Name - Baishnu Prayash Mohanty
USN - 1CR17ME014
SECTION - B
SUBJECT - OR

IAT-1

Q.1) a)

		sofa (Rs.)	chair (Rs.)
A	Mat. X	$2.50 \times 2 = 5$	$2.50 \times 3 = 7.50$
B	Mat. Y	$0.25 \times 4 = 1$	$0.25 \times 2 = 0.50$
C	Total (A+B)	6	8

i) Now,

minimise = $Z = 6x + 8y$

subject to constraints -

$2x + 3y \leq 16$ (max. material X constraint)

$4x + 2y \leq 16$ (max. material Y constraint)

$x \geq 2$ (max. sofa constraint)

$x, y \geq 0$ (Non-negativity constraint)

ii) $2x + 3y = 16$

when $X = 0$, $Y = \frac{16}{3}$

when $Y = 0$, $X = 8$

Thus, the vertices are 0, 16 & (8, 0)

[3]

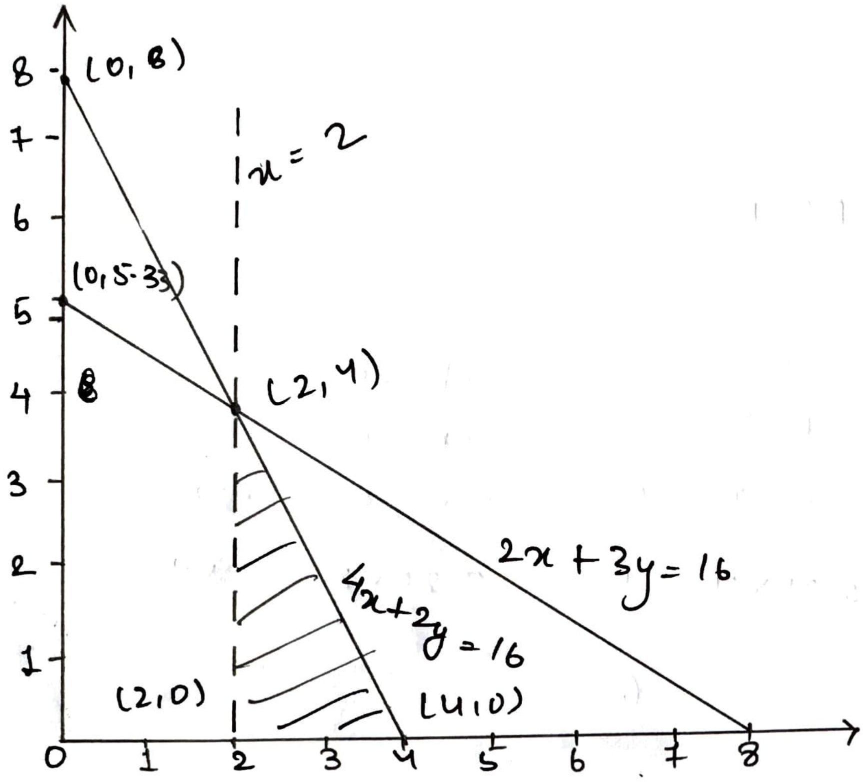
Now,

$4x + 2y = 16$

when $X = 0$ $Y = 8$

when $Y = 0$ $X = 4$.

Thus the vertices are (0, 8) & (4, 0)



So points = (2, 0), (2, 4), (4, 0)

i) $6(2) + 8(0) = 12$

ii) $6(2) + 8(4) = 44$

iii) $6(4) + 8(0) = 24$.

∴ Smallest value to be taken of Z for minimization, Z = 12

∴ X = 2 & Y = 0.

Q.1) b) Assumptions in LPP :

i) Proportionality

ii) Additivity.

iii) Divisibility.

iv) certainty

v) finiteness

vi) optimality.

Q.2) $\min z = x_1 - 3x_2 + 2x_3$

subj to

$3x_1 - x_2 + 3x_3 \le 7$

$-2x_1 + 4x_2 + 0x_3 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0.$

Converting min. to max.

$\max Z = -x_1 + 3x_2 - 2x_3$

converting inequalities to equalities adding slack variable -

$3x_1 - x_2 + 3x_3 + S_1 = 7$

$-2x_1 + 4x_2 + 0x_3 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1, x_2, x_3, S_1, S_2, S_3 \ge 0.$

new obj. funcn.

