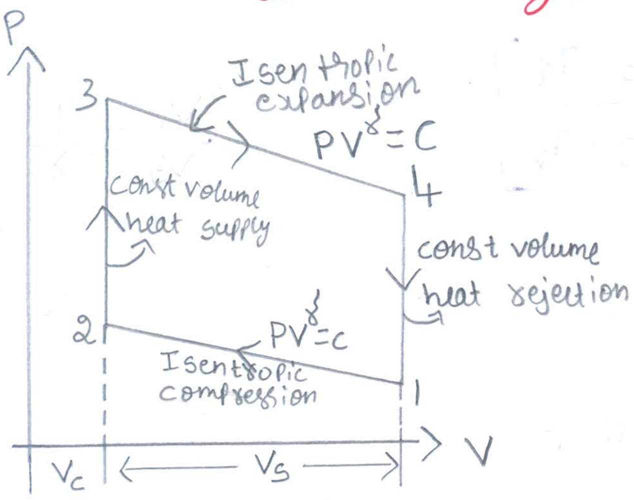
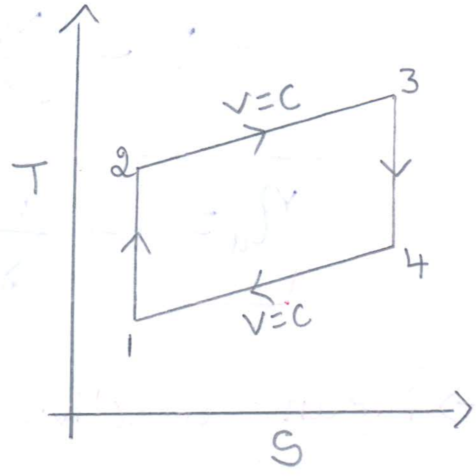
Gas Power cycles

1) OTTO CYCLE

→ Air standard efficiency.



P-V Diagram



T-S Diagram

$$\eta = \frac{W}{Q_s}$$

$$W = Q_s - Q_r = C_v(T_3 - T_2) - C_v(T_4 - T_1)$$

$$\eta = \frac{C_v(T_3 - T_2) - C_v(T_4 - T_1)}{C_v(T_3 - T_2)}$$

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \longrightarrow 1$$

applying isentropic law to process 3-4 & 1-2.

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} \Rightarrow T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \Big| \quad \begin{matrix} V_2 = V_3 \\ V_1 = V_4 \end{matrix}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_4}\right)^{\gamma-1} \Rightarrow T_1 = T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

substitute the above eqns. in eqn. 1

$$\eta_a = 1 - \left[\frac{T_3\left(\frac{V_2}{V_1}\right)^{\gamma-1} - T_2\left(\frac{V_2}{V_1}\right)^{\gamma-1}}{T_3 - T_2}\right]$$

$$= 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}\left[\cancel{\frac{T_3 - T_2}{T_3 - T_2}}\right]$$

$$\frac{V_1}{V_2} = \frac{1}{r}$$

$$\boxed{\eta_a = 1 - \frac{1}{r^{\gamma-1}}}$$

→ Mean effective pressure :-

$$MEP = \frac{\text{work done}}{\text{stroke volume}}$$

$$W.D = \frac{P_1V_1 - P_2V_2}{\gamma-1} - \frac{P_4V_4 - P_3V_3}{\gamma-1}$$

WKT $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ & $P_3V_3^{\gamma} = P_4V_4^{\gamma}$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} \qquad \frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^{\gamma}$$

$$\frac{P_1}{P_2} = (r)^{\gamma} \qquad \frac{P_4}{P_3} = (r)^{\gamma}$$

Now

$$W.D = \frac{1}{\gamma-1}\left[(P_1V_1 - P_2V_2) - (P_4V_4 - P_3V_3)\right]$$

$$= \frac{1}{\gamma-1}\left\{P_2V_2\left[\frac{P_1V_1}{P_2V_2} - 1\right] - P_3V_3\left[\frac{P_4V_4}{P_3V_3} - 1\right]\right\}$$

$$= \frac{1}{\gamma-1}\left\{P_2V_2\left[\frac{r^{\gamma}}{r} - 1\right] - P_3V_3\left[\frac{r^{\gamma}}{r} - 1\right]\right\}$$

$$= \frac{1}{\gamma-1}\left\{(P_2V_2 - P_3V_3)\left(\frac{r^{\gamma}}{r}-1\right)\right\}$$

