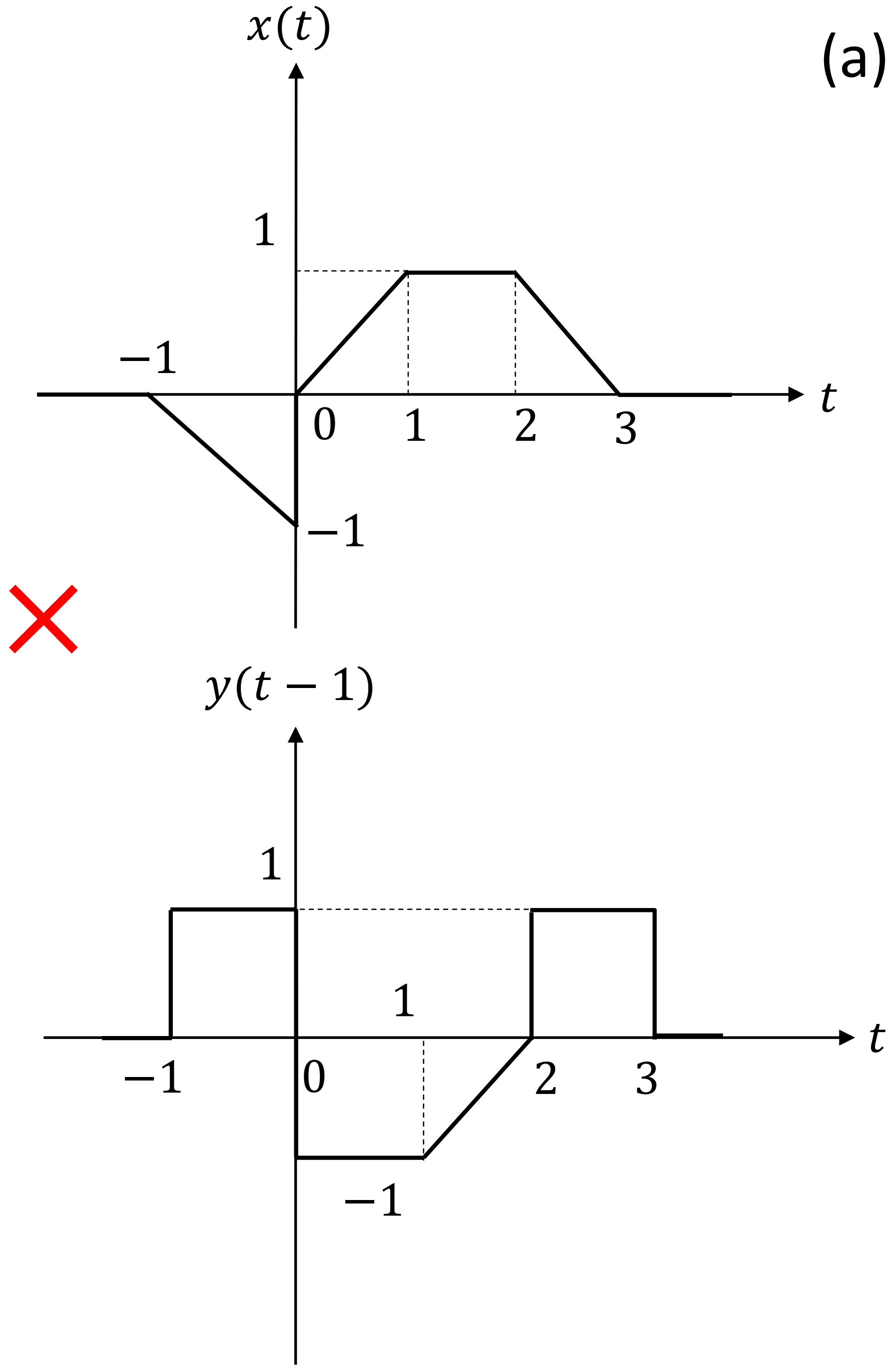
(c) $x(2t)y\left(\frac{1}