

Internal Assessment Test – 2

Sub: Kinematics of Machines			Code: 18ME44	
Date: 24/06/2021	Duration: 90 mins	Max Marks: 50	Sem: 4	Branch (sections): ME (A,B)

Question 1 is compulsory and answer any THREE questions from part B. Good luck!

PART A		Marks	CO	RBT
1	Draw to full size the profile of a cam which will give a lift of 38 mm to a follower carrying a roller of 25 mm diameter. The axis of the follower is offset by 18 mm to the right of the axis of cam. Ascent of the follower takes place with SHM in 0.05 second followed by a period of rest 0.0125 second. The follower by then descent with UARM during 0.125 second, the acceleration being 3/5 times retardation. The cam rotates in clockwise direction at a constant speed of 240 rpm and the base circle radius is 50 mm.	[20]	CO5	L3
<b>PART B</b>				
2	Derive an expression for length of path of contact.	[10]	CO3	L3
3	Explain with neat sketch i) Simple gear train, ii) Compound gear train, iii) Reverted gear train and iv) Epicyclic gear train	[10]	CO3	L2
4	Two gear wheels mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6 mm and standard addendum one module. Pressure angle = 18°. Pinion rotates at 90 rpm. Find i) Number of teeth on each wheel so that interference is just avoided ii) Length of path of contact, iii) Length of arc of contact iv) Maximum velocity of sliding between teeth. v) Number of pairs of teeth in contact.	[10]	CO3	L3
5	Two mating spur gears with module of 6.5 mm have 19 and 47 teeth of 20° pressure angle and 6.5 mm addendum. Determine the number of pairs of teeth in contact. Also determine the sliding velocity at the instant i) engagement begins ii) engagement terminates. The pitch velocity is 1.2 m/s.	[10]	CO5	L3

CI

CCI

HOD

Draw to full size the profile of a Cam which will give a lift of 38 mm to a follower carrying a roller of 25 mm diameter. The axis of the follower is offset by 12 mm to the right of the axis of Cam. Ascent of the follower takes place with SHM in 0.05 second followed by a period of rest 0.0125 second. The follower then descends with UARM during 0.125 second, the acceleration being  $\frac{3}{5}$  times retardation. The Cam rotates in clockwise direction at a constant speed of 240 rpm & the base circle radius is 50 mm.

Speed = 240 rpm. to convert to seconds.

$$\frac{240}{60} = 4 \text{ rps.}$$

$$\text{Ascent} = 4 \times 360^\circ \times 0.05 = 72^\circ$$

$$\text{Dwell} = 4 \times 360^\circ \times 0.0125 = 18^\circ$$

$$\text{Return / Descent} = 4 \times 360^\circ \times 0.125 = 180^\circ$$

$$\text{Acceleration} = \frac{3}{5} \text{ Retardation}$$

Therefore time for acceleration must be  $\frac{5}{3}$  times the time of retardation.

$$\text{Acceleration period} = \frac{5}{3} \times \text{Retardation period}$$

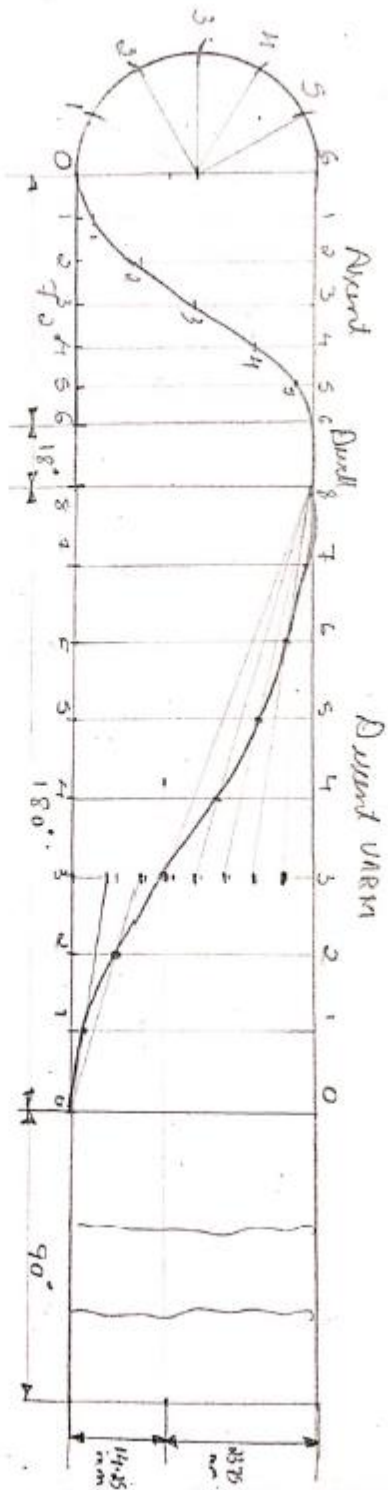
$$\text{Acceleration period} = \frac{5}{3} + \frac{3}{5} = \frac{25+9}{8} = \frac{34}{8}$$