$$\boxed{W.D = \frac{1}{\gamma-1}\left(\frac{r^{\gamma}}{r}-1\right)(P_2V_2 - P_3V_3)}$$

$$MEP = \frac{r^{\gamma-1}-1}{\gamma-1}\left[\frac{P_2V_2 - P_3V_3}{V_3 - V_1}\right]$$

$$= \frac{r^{\gamma-1}-1}{\gamma-1}\;\frac{P_3V_3}{V_3-V_1}\left[\frac{P_2\cancel{V_2}}{P_3\cancel{V_3}}-1\right]$$

$$= \frac{r^{\gamma-1}-1}{\gamma-1}\;\frac{P_3V_2}{V_2-V_1}\left[\beta-1\right]$$

$$= \frac{r^{\gamma-1}-1}{\gamma-1}\left[\beta-1\right]\frac{P_3V_2}{\left(\frac{V_2}{V_1}-1\right)V_1}$$

$$= \frac{r^{\gamma-1}}{\gamma-1}(\beta-1)\frac{P_3 \cdot r}{(r-1)}$$

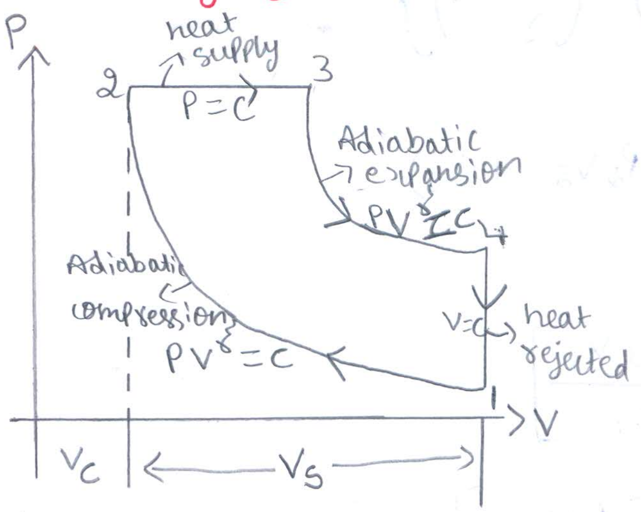
$$\boxed{MEP = \left[\frac{(r^{\gamma-1}-1)(\beta-1)(P_3 \cdot r)}{(\gamma-1)(r-1)}\right]}$$

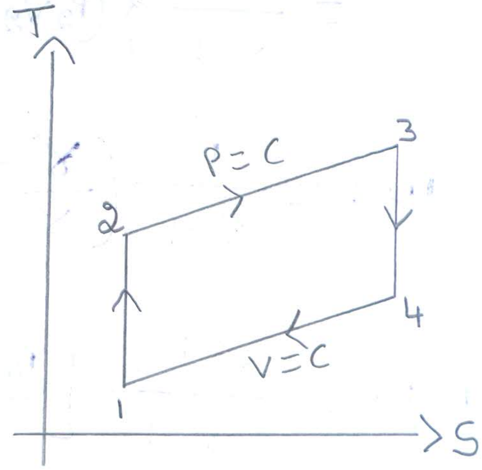
$V_2 = V_3$

$\frac{P_2}{P_3} = \beta$

$\frac{V_2}{V_1} = r$

2) Diesel cycle





→ Efficiency :-

$$\eta = \frac{W}{Q_s} = \frac{Q_s - Q_R}{Q_s} = 1 - \frac{Q_R}{Q_s}$$

Q_s = process 2-3 = $C_p(T_3 - T_2)$

Q_R = process 4-1 = $C_v(T_4 - T_1)$

$$\eta = 1 - \frac{C_v(T_4 - T_1)}{C_p(T_3 - T_2)} \quad \Bigg| \quad \frac{C_v}{C_p} = \frac{1}{\gamma}$$

$$\eta = 1 - \frac{1}{\gamma}\left[\frac{T_4 - T_1}{T_3 - T_2}\right] \longrightarrow (1)$$

Applying isentropic law to process 1-2 & 3-4

$$\frac{T_1}{T_2} = \left[\frac{V_2}{V_1}\right]^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} \Rightarrow T_1 = \frac{T_2}{r^{\gamma-1}}$$

$$\frac{T_4}{T_3} = \left[\frac{V_3}{V_4}\right]^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1} \Rightarrow T_4 = T_3\left(\frac{\rho}{r}\right)^{\gamma-1}$$