{2}t+1\right)$

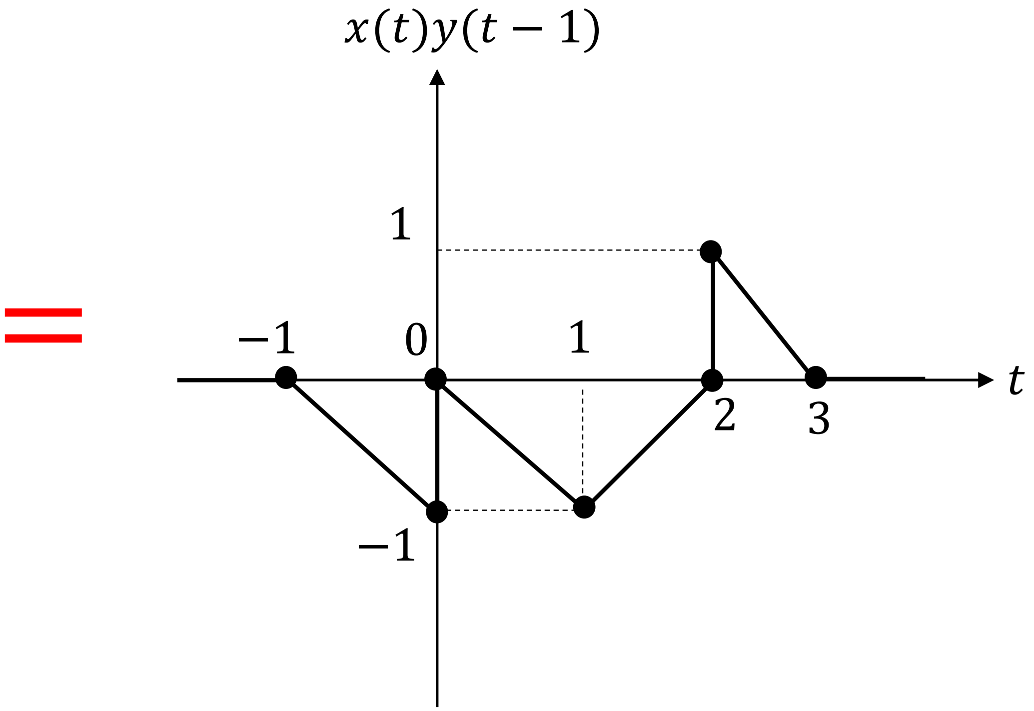




Sol:

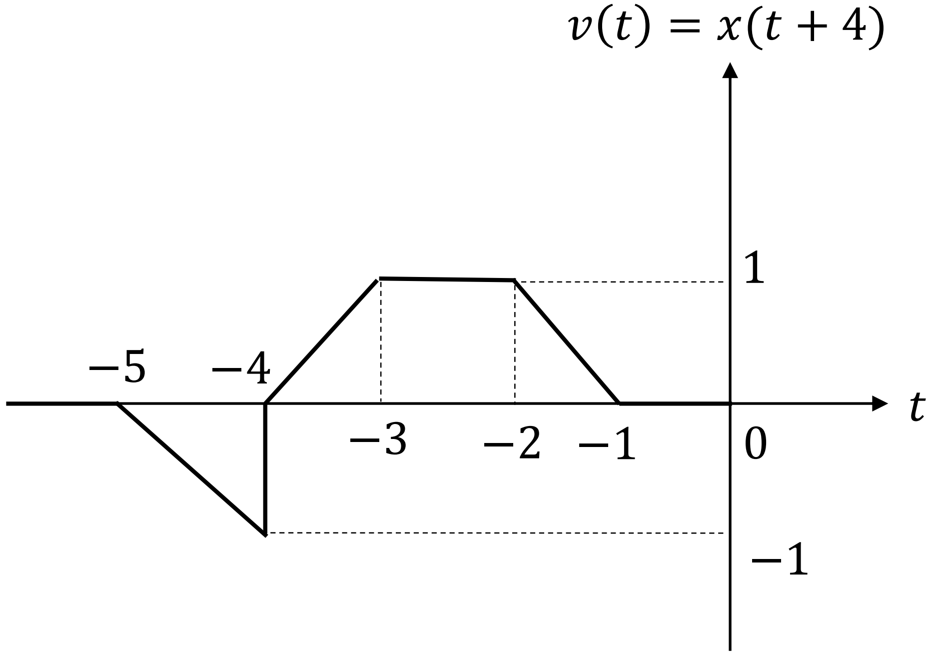
(a) $x(t)y(t-1)$





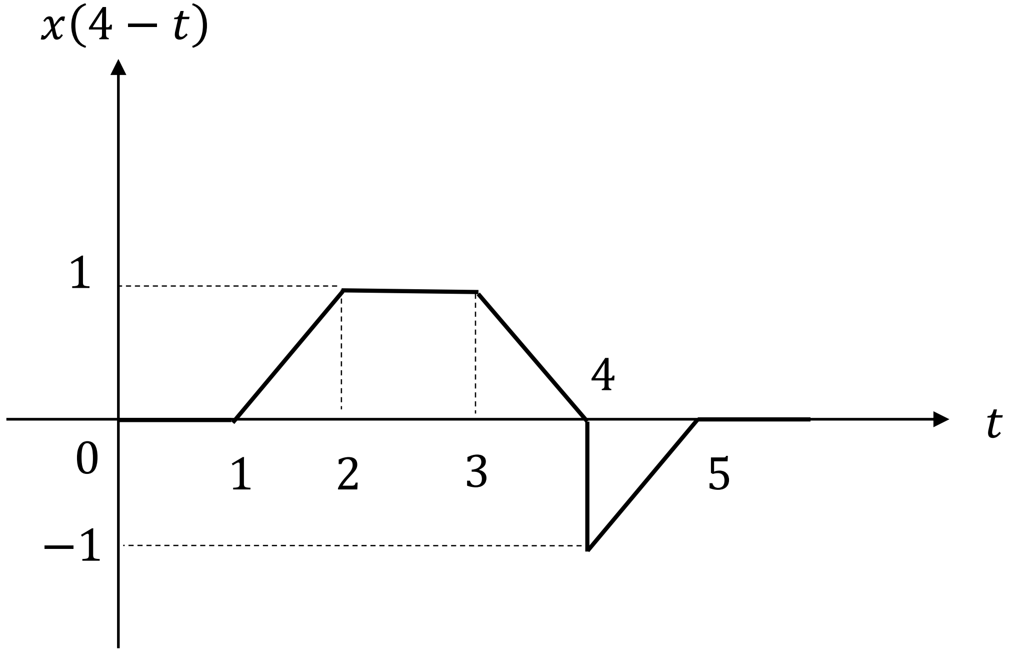
(b) Obtain the signal $x(4-t) = x(-t+4)$