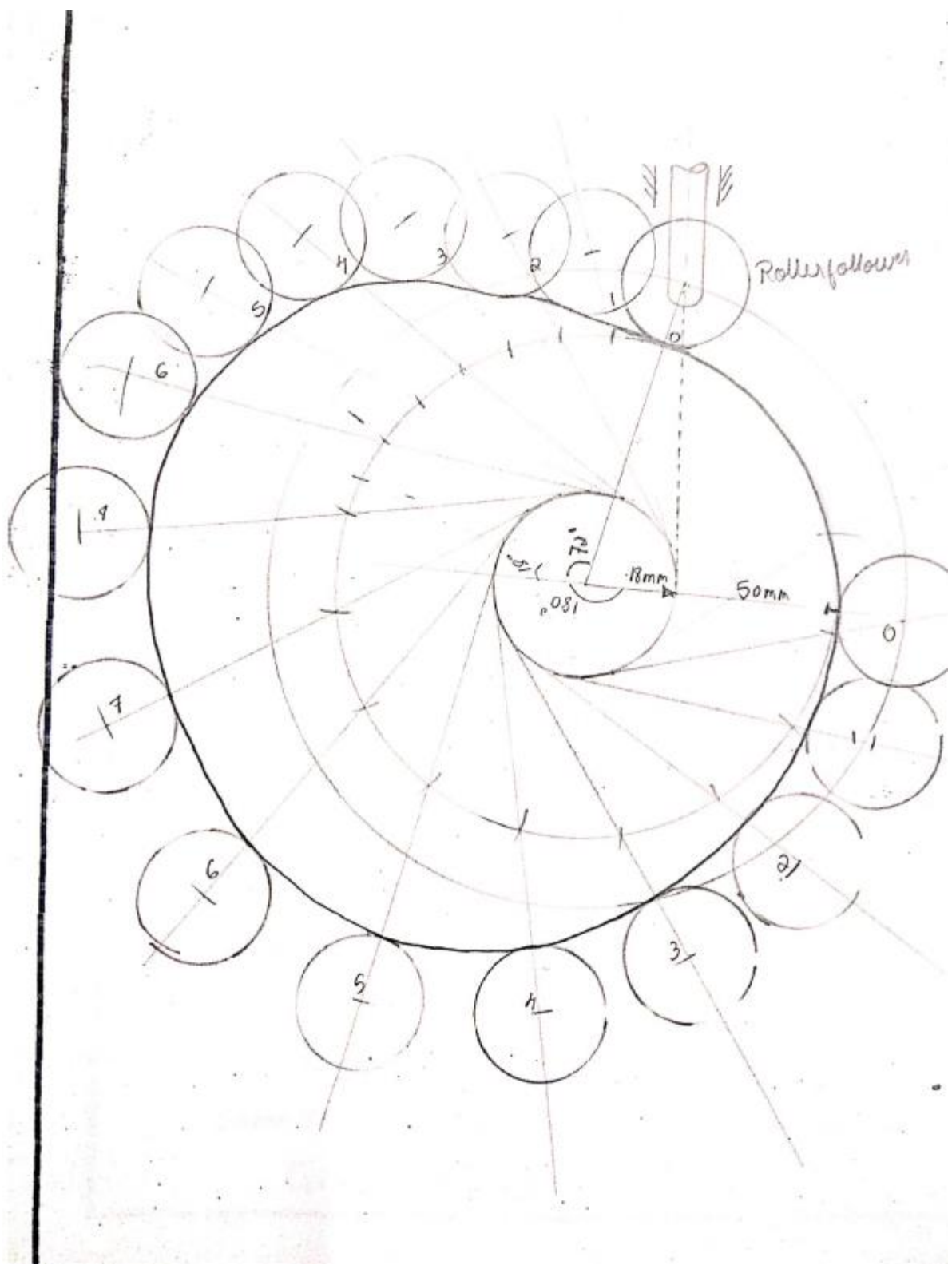
$$\text{Acceleration period} = \frac{5}{8} \times 180^\circ = 112.5^\circ$$

