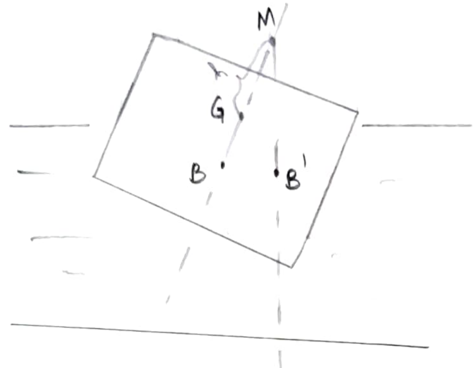
IAT-1 (AH)

1. Explain the terms with expression for the following.

a) <u>META CENTRIC HEIGHT</u>.

The distance MG, ie the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta centre



$MG = \frac{I}{V}$ – distance between G and B.

b) <u>Kinematic similitude</u> –

when the ratio of the velocity and acceleration at the corresponding points in the prototype and model are same, it is said to have Kinematic similarity / Kinematic similitude. But the direction of the vector quantities should be same.

Let,

V_{m_1} = velocity at p_1 in model

V_{m_2} = velocity at p_2 in model

a_{m_1} = acceleration at p_1 in model

a_{m_2} = acceleration at p_2 in model

V_{p_1} = velocity at p_1 in prototype

V_{p_2} = velocity at p_2 in prototype

a_{p_1} = acceleration at p_1 in prototype

a_{p_2} = acceleration at p_2 in prototype.

according to statement,

$$\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = V_r$$ where V_r = velocity ratio.

$$\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r$$ where a_r = acceleration ratio.

c) Dynamic similitude -

When ratio of the forces acting at the corresponding points in model and prototype are same, it is said to have dynamic similitude.

But the direction of forces should be same.

Let,

$(F_i)_m$ = Inertial force in model
$(F_v)_m$ = Viscous force in model
$(F_g)_m$ = gravity force in model

$(F_i)_p$ = Inertial force in prototype
$(F_v)_p$ = viscous force in prototype
$(F_g)_p$ = gravity force in prototype

According to statement,

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r$$

where F_r = force ratio.

d) Reynold's model -

Model based on reynold's number include.

i) Pipe flow
ii) Resistance experienced by sub-marines, airplanes, fully immersed bodies.

$$[Re]_m = [Re]_p$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

where

ρ_m = density of fluid in model
V_m = velocity of fluid in model
L_m = length of model
μ_m = viscosity of fluid in model

ρ_p = density of fluid in prototype
V_p = velocity of fluid in prototype
L_p = length of prototype
μ_p = viscosity of fluid in prototype

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\Rightarrow \frac{\rho_p}{\rho_m} \times \frac{V_p}{V_m} \times \frac{L_p}{L_m} \times \frac{1}{\mu_p/\mu_m} = 1 \quad \Rightarrow \frac{\rho_r V_r L_r}{\mu_r} = 1$$

d) Froude's model —

The model is based on Froude's number. It includes :

i) Flow of jet from an orifice or nozzle
ii) Force surface flows such as flow over spillway, weir, orifice.
iii) where water are formed.
iv) where fluids of different density flows over one another.

It is applicable where gravity force is predominant.

$$[F_r]_m = [F_r]_p$$

$$\frac{V_m}{\sqrt{L_m g_m}} = \frac{V_p}{\sqrt{L_p g_p}}$$

$$\frac{V_m}{\sqrt{L_m}} = \frac{V_p}{\sqrt{L_p}}$$

$$\Rightarrow \frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}} = \sqrt{L_r}$$

$$V_r = \sqrt{L_r}$$

where V_r = velocity ratio
L_r = length ratio.