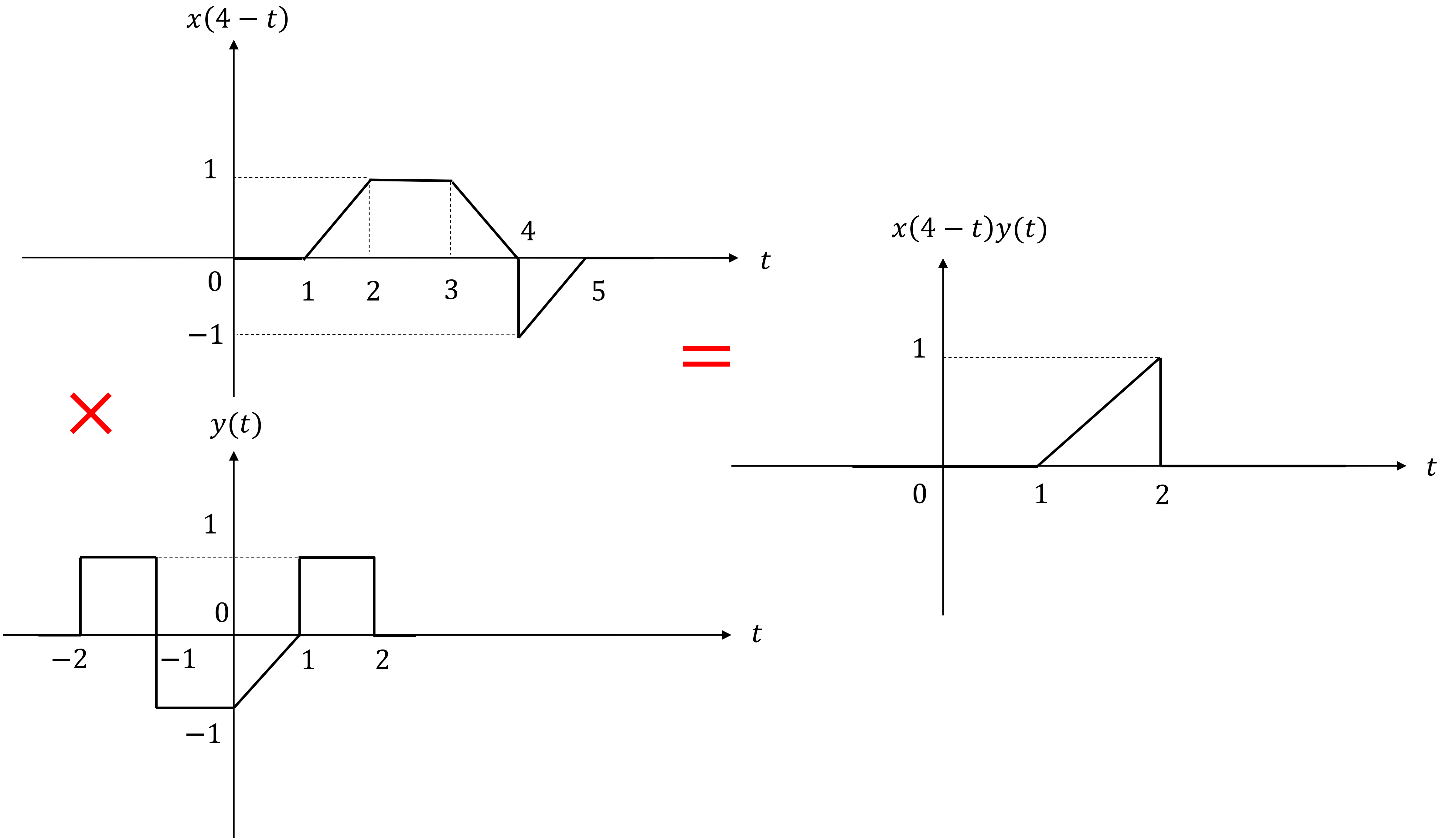
Step 1: From $x(t)$ obtain the time shifted signal $v(t) = x(t+4)$



