HEAT TRANSFER (18ME73/17ME73)
IAT 1

Instructions:
Each question carries 10 Marks
Attempt all 5 questions
Use of heat transfer data hand book is permitted

Max Marks: 50
Duration: 1.5 hrs

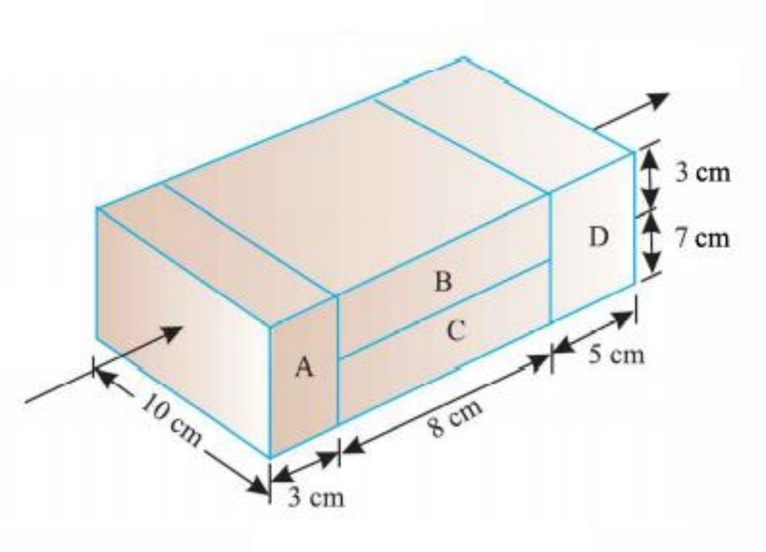
1. Define the following terms in brief:
   i. Thermal conductivity   ii. Thermal Diffusivity   iii. Thermal resistance
   iv. Thermal contact resistance   v. Overall heat transfer coefficient

2. A furnace wall is made up of three layers, first layer of insulation brick of 12 cm thickness of conductivity 0.6 W/mK. The face is exposed to gases at 870°C with a convection coefficient of 110 W/m^2K. This layer is backed by a 10 cm layer of firebrick of conductivity 0.8 W/mK. There is a contact resistance between the layers of 2.6×10^{-4} m^2 °C/W. The third layer is the plate backing of 10 mm thickness of conductivity 49 W/mK. The contact resistance between the second and third layers is 1.5×10^{-4} m^2 °C/W. The plate is exposed to air at 30°C with a convection coefficient of 15 W/m^2K. Determine the rate of heat flow, the surface temperatures and the overall heat transfer coefficient.

3. A furnace has a composite wall constructed of a refractory material for the inside layer and an insulating material on the outside. The total wall thickness is limited to 60cm. The mean temperature of the gases within the furnace is 850^{o}C, the external temperature is 30^{o}C and the material interface temperature is 500^{o}C. The thermal conductivities of refractory and insulating materials are 2 W/mK and 0.2 W/mK respectively. The combined coefficient of heat transfer by convection and radiation between gases and refractory surface is 200 W/m^2K and between outside surface and atmosphere is 40 W/m^2K. Find: (i) Thickness of each material. (ii) Rate of heat loss to atmosphere. (iii) Temperatures of external and internal surfaces.

4. A furnace wall consists of 200 mm of refractory fire clay brick, 100 mm of kaolin brick and 6 mm of steel plate. The fire side of the refractory is at 1150^{o}C and the temperature at the outside surface of the wall is 30^{o}C. An accurate heat balance over the furnace shows the heat loss from the wall to be 300 W/m^2.
   It is known that there may be thin layers of air between the layers of brick and steel. To how many millimeters of kaolin are these air layers equivalent?
   The thermal conductivities are as follows:
   $k_{\text{refractory fire clay brick}}$ = 1.7 W/m^{o}C
   $k_{\text{kaolin brick}}$ = 0.17 W/m^{o}C
   k_{steel} = 17 W/m^{o}C

5. Determine the equivalent thermal conductivity for the composite wall given below. Consider: k_A=150 W/mK, k_B=30 W/mK, k_C=65 W/mK and k_D=50 W/mK.