Q2. Differentiate between

MODEL	PROTOTYPE
★ A model is a scaled replica of the actual structure on which case study etc can be done	★ The actual structure or the machine itself is called the prototype.

b)

DISTORTED MODEL	UNDISTORTED MODEL
* The models which are not geometrically similar to their prototype is called distorted model. * For distorted model, different scale ratio for linear dimension are adopted / taken. * cost of model can be reduced	* These are models which are geometrically similar to the prototype. * The prototype behaviour, reaction etc can be predicted from the result of undistorted model

c)

DIMENSIONAL HOMOGENOUS	NON-HOMOGENOU EQUATON
* It means that, the dimension of each terms in an equation on both sides are equal. Such equation are independent of the system of units. EXAMPLE – $V = \sqrt{2gH}$. LHS – $V = \frac{L}{T} \rightarrow [LT^{-1}]$ — ① RHS – $g = \frac{L}{T^2} \rightarrow [LT^{-2}]$ — ② – $H = L \rightarrow [L]$ — ③ $gH = [LT^{-1}]$ from ② & ③ $\sqrt{2gH} = [LT^{-1}]$ $\Rightarrow$ LHS = RHS	* An equation that is dimensionally non homogenous cannot possibly be valid. * It means that the dimension of each term in equation on both sides are not equal. Such equation is called dimensionally non-homogenous. * It occurs through mistake Example – $u = u_0 + gt$. by chance if you miss (t). $u(m/s) = u_0(m/s) + g(m/s^2)$ $[MT^{-1}] = [MT^{-1}] + [MT^{-2}]$ LHS $\neq$ RHS.

Q3. Using Buckingham's π theorem, show that velocity through a circular orifice is given by.

$$V = \sqrt{2gH}\ \phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$$

→ $V = f(g, H, D, \mu, \rho)$

$f_1(V, g, H, D, \mu, \rho) = 0.$

$V = m/s \rightarrow [LT^{-1}]$

$g = m/s^2 \rightarrow [LT^{-2}]$

$H = m \rightarrow [L]$

$D = m \rightarrow [L]$

$\mu = [ML^{-1}T^{-1}]$ from eq $\frac{F}{A} = \mu \frac{dx}{dy}$

$\rho = kg/m^3 . [ML^{-3}]$

No. of fundamental dimension = 3.

No. of π terms = 6 − 3 = 3.

$\therefore f_1(\pi_1, \pi_2, \pi_3) = 0$

each π term contain 3 + 1 variables $(m+1)$

3 are repeating variable.

choosing ρ, H and g as repeating term.

$\pi_1 = \rho^{a_1} . H^{b_1} g^{c_1} . V$

$M^0L^0T^0 = [ML^{-3}]^{a_1} [L]^{b_1} [LT^{-2}]^{c_1} [LT^{-1}]$

$0 = a_1$ $\quad a_1 = 0$

$0 = -3a_1 + b_1 + c_1 + 1$ $\quad b_1 = -1/2.$

$0 = -2c_1 - 1$ $\quad c_1 = -1/2$

$$\pi_1 = \rho^0 H^{-1/2} g^{-1/2} \cdot V$$

$$\boxed{\pi_1 = \frac{V}{\sqrt{gH}}}$$

$$\pi_2 = \rho^{a_2} H^{b_2} g^{c_2} \cdot \mu$$

$$M^0 L^0 T^0 = [ML^{-3}]^{a_2} [L]^{b_2} [LT^{-2}]^{c_2} [ML^{-1}T^{-1}]$$

$0 = a_2 + 1$ $\qquad a_2 = -1$

$0 = -3a_2 + b_2 + c_2 - 1$ $\qquad b_2 = -3/2$

$0 = -2c_2 - 1$ $\qquad c_2 = -1/2$

$$\pi_2 = \rho^{-1} H^{-3/2} g^{-1/2} \mu$$

$$\pi_2 = \frac{\mu}{\rho H^{3/2} g^{1/2}}$$

$$\boxed{\pi_2 = \frac{\mu}{\rho H \sqrt{gH}}}$$

$$\pi_3 = \rho^{a_3} H^{b_3} g^{c_3} \cdot D.$$

$$M^0 L^0 T^0 = [ML^{-3}]^{a_3} [L]^{b_3} [LT^{-2}]^{c_3} [L]$$

$0 = a_3.$ $\qquad a_3 = 0$

$0 = -3a_3 + b_3 + c_3 + 1$ $\qquad b_3 = -1.$

$0 = -2c_3$ $\qquad c_3 = 0.$

$$\pi_3 = \rho^0 H^{-1} g^0 \cdot D$$

$$\boxed{\pi_3 = \frac{D}{H}}$$

Applying principle ④.

Any constanst can be put in the equation

$$\pi_1 = \frac{V}{\sqrt{gH}} \rightarrow \pi_1 = \frac{V}{\sqrt{2gH}}.$$

Applying principle ⑥.