$$\text{Retardation Period} = \frac{3}{8} \times 180^\circ = 67.5^\circ$$

$$\text{Return of the follower during acceleration} = \frac{5}{8} \times 38 = 23.75$$

$$\text{Return of the follower during retardation} = \frac{3}{8} \times 38 = 14.25$$



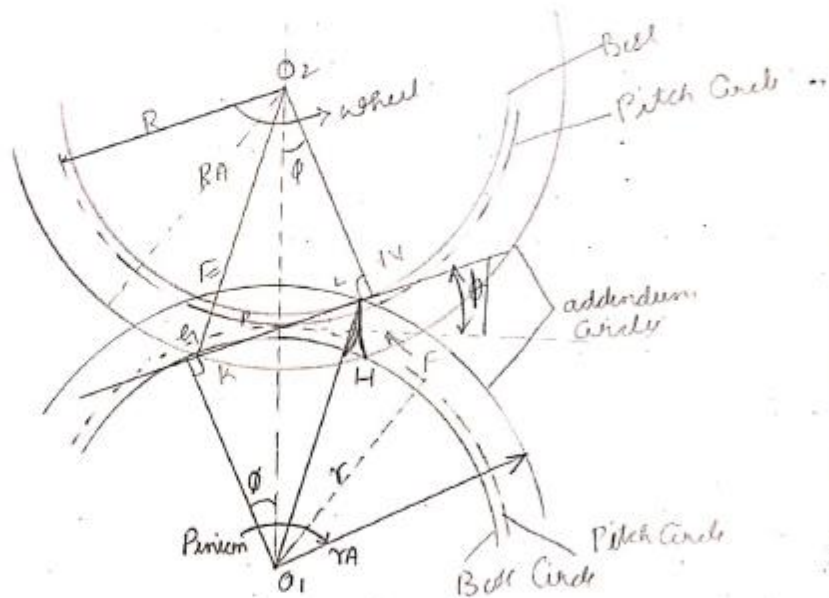




PART-B

24  
Ans

Derive an Expression for length of path of Contact  
 Consider a pinion driving the wheel as shown in fig. When the pinion rotates in clockwise direction, the Contact between a pair of involute teeth begins at K on the flank near the base circle of the pinion or the outer end of the tooth face on the wheel & ends at L. MH is the Common normal at the Point of Contact A, the Common tangent to the base Circle. The point K is the intersection of the addendum Circle of pinion & Common tangent



Let  $r_A = O_1 L =$  Radius of addendum Circle of pinion

$R_A = O_2 K =$  Radius of addendum Circle of wheel

$r = O_1 P =$  Radius of pitch Circle of pinion

$R = O_2 P =$  Radius of pitch Circle wheel

From Fig, we find that radius of base Circle of pin

$$O_1 M = O_1 P \cos \phi = r \cos \phi$$

4 radius of the base circle of wheel

$$O_2N = O_2P \cos \phi = R \cos \phi$$

Now from right angled triangle  $O_2KN$

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(RA)^2 - (R)^2 \cos^2 \phi}$$