HT IAT-I

(1)

i) Thermal conductivity:

* Thermal conductivity is the measure of ability of the material to conduct heat
* For gases k is proportional to the square root of absolute temperature
* For liquids k decreases with increasing temperature
* In solids it is obtained by adding the lattice and electronic components.

ii) Thermal Diffusivity:

* Thermal diffusivity is a property of a material which tells us about the rate of heat diffusion through the material. It is denoted by α
* Larger the value of α, faster the heat will diffuse through the material and its temperature will change with time.

$$\alpha = \frac{k}{\rho c}$$

k - Thermal conductivity $\qquad$ ρ - Density

c → Specific heat capacity

iii) <u>Thermal resistance:</u>

* It is defined as the ratio of the temperature difference between the two faces of a material to the rate of heat flow per unit area.
* Higher the thermal resistance, the lower is the heat loss.

iv) <u>Thermal contact resistance:</u>

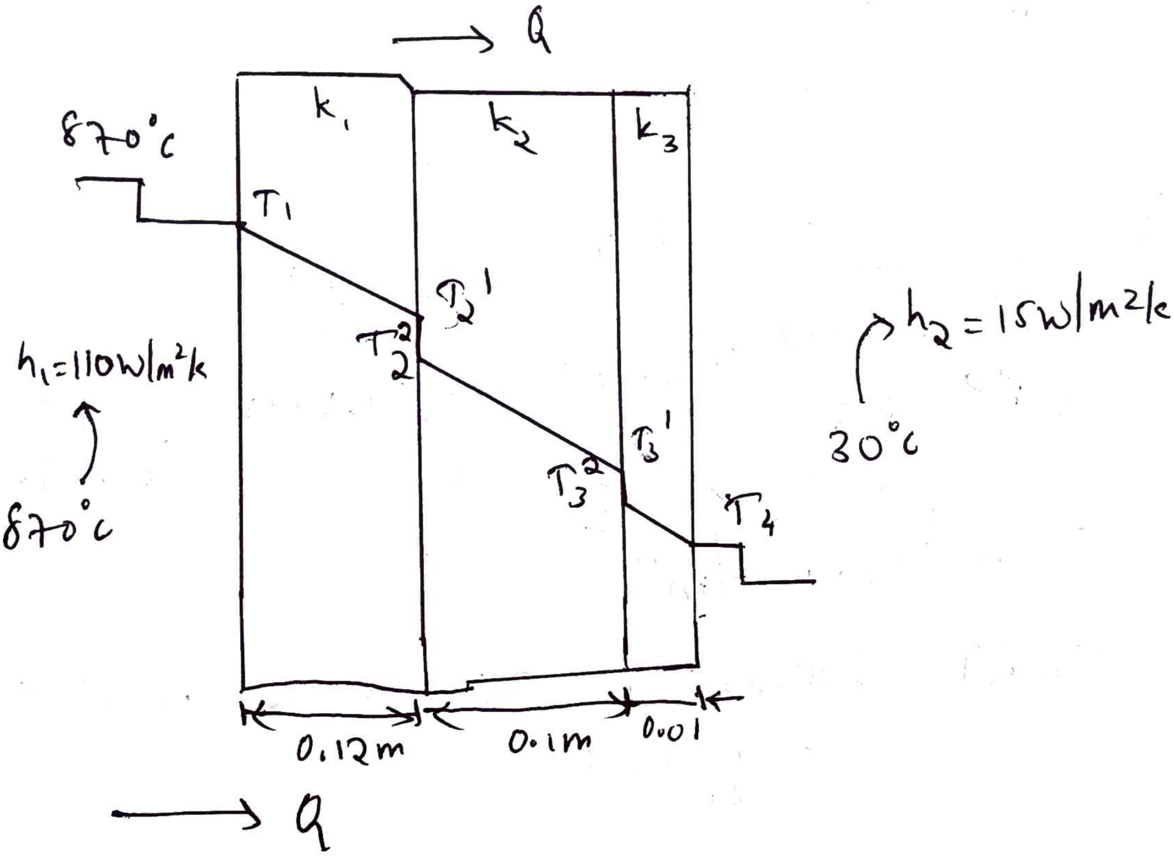
* It is defined as the ratio between ~~the~~ the temperature drop and the average heat flow across the interface.

v) <u>Overall heat transfer coefficient:</u>

Overall heat transfer coefficient represents the intensity of heat transfer when heat is transferred from one fluid to another through a wall separating them. Numerically it is equal to the quantity of heat passing through unit area of wall surface in unit time with a temperature difference of unit degree.

It has the dimension of $W/m^2 °C$ or $W/m^2 K$.