New $\pi_2 \rightarrow \frac{\pi_2}{\pi_1}$ Now multiply & divide by V

$$\pi_2 = \frac{\mu}{\rho H\sqrt{gH}} \times \frac{V}{V}$$

$$= \frac{\mu}{\rho H V} \times \frac{V}{\sqrt{gH}}$$

$$\pi_2 = \frac{\mu}{\rho H V} \times \pi_1$$

$$\frac{\pi_2}{\pi_1} = \frac{\mu}{\rho H V} \longrightarrow \pi_2 (New)$$

$$f_1(\pi_1, \pi_2, \pi_3) = 0.$$

$$f_1\left(\frac{V}{\sqrt{2gH}}, \frac{\mu}{\rho H V}, \frac{D}{H}\right) = 0.$$

$$\frac{V}{\sqrt{2gH}} = f\left(\frac{\mu}{\rho H V}, \frac{D}{H}\right)$$

$$\boxed{V = \sqrt{2gH}\,\phi\left[\frac{\mu}{\rho H V}, \frac{D}{H}\right]}$$

Q4. A ship of 300m long moves in a sea water, whose density is 1030 kg/m³. a 1:100 model of this ship is to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30m/s and the resistance of model is 60N. Determine velocity of ship in sea water and also the resistance of ship in sea water. The density of air is 1.24 kg/m³. Take ν as 0.012 stokes and 0.018 stokes.

→ Given

model

$L_m = 3\,m$ [ratio 1:100]

$\rho_m = 1.24\ kg/m^3$

$V_m = 30\,m/s$.

$\nu_m = 0.018$ stoke $= 0.018 \times 10^{-4}\ m^2/s$

Prototype.

$L_p = 300\,m$. ~~scale~~

$\rho_p = 1030\ kg/m^3$.

$V_p = ?$

$\nu_p = 0.012$ stokes $= 0.012 \times 10^{-4}\ m^2/s$.

$F_m = 60\,N$.

Reynold's model.

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}$$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p} \qquad \because \nu = \frac{\mu}{\rho}$$

$$\frac{30 \times 3}{0.018 \times \cancel{10^{-4}}} = \frac{V_p \times 300}{0.012 \times \cancel{10^{-4}}}$$

$$V_p = 0.2\,m/s.$$

$$F_r = \rho_r V_r^2 L_r^2$$

$$\frac{F_p}{F_m} = \frac{\rho_p}{\rho_m} \times \left(\frac{V_p}{V_m}\right)^2 \times \left(\frac{L_p}{L_m}\right)^2$$

$$\frac{F_p}{60} = \frac{1030}{1.24} \times \left(\frac{0.2}{30}\right)^2 \times \left(\frac{300}{3}\right)^2$$

$$F_p = 22150.54\,N$$

Q.5. A solid cylinder 4m in diameter and 4m high is floating in water with its axis vertically. If the specific gravity of the material of cylinder is 0.65, find its meta centric height. State also whether it is stable or unstable

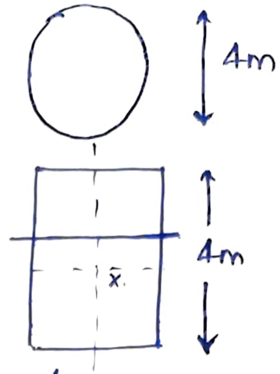
Given -

$d = 4m$

$h = 4m$.

$S = 0.65$.

$\rho = 0.65 \times 1000 = 650 \ kg/m^3$.



$I = \frac{\pi}{64} d^4 = \frac{\pi}{64} \times 4^4 = 12.56 \ m^4$.

depth of cylinder in water $h' = S \times h$

$= 0.65 \times 4$.

$= 2.6m$.

volume $= \frac{\pi}{4} d^2 \times h' = \frac{\pi}{4} \times 4^2 \times 2.6$

$V = 32.67 \ m^3$.

distance of centre of buoyance(B) from bottom $= \frac{2.6}{2} = 1.3 \ m$.

distance of centre of gravity(G) from bottom $= \frac{4}{2} = 2m$.

distandce between (G) and (B) $= 2 - 1.3 = 0.7m$.

meta centric height $= \frac{I}{V}$ - distance btw B and G.

$= \frac{12.56}{32.67} - 0.7$

$= -0.315 \ m$.

since meta centric height is -ve it is unstable.