$$\text{4 } PN = O_2P \sin \phi = R \sin \phi$$

Therefore length of the part of the path of contact

$$KP = KN - PN = \sqrt{(RA)^2 - (R)^2 \cos^2 \phi} - R \sin \phi$$

Similarly from right angled triangle  $O_1ML$

$$\text{4 } ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(rA)^2 - (r)^2 \cos^2 \phi}$$

$$MP = O_1P \sin \phi = r \sin \phi$$

Therefore length of the part of the path of contact

$$PL = ML - MP = \sqrt{(rA)^2 - (r)^2 \cos^2 \phi} - r \sin \phi$$

Therefore length of path of contact

$$KL = KP + PL = \sqrt{(RA)^2 - (R)^2 \cos^2 \phi} + \sqrt{(rA)^2 - (r)^2 \cos^2 \phi} - (R + r) \sin \phi$$

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Explain with neat sketch the following

i) Simple gear train



\* In simple gear train each shaft carries only one gear

\* All the gears revolve about fixed axis

\* Velocity ratio (V.R)

Let  $n_1, n_2$  &  $n_3$  are speeds of gears 1, 2 & 3 respectively

$Z_1, Z_2$  &  $Z_3$  are numbers of teeth on gears 1, 2, & 3 respectively

$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1} \rightarrow \textcircled{1}$$

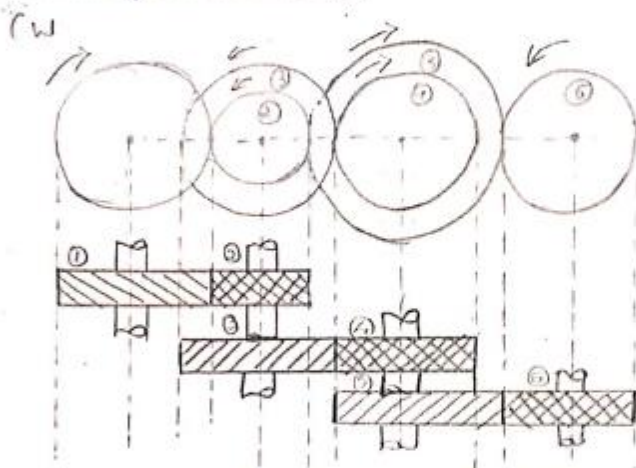
$$\Rightarrow \frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{Z_2}{Z_1} \times \frac{Z_3}{Z_2}$$

$$\frac{N_1}{N_3} = \frac{Z_3}{Z_1} \rightarrow \textcircled{2}$$

$$\frac{N_1}{N_3} = \frac{Z_3}{Z_1}$$

$$\text{Speed ratio} = \frac{\text{Speed of driving}}{\text{Speed of driven}} = \frac{\text{No of teeth on followers}}{\text{No of teeth on driver}}$$

### Compound Gear train



- \* In Compound gear train each shaft carries two or more gears except the first & last, one of which acts as a follower & the other as the driver.
- All the gears revolve about a fixed axis.

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \rightarrow \textcircled{1} \quad \frac{N_3}{N_4} = \frac{T_4}{T_3} \rightarrow \textcircled{2} \quad \frac{N_5}{N_6} = \frac{T_6}{T_5} \rightarrow \textcircled{3}$$

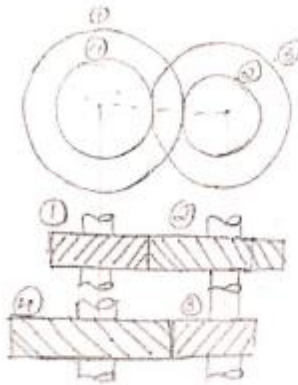
$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\text{Speed ratio} = \frac{\text{Speed of driver}}{\text{Speed of follower}} = \frac{\text{Product of teeth on followers}}{\text{Product of teeth on driver}}$$



## \* Reverted gear train



- \* In reverted gear train the first & last gears are on the <sup>same</sup> axis.
- \* In reverted gear train, the Centre distances of the two pair of gears must be the same.