② The equivalent circuit is →



Using eqn → $Q = \frac{\Delta T}{\Sigma R}$

The resistances (°C/W) are : Taking $A = 1.0\ m^2$,

$R_{c_1} = \frac{1}{h_1 A} = \frac{1}{110}$ ⇒ $\Delta T = T_{w_1} - T_{w_2}$

$= 870 - 30 = 840°C$

$R_1 = \frac{L_1}{k_1 A}$ $(0.12/0.6)$ $R_{s_1} = 2.6 \times 10^{-4}$

$R_2 = \frac{L_2}{K_2 A} =$ $(0.1/0.8)$ $R_{s_2} = 1.5 \times 10^{-4}$

$R_3 = \frac{L_3}{k_3 A} = \frac{0.01}{49}$. $R_{c_2} = 1/15$

$$Q = \frac{840}{\frac{1}{110} + \frac{0.12}{0.6} + 2.6\times10^{-4} + \frac{0.1}{0.8} + 1.5\times10^{-4} + \frac{0.01}{49} + \frac{1}{15}}$$

$= 2092.8 \; w/m^2$

$Q = UA\Delta T$. Where U is the overall heat transfer coefficient

As $A = 1$ $\therefore U = \frac{Q}{\Delta T} = \frac{2092.8}{840} = 2.491 \; w/m^2 °C$

Surface temperatures:

$$Q = \frac{T_{\infty,1} - T_1}{1/h_1}$$

$\therefore Q \times \frac{1}{h_1} = T_{\infty,1} - T_1$ or $\frac{2092.8}{110} = 870 - T_1$

$T_1 = 850.97 °c$

Similarly $Q = \frac{T_1 - T_2'}{\frac{0.12}{0.6}}$ $T_2' = 432.40°C$

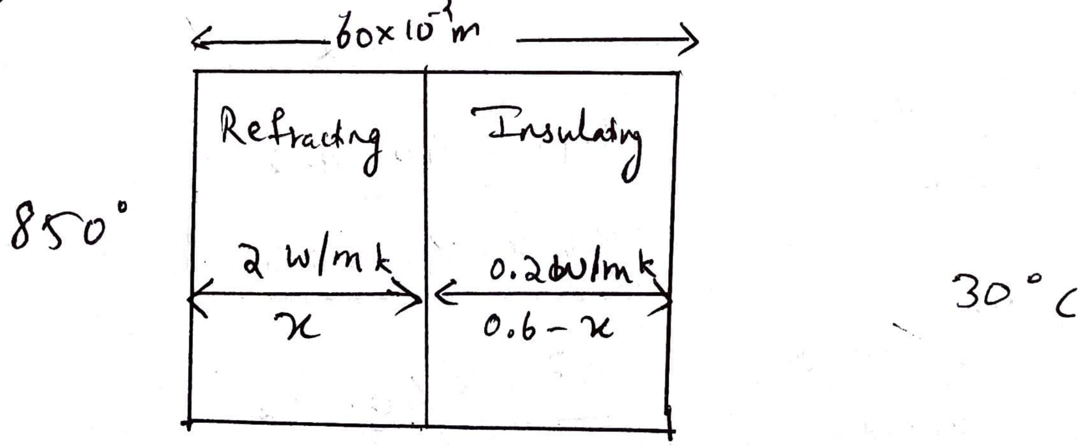
$Q \times R_{s_1} = (T_2^1 - T_2^2)$ $\therefore T_2^2 = 431.86°C$

$Q = \dfrac{T_2^2 - T_3^1}{0.1/0.8}$ $\therefore T_3^1 = 170.26°C$

$Q \times R_{s_2} = T_3^1 - T_3^2$ $\therefore T_3^2 = 169.95°C$

$Q = \dfrac{T_3^2 - T_4}{0.01}$ $\therefore T_4 = 169.52°C$.