Put these eqns in eqn 1
we get

$$\eta = 1 - \frac{1}{\gamma}\left[\frac{T_3\left(\frac{\rho}{r_c}\right)^{\gamma-1} - \frac{T_2}{r^{\gamma-1}}}{T_3 - T_2}\right]$$

$$= 1 - \frac{1}{\gamma}\left[\frac{T_3\,\rho^{\gamma-1} - T_2}{T_3 - T_2}\right]\frac{1}{r^{\gamma-1}}$$

WKT $\frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3}$ $\quad | \; P_2 = P_3$

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

$$T_3 = T_2 \cdot \frac{V_3}{V_2}$$

$$T_3 = T_2 \cdot \rho$$

$$= 1 - \frac{1}{\gamma} \cdot \frac{1}{r^{\gamma-1}}\left[\frac{T_2\,\rho\,\rho^{\gamma-1} - T_2}{T_2\rho - T_2}\right]$$

$$= 1 - \frac{1}{\gamma} \cdot \frac{1}{r^{\gamma-1}}\,\frac{T_2}{T_2}\left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right]$$

$$\boxed{\eta_a = 1 - \left[\frac{1}{\gamma}\right]\left[\frac{1}{r^{\gamma-1}}\right]\left[\frac{\rho^{\gamma}-1}{\rho-1}\right]}$$

→ Mean effective Pressure

$$MEP = \frac{\text{work done}}{\text{stroke volume}}$$

$$W = W_{2-3} + W_{3-4} - W_{1-2}$$

$$= P_2(V_3 - V_2) + \left[\frac{P_3V_3 - P_4V_4}{\gamma - 1}\right] - \left[\frac{P_2V_2 - P_1V_1}{\gamma - 1}\right]$$

$V_3 = \rho V_2$

$V_1 = V_4 = r V_2$

$$= P_2(\rho V_2 - V_2) + \left[\frac{P_3 \rho V_2 - P_4 r V_2}{\gamma - 1}\right] - \left[\frac{P_2V_2 - P_1 r V_2}{\gamma - 1}\right]$$

$$= P_2V_2(\rho - 1) + V_2\left(\frac{P_3\rho - P_4 r}{\gamma - 1}\right) - V_2\left(\frac{P_2 - P_1 r}{\gamma - 1}\right)$$

$$= \frac{V_2\left[P_2(\rho - 1)(\gamma - 1) + P_3\left(\rho - \frac{P_4}{P_3} r\right) - P_2\left(1 - \frac{P_1}{P_2} r\right)\right]}{\gamma - 1}$$

$$= \frac{P_2V_2\left[(\rho - 1)(\gamma - 1) + \left(\rho - \frac{P_4}{P_3} r\right) - \left(1 - \frac{P_1}{P_2} r\right)\right]}{\gamma - 1}$$

But $\frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^\gamma = \frac{1}{r^\gamma} = \left(\frac{\rho}{r}\right)^\gamma = \rho^\gamma r^{-\gamma}$

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = r^\gamma \Rightarrow P_2 = P_1 r^\gamma$ & $V_2 = V_1 r^{-1}$

$$W = \frac{P_1V_1 r_c^{\gamma - 1}\left[(\rho - 1)(\gamma - 1) + (\rho - \rho^\gamma r^{1-\gamma}) - (1 - r^{1-\gamma})\right]}{\gamma - 1}$$

$$W = \frac{P_1V_1 r^{\gamma - 1}\left[\gamma(\rho - 1) - r^{1-\gamma}(\rho^\gamma - 1)\right]}{\gamma - 1}$$

$$MEP = \frac{P_1 V_1 r^{\gamma-1}\left[\gamma(\rho-1) - r^{1-\gamma}(\rho^{\gamma}-1)\right]}{(\gamma-1)(V_1-V_2)}$$