Step 2: From $v(t)$ obtain the time scaled signal

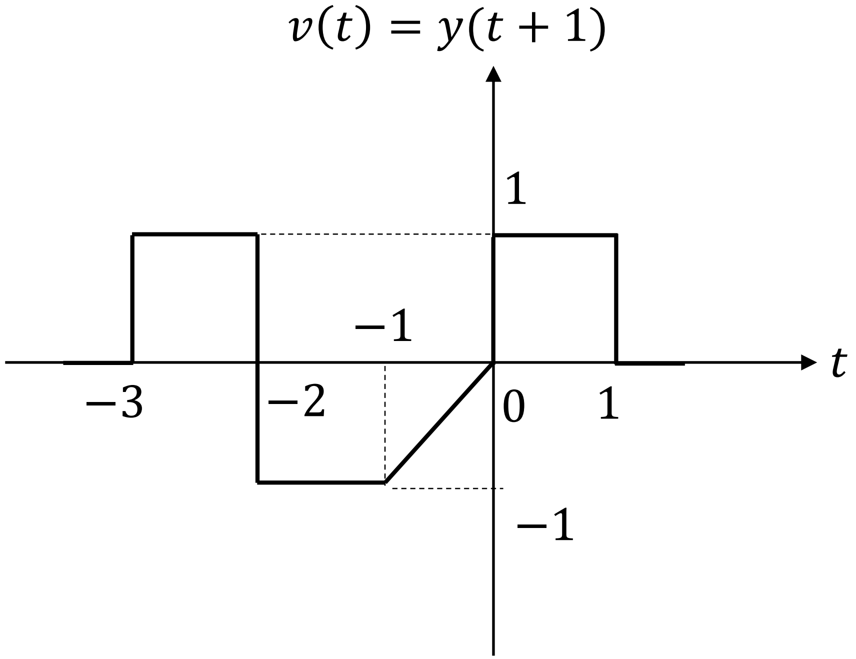
$$v(-t) = x(-t+4)$$




$x(4-t)$
1
0
−1
1
2
3
4
5
t
$y(t)$
1
0
−1
−2
−1
1
2
t
$x(4-t)y(t)$
1
0
1
2
t

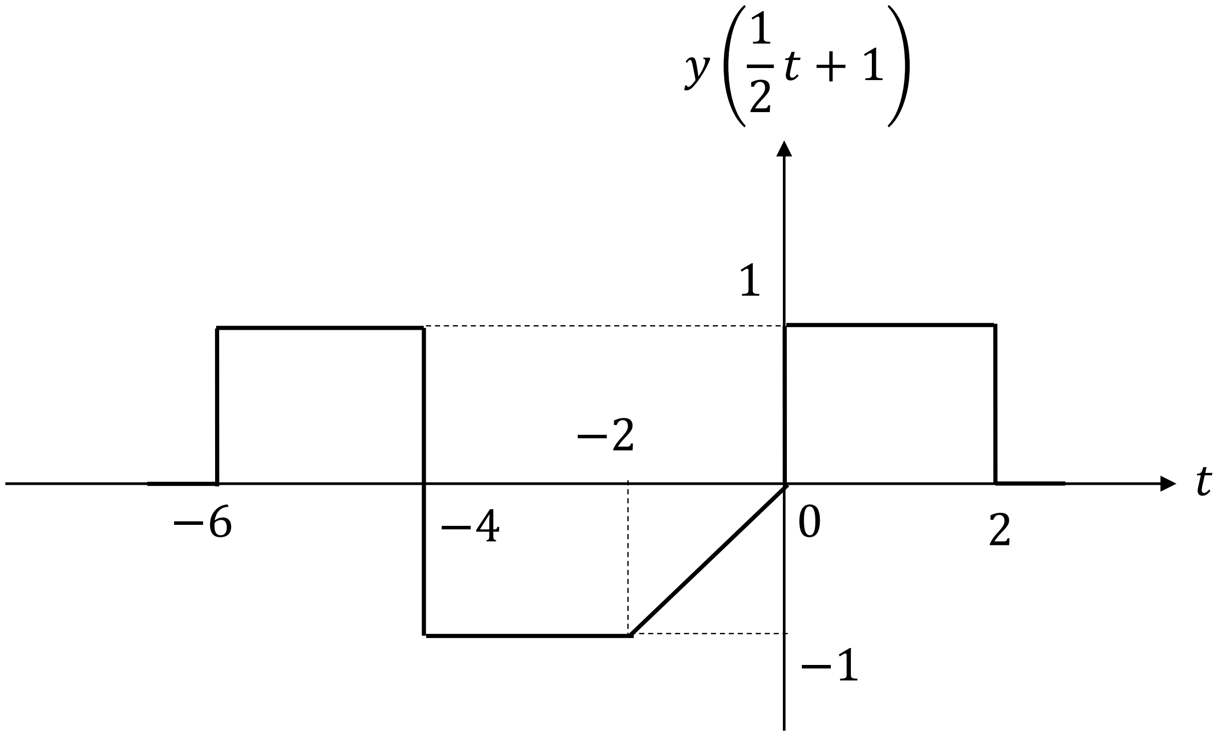
(c) Obtain the signal $y\left(\frac{1}{2}t+1\right)$