(3)



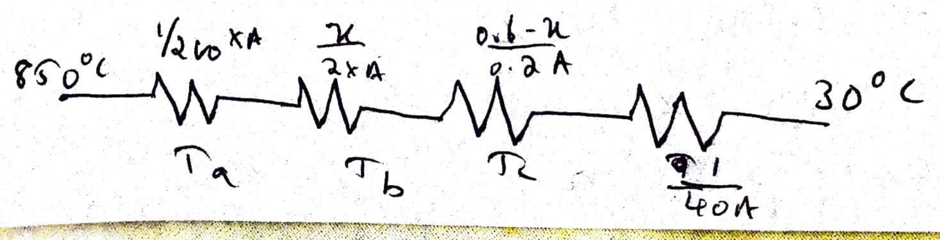
$h_1 = 200\ W/m^2k$ $h = 40W/m^2k$

$T_3 = 500°C$

$L = 0.6m$ $T_i = 850°C$ $T_o = 30°C$ $T_2 = 500°C$

$k = 2\ w/mk$ $k_2 = 0.2\ w/mk$ $h = 200\ W/m^2k$

$h = 40W/m^2k$ $L_1 = ?$ $L_2 = ?$ $Q = ?$ $T_1 = ?$ $T_2 = ?$



$$\frac{850-500}{\frac{1}{200A}+\frac{x}{2A}} = \frac{500-30}{\frac{0.6-x}{0.2A}+\frac{1}{40A}}$$

$$\frac{350}{\frac{1}{200}+\frac{x}{2}} = \frac{470}{\frac{0.6-x}{0.2}+\frac{1}{40}}$$

$$\frac{350}{\frac{101\times x}{200}} = \frac{470}{\frac{(0.6-x)40+0.2}{40\times 0.2}}$$

$$\frac{350\times 200}{101\times x} = \frac{470\times 40\times 0.2}{(0.6-x)40+0.2}$$

$$\frac{350\times 200}{101\times x} = \frac{470\times 40\times 0.2}{24-40x+0.2}$$

$$\frac{18.67}{101\times x} = \frac{1}{24.02-40x}$$

$$18.67(24.02-40x) = 101x$$

$$448.45-746.8x = 101x$$

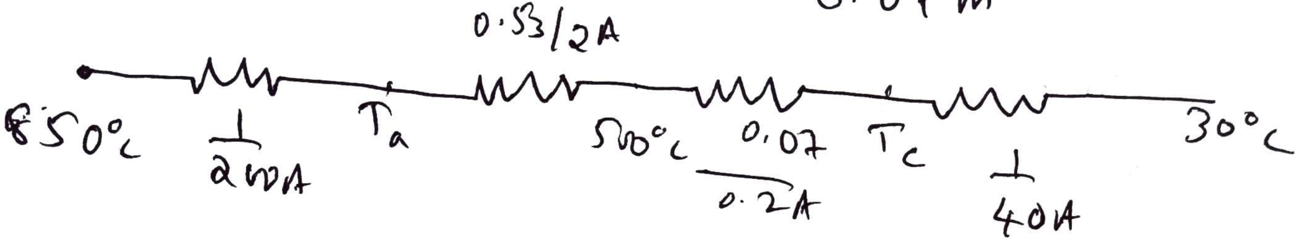
$$448.45 = 847.8x$$

$$x = 0.5289 \cong 0.53 m$$

Insulating material thickness $= 0.6 - x$

$= 0.6 - 0.53$

$= 0.07\,m$



$$Q = \frac{\Delta T}{R_{th}} = \frac{850 - 30}{\frac{1}{200A} + \frac{0.53}{2A} + \frac{0.07}{0.2A} + \frac{1}{40A}}$$

$$Q = \frac{820}{\frac{129}{200} \times \frac{1}{A}} = \frac{200 \times A \times 820}{129}$$

$$\frac{Q}{A} = 127.31 \text{ W/m}^2$$ [$\frac{Q}{A}$ is also heat flux]

Or $$\frac{Q}{A} = \frac{850 - 500}{\frac{1}{200} + \frac{0.53}{2}} = 1296.29 \text{ W/m}^2$$

or $$\frac{Q}{A} = \frac{500 - 30}{\frac{0.07}{0.2} + \frac{1}{40}} = ~~1256~~ \; 1256.3 \text{ /m}$$