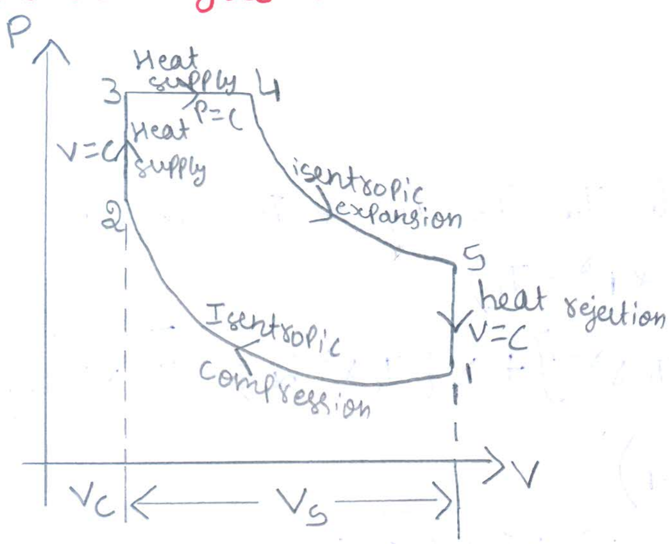
$$= \frac{P_1 V_1 r^{\gamma-1}\left[\gamma(\rho-1) - r^{1-\gamma}(\rho^{\gamma}-1)\right]}{(\gamma-1)\, V_1\left[1-\frac{V_2}{V_1}\right]}$$

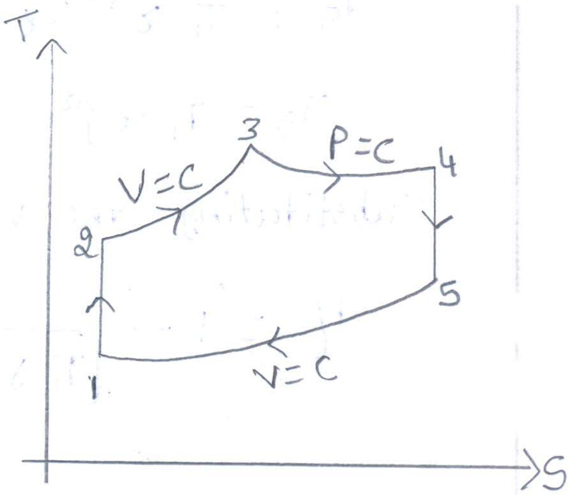
$$= \frac{P_1 r^{\gamma-1}\left[\gamma(\rho-1) - r^{1-\gamma}(\rho^{\gamma}-1)\right]}{(\gamma-1)\left(1-\frac{1}{r}\right)}$$

$$= \frac{P_1 r^{\gamma}\left[\gamma(\rho-1) - r^{1-\gamma}(\rho^{\gamma}-1)\right]}{\gamma-1\ (r-1)}$$

$$\boxed{MEP = \frac{P_1}{(\gamma-1)(r-1)}\left[r^{\gamma}\,\gamma(\rho-1) - r(\rho^{\gamma}-1)\right]}$$

③ DUAL Cycle :-





→ Efficiency :-

$$\eta = \frac{W}{Q_S} = 1 - \frac{Q_R}{Q_S}$$

$$= 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$= 1 - \frac{C_v(T_5 - T_1)}{C_v\left[(T_3 - T_2) + \frac{C_p}{C_v}(T_4 - T_3)\right]} \rightarrow (1)$$

Applying isentroic Law to 1-2

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \Rightarrow \underline{T_2 = T_1 r^{\gamma-1}}$$

Applying gas law to process 2-3 & process 3-4

$$\frac{P_3 \cancel{V_3}}{T_3} = \frac{P_2 \cancel{V_2}}{T_2}$$

$$T_3 = T_2 \frac{P_3}{P_2}$$

$$T_3 = T_2 \alpha$$

$$\underline{T_3 = T_1 r^{\gamma-1} \alpha}$$

$$\frac{P_4 V_4}{T_4} = \frac{P_3 V_3}{T_3}$$

$$T_4 = T_3 \frac{V_4}{V_3}$$

$$= T_3 \rho$$

$$\underline{T_4 = T_1 r^{\gamma-1} \alpha \rho}$$

Applying Isentropic law to 4-5.

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} \Rightarrow T_5 = T_4 \, r^{\gamma-1} = T_4 \left(\frac{\rho}{r}\right)^{\gamma-1}$$

$$T_5 = T_1 r^{\gamma-1} \alpha \rho \frac{\rho^{\gamma-1}}{r^{\gamma-1}}$$

$$\underline{T_5 = T_1 \alpha \rho^{\gamma}}$$

Substituting the values of T_2, T_3, T_4 & T_5 in (1)

$$\eta = 1 - \frac{(T_1 \alpha \rho^{\gamma} - T_1)}{[T_1 r^{\gamma-1} \alpha - T_1 r^{\gamma-1}] + \gamma [T_1 r^{\gamma-1} \alpha \rho - T_1 r^{\gamma-1} \alpha]}$$

$$= 1 - \frac{(\alpha \rho^{\gamma} - 1)}{r^{\gamma-1} (\alpha - 1) + r^{\gamma-1} \gamma \alpha (\rho - 1)}$$

$$\boxed{\eta_a = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{\alpha \rho^{\gamma} - 1}{(\alpha - 1) + \gamma \alpha (\rho - 1)}\right]}$$