$\max Z = x_1 - 3x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3.$

C_B	$C_j \rightarrow$ Basics	-1 x_1	3 x_2	-2 x_3	0 S_4	0 S_2	0 S_3	RHS	min. Ratio.
0	S_1	3	-1	3	1	0	0	7	-7
0	S_2	-2	4	0	0	1	0	12	3 →
0	S_3	-4	3	8	0	0	1	10	10/3
	$Z' = 0$	1	-3 ↑	2	0	0	0	0	Δj
0	S_1	5/2	0	3	1	1/4	0	10	20/5
3	X_2	-1/2	1	0	0	1/4	0	3	3/-1/2
0	S_3	5/2	0	8	0	-3/4	1	1	1/-5/2
	$Z = Z_1 G$	-1/2	0	-2	0	3/4	0		
1	x_1	1	0	6/5	2/5	1/10	0	4	
3	x_2	0	1	3/5	1/5	3/10	0	5	
0	S_3	0	0	11	1	-1/2	1	11	
	$Z = Z_1 - 4$	0	0	13/5	1/5	8/10	0		

$x_1 = 4,\ x_2 = 5,\ x_3 = 0.$

$\therefore$ Max $Z = 11$

& min $Z = -11$

$x_1 = 4,\ x_2 = 5,\ x_3 = 0.$

Q.4)

a) i) Slack - If the constraints are of the type $\leq$ in the LPP in order to use simplex Algorithms. we convert the inequality into eqn. by adding a variable say 's' towards the LHS of constraint which is called slack variables.

ii) Surplus & artificial variables - constraints of the type $\geq$ when given in LPP using simplex algorithm we convert this inequality into an eqn. by substracting a variable from its LHS, the variable which are substracted are called surplus variable & we add an artificial variable. However when an eqn. itself is given in order to complete logical requirement we add only artificial variables 'A'.

iii) Binding constraint - It is the constraint that has through the optimal point of the feasible region.

iv) Non-binding constraint - Are those that do not pass through the optimal point of feasible region.

1CE17ME014

Q.4) b) Principles of Duality -

a) If primal has large no of constraints and small no of variables, computation can be considerably reduced by converting problem to Dual and then solving it

b) Duality in linear programming has certain far reaching conseqeunces of economic nature. This can help managers answer question about alternative courses of action and their relatives values.

c) Calculation of dual checks the accuracy of the primal solution

d) Duality in linear programming shows that each linear programming is equivalent to person zero-sum game. This indicates that fairly close relationships exist between linear programming and theory of games.

4)b) Principles of Duality-

a) If primal has large no of constraints and small no of variables, computation can be considerably reduced by converting problem to Dual and then solving it

b) Duality in linear programming has certain far reaching consequences of economic nature. This can help managers answer question about alternative courses of action and their relatives values.

c) Calculation of dual checks the accuracy of the primal solution

d) Duality in linear programming shows that each linear programming is equivalent to person zero-sum game. This indicates that fairly close relationships exist between linear programming and theory of games.

1CR17ME014

Q.5)

$Z = 4x_1 + x_2$

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 > 6$

$x_1 + 2x_2 < 4$

& $x_1 > 0 \quad x_2 > 0$

soln. –

$\min Z = \max Z = 4x_1 - x_2$

$3x_1 + x_2 + a_1 = 3$

$4x_1 + 3x_2 \cancel{*} - s_1 + a_2 = 6$

$x_1 + 2x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2\ a_1 a_2 > 0.$

Auxillary LPP

$\text{Max } Z = 0x_1 + 0x_2 + 0s_1 + 0s_2 - 1a_1 - 1a_2$

Now,

$3x_1 + x_2 + a_1 = 3$

$4x_1 + 3x_2 - s_1 + a_2 = 6$

$x_1 + 2x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2, a_1, a_2 > 0.$

Phase 1 –

Basic	$c_j \rightarrow$		0	0	0	0	-1	-1	
	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	min ratio
a_1	-1	3	3	1	0	0	1	0	1 →
a_2	-1	6	4	3	-1	0	0	1	6/4.
s_2	0	4	1	2	0	1	0	0	4
	Z_j^* =	-9	-7	-4	1	0	0	0	
x_1	0	1	1	1/3	0	0	x	0	3
a_2	-1	2	0	5/3	-1	0	x	1	6/5
s_2	0	3	0	5/3	0	1	x	0	9/5

z^*	$= -2$	0	0	$-5/2$	1	0	x	0
x_1	0	$3/5$	1	0	$1/5$	0	x	x
x_2	0	$6/5$	0	1	$-3/5$	0	x	x
s_2	0	1	0	0	1	1	x	x
$Z^* = 0$		0	0	0	0	x	x	x