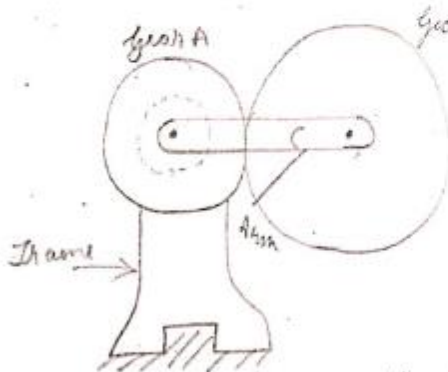
$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \rightarrow \textcircled{1} \quad \frac{N_3}{N_4} = \frac{T_4}{T_3} \rightarrow \textcircled{2}$$

$$\frac{N_1}{N_4} = \frac{N_3}{N_2} = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Speed ratio =  $\frac{\text{Speed of driver}}{\text{Speed of follower}} = \frac{\text{Product of no of teeth on follow}}{\text{Product of no of teeth on driver}}$

## + Epicyclic gear train



In epicyclic gear train, the axes of rotation of all the wheels are not fixed. In epicyclic gear train, axis of some gears having relative motion with respect to other or relative to the frame.

The gear B revolves about its own

axis as well as about the centre of the fixed gear B, Epicyclic gear train is also called planetary gear train.

Application = Used in automobile differential, watches, etc.



44 Two gear wheels mesh externally and are to give a velocity ratio of 3. The teeth are of involute form of module 6 mm of standard addendum one module. Pressure angle =  $18^\circ$ . Pinion rotates at 90 rpm. Find (1) Number of teeth on each wheel so that interference is just avoided (2) Length of path of Contact (3) Length of arc of Contact (4) Maximum velocity of sliding b/w teeth (5) Number of pairs of teeth in Contact.

Solu

No of teeth on pinion to avoid interference

$$a_w = 1$$

$$\text{Velocity ratio, } G = 3$$

$$\text{Addendum} = 1m \\ = 1 \times 6 = 6 \text{ mm}$$

$$\text{module} = 6 \text{ mm}$$

$$\phi = 18^\circ$$

$$t = \frac{2a_w}{9 \left[ \left( 1 + \frac{1}{G} \sin^2 \phi \left( \frac{1}{G} + 2 \right)^{1/2} - 1 \right) \right]}$$

$$t = \frac{2 \times 1}{3 \left[ \left( 1 + \frac{1}{3} \sin^2 18^\circ \left( \frac{1}{3} + 2 \right)^{1/2} - 1 \right) \right]}$$

$$t = \frac{2}{0.111406} = 18.01 \approx 19$$

$$t = 19 \text{ teeth}$$

$$\text{W.K.T Gear Ratio } G = \frac{T}{t}$$

$$3 = \frac{T}{19} \Rightarrow T = 57 \text{ teeth}$$

$$\text{Pitch Circle radius of Pinion} = \frac{mt}{2} = \frac{6 \times 19}{2} \Rightarrow r = 57 \text{ mm}$$

$$\text{Pitch Circle radius of Gear} = \frac{mT}{2} = \frac{6 \times 57}{2} \Rightarrow R = 171 \text{ mm}$$

Addendum Circle radius of pinion

$$r_a = r + \text{addendum}$$

$$r_a = 57 + 6$$

$$r_a = 63 \text{ mm}$$

Addendum Circle radius of Gear

$$R_a = R + \text{addendum}$$

$$R_a = 171 + 6$$

$$R_a = 177 \text{ mm}$$

$$\text{Path of Contact} = \left( R_a^2 - r^2 \cos^2 \phi \right)^{1/2} - r \sin \phi + \left( r_a^2 - R^2 \cos^2 \phi \right)^{1/2} - R \sin \phi$$

$$= 17.01 + 14.48$$

$$= 31.49 //$$

$$\text{Length of arc of Contact} = \frac{\text{Path of Contact}}{\cos \phi} = \frac{31.49}{\cos 18} = 33.11054$$

$$\text{Angular speed } \omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 90}{60} = 9.424$$

$$\frac{\omega_1}{\omega_2} = \frac{T}{t} \Rightarrow \frac{9.424}{\omega_2} = \frac{5t}{19}$$

$$\omega_2 = 3.141$$

$$\begin{aligned} \text{Max sliding Velocity} &= (\omega_1 \times \omega_2) \text{ Path of Contact} \\ &= (9.424 \times 3.141) \times 31.49 \\ &= 932.1286 \end{aligned}$$