Mean effective pressure :-

$$MEP = \frac{\text{work done}}{\text{stroke volume}}$$

$$W = P_3(V_4 - V_3) + \frac{(P_4V_4 - P_5V_5)}{\gamma - 1} - \frac{(P_2V_2 - P_1V_1)}{\gamma - 1}$$

WKT $\frac{V_4}{V_3} = \rho \qquad V_4 = \rho V_3$, $\frac{V_1}{V_2} = \gamma_c \Rightarrow V_1 = \gamma_c V_2 = \gamma_c V_3$

$V_2 = V_3$

$$\frac{V_5}{V_4} = \gamma_e \Rightarrow V_5 = \gamma_e V_4 = \frac{\gamma_e}{\rho} V_4 = \frac{\gamma_c}{\rho} \times \rho V_3 = \gamma_c V_3$$

$$W = \frac{P_3V_3(\rho-1)(\gamma-1) + P_4V_3\left(\rho - \frac{P_5}{P_4}\gamma_c\right) - P_2V_3\left(1 - \frac{P_1}{P_2}\gamma_c\right)}{\gamma - 1}$$

$$\frac{P_5}{P_4} = \left(\frac{V_4}{V_5}\right)^\gamma = \left(\frac{\rho}{\gamma}\right)^\gamma \quad , \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \gamma_c^\gamma$$

$$W = \frac{V_3\left[P_3(\rho-1)(\gamma-1) + P_3\left(\rho - \frac{\rho^\gamma}{\gamma_c^\gamma}\gamma_c\right) - P_2\left(1 - \frac{1}{\gamma_e^\gamma}\gamma_e\right)\right]}{\gamma - 1}$$

$$= \frac{V_3\left[P_3(\rho-1)(\gamma-1) + \frac{P_3}{P_2}(\rho - \rho^\gamma \gamma_c^{1-\gamma}) - (1 - \gamma_e^{1-\gamma})\right]}{\gamma - 1}$$

$$= \frac{P_2V_2\left[\frac{P_3}{P_2}(\rho-1)(\gamma-1) + P_3/P_2(\rho - \rho^\gamma \gamma_c^{1-\gamma}) - (1 - \gamma_e^{1-\gamma})\right]}{\gamma - 1}$$

$$= \frac{P_1\gamma_c^\gamma \frac{V_1}{\gamma_c}\left[\alpha(\rho-1)(\gamma-1) + \alpha(\rho - \rho^\gamma \gamma_c^{1-\gamma}) - (1 - \gamma_c^{1-\gamma})\right]}{\gamma - 1}$$

$$= \frac{P_1\gamma_c^{\gamma-1}V_1\left[\alpha\gamma\rho - \alpha\gamma - \alpha\rho + \alpha + \alpha\rho - \alpha\rho^\gamma + \gamma_c^{1-\gamma} - 1 + \gamma_c^{1-\gamma}\right]}{\gamma - 1}$$

$$W = \frac{P_1V_1\gamma_c^{\gamma-1}\left[\alpha\gamma(\rho-1) + (\alpha-1) - \gamma_c^{1-\gamma}(\alpha\rho^\gamma - 1)\right]}{\gamma - 1}$$

$$MEP = \frac{W}{V_s} = \frac{W}{V_1 - V_2} = \frac{W}{V_1\left(1 - \frac{V_2}{V_1}\right)} = \frac{W}{V_1\left(1 - \frac{1}{r_c}\right)}$$

$$= \frac{W}{\left(\frac{V_1 (r_c - 1)}{r_c}\right)}$$

$$MEP = \frac{P_1 V_1 r_c^{\gamma-1}\left[\alpha\gamma(\rho - 1) + (\alpha - 1) - r_c^{1-\gamma}(\alpha\rho^{\gamma} - 1)\right]}{(\gamma - 1)\ \frac{V_1 (r_c - 1)}{r_c}}$$

$$\boxed{MEP = \frac{P_1 r_c^{\gamma}\left[\alpha\gamma(\rho - 1) + (\alpha - 1) - r_c^{1-\gamma}(\alpha\rho^{\gamma} - 1)\right]}{(\gamma - 1)(r_c - 1)}}$$