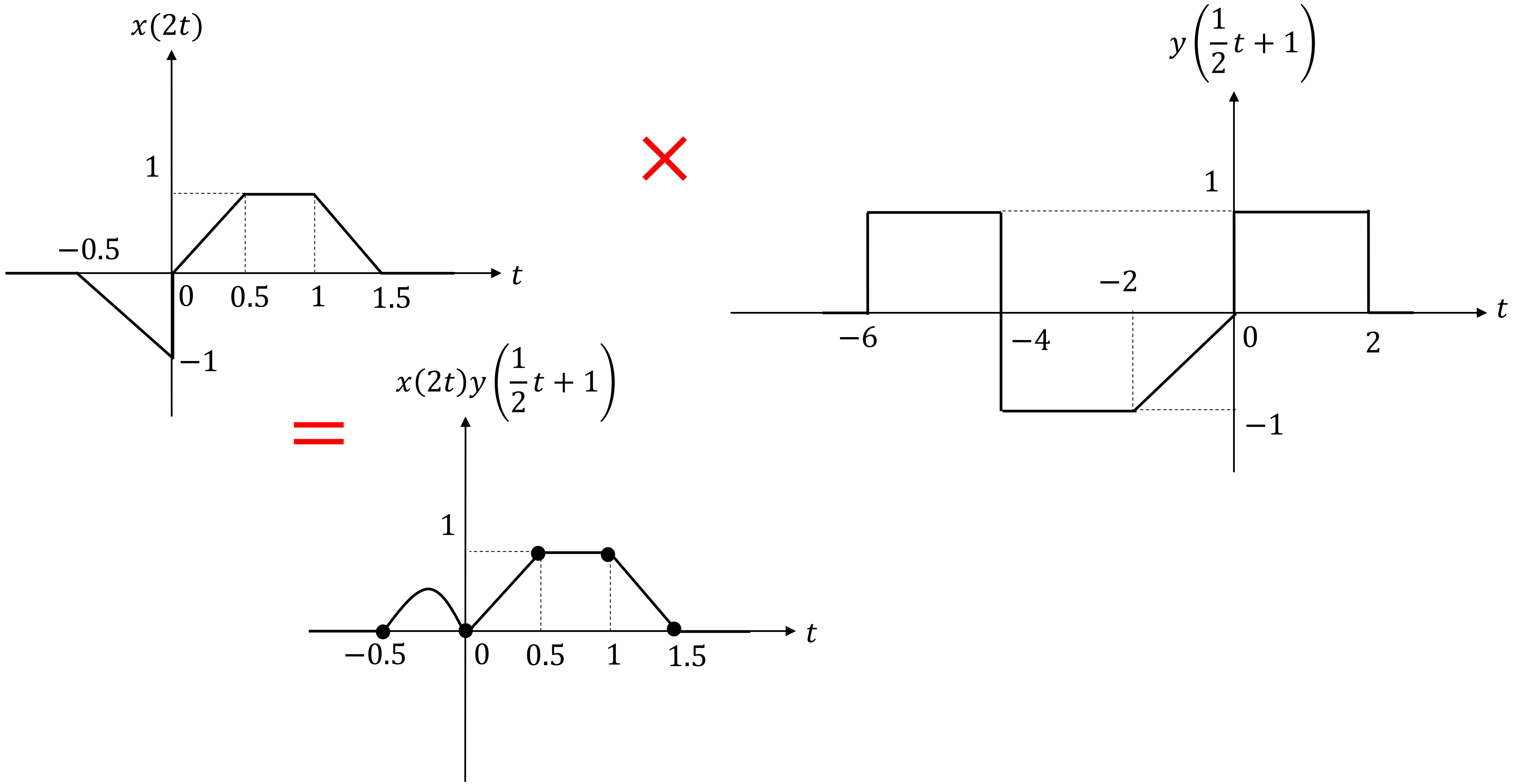
Step 1: From $y(t)$ obtain the time shifted signal $v(t) = y(t+1)$



Step 2: From $v(t)$ obtain the time scaled signal

$$v\left(\frac{1}{2}t\right) = y\left(\frac{1}{2}t+1\right)$$





$x(2t)$
1
-0.5
t
0
0.5
1
1.5
-1
$y\left(\frac{1}{2}t+1\right)$
1
-2
t
-6
-4
0
2
-1
$x(2t)y\left(\frac{1}{2}t+1\right)$
1
t
-0.5
0
0.5
1
1.5