$$\therefore \frac{Q}{A} = 1.27 \text{ kW/m}^2$$

$$Q = \frac{850 - T_a}{\frac{1}{200}A} \qquad Q = \frac{T_a - 30}{R_{th}}$$

$$\frac{Q}{A} = (850 - T_a) \times 200$$

$$\frac{1.27}{200} = 850 - T_a$$

$$6.35 \times 10^{-3} - 850 = -T_a$$

$$T_a = 850 - 6.35 \times 10^{-3}$$

$$= ~~84.364~~ °C = 843.64 °C$$

$$Q = \frac{T_c - 30}{\frac{1}{40A}}$$

$$Q = (T_c - 30)\, 40A$$

$$\frac{Q}{A} = (T_c - 30)\, 40$$

$$\frac{1271.3}{40} = T_c - 30$$

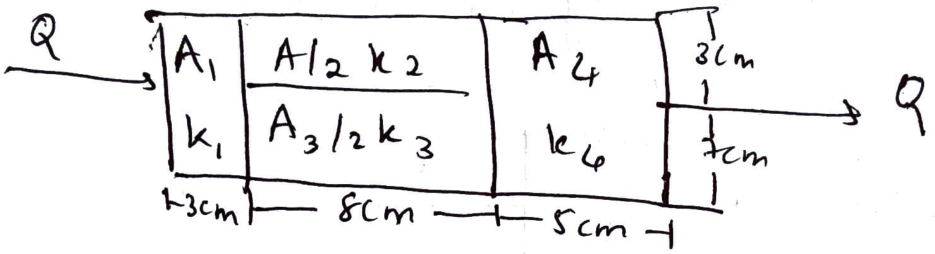
$$\boxed{T_c = 61.7825}$$

(5) Given $k_A = 150\ W/m^\circ C$

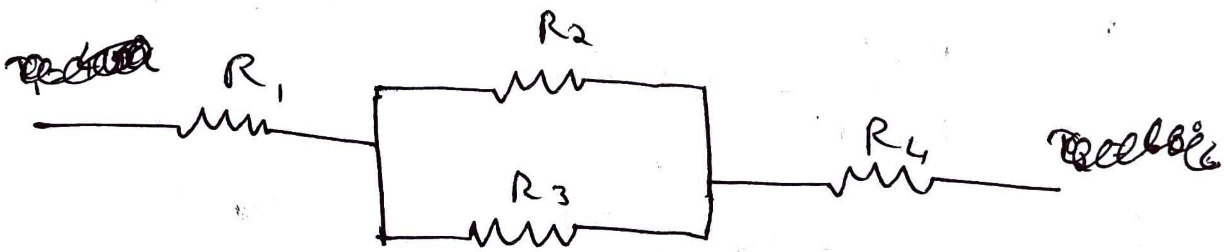
$k_B = 30\ W/m^\circ C$

$k_C = 65\ W/m^\circ C$

$k_D = 50\ W/m^\circ C$



Circuit diagram



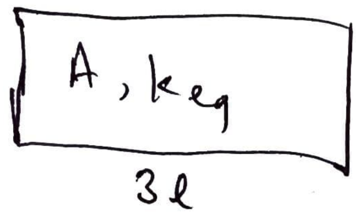
Where $R_1 = \dfrac{l_A}{k_A A_A} = \dfrac{0.03}{150 \times 0.01} = 0.02$

$R_2 = \dfrac{l_B}{k_B A_B} = \dfrac{0.08}{30 \times 0.03} \simeq 0.89$

$R_3 = \dfrac{l_C}{k_C A_C} = \dfrac{0.08}{65 \times 0.007} = 0.176$

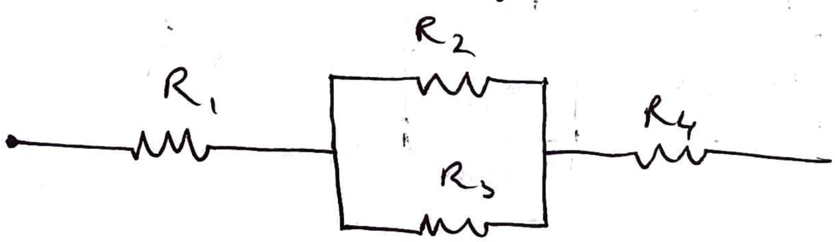
$R_4 = \dfrac{l_D}{k_D A_D} = \dfrac{0.05}{50 \times 0.01} = 0.176$

By assuming another material with area 'A' and with an equivalent thermal conductivity



& $R_{eq} = \dfrac{3l}{k_{eq} A}$ —— ①

from the circuit diagram we get



$$R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4$$

$$\frac{L_1}{k_1 A} + \frac{\dfrac{2L_2}{k_2 A} \times \dfrac{2L_3}{k_3 A}}{\dfrac{2L_2}{k_2 A} + \dfrac{2L_3}{k_3 A}} + \frac{L_4}{k_4 A}$$

On simplification we get

$$A \left[\frac{1}{k_1} + \frac{2}{k_2 + k_3} + \frac{1}{k_4} \right] \quad \text{—— ②}$$