5y  
sol  
Two mating spur gears with module of 6.5 mm have 19 & 47 teeth of  $20^\circ$  pressure angle & 6.5 mm addendum. Determine the number of pairs of teeth in contact. Also determine the sliding velocity at the instant (i) Engagement begins (ii) Engagement terminates. The pitch velocity is 1.2 m/s

$$\text{Given } m = 6.5 \text{ mm}$$

$$t = 19$$

$$T = 47$$

$$\phi = 20^\circ$$

$$v = 1.2 \text{ m/sec}$$

$$\text{addendum} = 6.5 \text{ mm}$$

$$\text{Pitch Circle radius of pinion } r = \frac{m \times t}{2} = \frac{6.5 \times 19}{2} = 61.75 \text{ mm}$$

$$\text{Pitch Circle radius of gear } R = \frac{m T}{2} = \frac{6.5 \times 47}{2} = 152.75 \text{ mm}$$

$$\begin{aligned} \text{Radius of addendum of pinion } r_a &= r + \text{addendum} \\ &= 61.75 + 6.5 = 68.25 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Radius of addendum of gear } R_a &= R + \text{addendum} \\ &= 152.75 + 6.5 = 159.25 \text{ mm} \end{aligned}$$

W.K.T

$$\begin{aligned} \text{Length of Path of approach} &= \sqrt{R_a^2 - R^2 \cos^2 \phi} - R \sin \phi \\ &= \sqrt{159.25^2 - 152.75^2 \cos^2 20^\circ} - 152.75 \sin 20^\circ \\ &= 68.97 - 52 - 24 \\ &= 16.73 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Length of Path of Recess} &= \sqrt{r_1^2 - r^2 \cos^2 \phi} - r \sin \phi \\ &= \sqrt{65.25^2 - 61.75^2 \cos^2 20^\circ} - 61.75 \times \sin 20^\circ \\ &= 35.93 - 21.12 \\ &= 14.81 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Length of Path of Contact} &= \text{Length of Path of Approach} + \text{Length of Path of Recess} \\ &= 16.73 + 14.81 \\ &= 31.54 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Length of arc of Contact} &= \frac{\text{Length of Path of Contact}}{\cos \phi} \\ &= \frac{31.54}{\cos 20^\circ} \\ &= 33.58 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{① No of pairs of teeth in Contact} &= \frac{\text{Length of arc of Contact}}{\text{Circular Pitch}} \\ &= \frac{33.58}{\pi \times m} = \frac{33.58}{\pi \times 6.5} = 1.645 \approx 2 \text{ pairs} \end{aligned}$$

② Angle turned through by the gear for one pair of teeth in Contact

$$\begin{aligned} \text{W.K.T Angle turned through by the gear} &= \frac{\text{Length of arc of Contact} \times 360^\circ}{\text{Circumference of Gear}} \\ &= \frac{33.58 \times 360^\circ}{2 \pi R} = \frac{33.58 \times 360^\circ}{2 \pi \times 152.75} \\ &= 12.60^\circ \end{aligned}$$

③ Sliding Velocity

$$\text{Let } \omega_1 = \frac{V}{Y} = \frac{1.2 \times 1000}{61.75} = 19.43 \text{ rad/sec}$$

$$\omega_2 = \frac{V}{R} = \frac{1.2 \times 1000}{152.75} = 7.85 \text{ rad/sec}$$

by The sliding Velocity at the Engagement Commences

$$V_{s1} = (\omega_1 + \omega_2) \times \text{Length of Path of Approach}$$

$$= (19.43 + 7.85) \times 16.73$$

$$V_{s1} = 456.39 \text{ mm/sec}$$

by The sliding Velocity at the Engagement terminates

$$V_{s2} = (\omega_1 + \omega_2) \times \text{Length of Path of Recess}$$

$$V_{s2} = (19.43 + 7.85) \times 14.81$$

$$V_{s2} = 404 \text{ mm/sec}$$