Phase II –

Basics	$c_j \rightarrow$		-4	-1	0	0	
	C_B	X_B	x_1	x_2	s_1	s_2	min ratio
x_1	-4	$3/5$	1	0	$1/5$	0	3
x_2	-1	$6/5$	0	1	$-3/5$	0	–
s_2	0	1	0	0	1	1	$1 \rightarrow$
	$Z = -18/5$		0	0	$-1/5$	0	
x_1	-4	$2/5$	1	0	0	$-1/5$	
x_2	-1	$8/5$	0	1	0	$3/5$	
s_1	0	1	0	0	1	1	
	$z^* = -17/5$		0	0	0	$1/5$	$\leftarrow \Delta_j$

Since All $\Delta_j \geq 0$ optimal basic feasible soln. is obtained.

$\therefore$ Max $Z = -17/5$.

Min $Z = 17/5$, $X_1 = 2/5$ $X_2 = 9/5$.

Q.3)

1CR17ME014

$Z = 4x_1 + x_2$

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 > 6$

$x_1 + 2x_2 < 4$

& $x_1 > 0 \quad x_2 > 0$

Soln. -

$\min Z = \max Z = 4x_1 - x_2$

$3x_1 + x_2 + a_1 = 3$

$4x_1 + 3x_2 - S_1 + a_2 = 6$

$x_1 + 2x_2 + S_2 = 4$

$x_1, x_2, S_1, S_2\ a_1 a_2 > 0.$

Auxillary LPP

$\text{Max } Z = 0x_1 + 0x_2 + 0S_1 + 0S_2 - 1a_1 - 1a_2$

Now,

$3x_1 + x_2 + a_1 = 3$

$4x_1 + 3x_2 - S_1 + a_2 = 6$

$x_1 + 2x_2 + S_2 = 4$

$x_1, x_2, S_1, S_2, a_1, a_2 > 0.$

Phase 1 -

Basic	$c_j \rightarrow$		0	0	0	0	-1	-1	
	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	min ratio
a_1	-1	3	3	1	0	0	1	0	1 →
a_2	-1	6	4	3	-1	0	0	1	6/4.
S_2	0	4	1	2	0	1	0	0	4
	$Z_X^* =$	-9	-7	-4	1	0	0	0	3
x_1	0	1	1	1/3	0	0	x	0	
a_2	-1	2	0	5/3	-1	0	x	1	6/5
S_2	0	3	0	5/3	0	1	x	0	9/5

1CR17ME014

Z^*		$= -2$	0	0	$-5/2$	1	0	X
x_1	0	$3/5$	1	0	$1/5$	0	X	X
x_2	0	$6/5$	0	1	$-3/5$	0	X	X
s_2	0	1	0	0	1	1	X	X
$Z^* = 0$		0	0	0	0	X	X	X

Phase II –

Basics	$c_j \rightarrow$		-4	-1	0	0	
	C_B	X_B	x_1	x_2	s_1	s_2	min ratio
x_1	-4	$3/5$	1	0	$1/5$	0	3
x_2	-1	$6/5$	0	1	$-3/5$	0	–
s_2	0	1	0	0	1	1	$1 \rightarrow$
	$Z =$	$-18/5$	0	0	$-1/5$	0	
x_1	-4	$2/5$	1	0	0	$-1/5$	
x_2	-1	$8/5$	0	1	0	$3/5$	
s_1	0	1	0	0	1	1	
	Z^*	$= -17/5$	0	0	0	$1/5 \leftarrow \Delta_j$	

Since All $\Delta_j \geq 0$ optimal basic feasible soln. is obtained.

$\therefore$ Max $Z = -17/5$.

Min $Z = 17/5$, $x_1 = 2/5$ $x_2 = 9/5$.