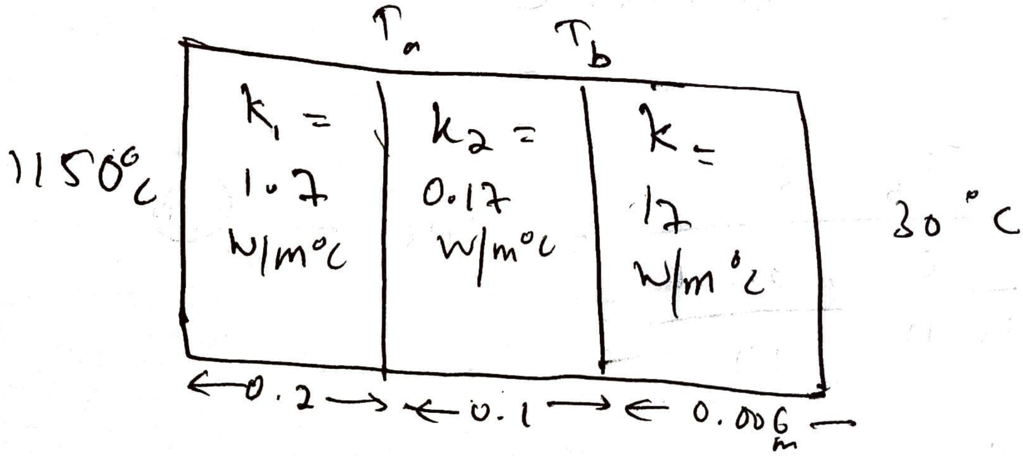
On equating we get

$$\frac{1}{k_1} + \frac{2}{k_2+k_3} + \frac{1}{k_4} = \frac{3}{k_{eq}}$$

$$\frac{3}{k_{eq}} = \frac{(k_2+k_3)k_4 + 2k_1 k_4 + k_1(k_2+k_3)}{k_1(k_2+k_3)k_4}$$

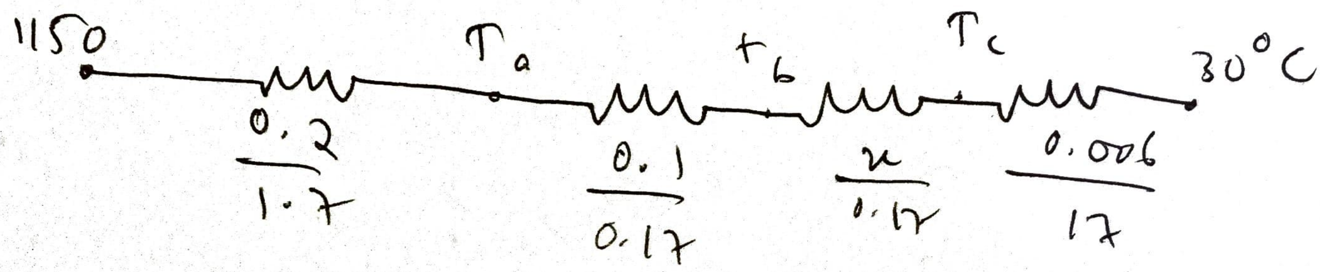
$$\boxed{k_{eq} = 62.86 \text{ W/m°C}}$$

(4)



$$\frac{Q}{A} = 300 \text{ W/m}^2$$

electrical Analogy



We know

$$\frac{Q}{A} = \frac{1150-30}{R_{th}} = 300$$

Also

$$\frac{Q}{A} = \frac{T_c - 30}{\frac{0.006}{17}} = 300$$

$$300 = \frac{T_c - 30}{\frac{0.006}{17}}$$

$$T_c = 30.105\ ^\circ C$$

Also

$$\frac{T_b - 30}{\frac{x}{10.17} + \frac{0.006}{17}} = 300 \quad \text{—— (1)}$$

we have

$$\frac{1150 - T_b}{\frac{0.2}{1.7} + \frac{0.1}{0.17}} = 300$$

$T_b = 938.23\,°C$

Sub in ① we got

$$\frac{938.23 - 30}{\frac{x}{0.17} + \frac{0.006}{17}} = 300$$

$$\frac{908.235}{5.882x + 3.52 \times 10^{-4}} = 300$$

$908.235 = 1764.70x + 0.10588$

$1764.70x = 908.1281$

$\boxed{x = 0.5146m}$