IAT - III

Operations Research

Deeptha Sabarish
1CR17ME023

1)
1A) Queue discipline - This indicates the the sequence or the order in which the customers from the queue are selected for the service. Some of the well known disciplines are

→ SIRO discipline - ~~Rando~~ Service in Random Order where the customers are randomly chosen regardless of their order of arrival.

→ FCFS discipline - First come First Serve where the order of the customer's arrival decides their turn as the name states.

→ LCFC discipline - Last come First Serve where the last customer recieves immediate attention.

Customer population - The number of customers at any given time ~~as~~ isn't exactly propostional, quite random. This can be classified as either poisson or exponential distribution.

Jockeying - A customer can move to a different queue if they predict quicker service.

Renegaging - A customer after joining the leave due to the crowd thats alreddy at the queue because they are impatient. ~~to~~

Balking - A customer who doesn't join a queue or gives up their place in a queue because of either the crowd or physical space restriction and they don't think they will be able get the expected service.

1B) Given data

$\lambda = 8/hr = \cancel{10}\, 8$

$\mu = 6\ min = \frac{1}{6} \times 60 = 10$

a) $P_0 = 1 - \rho$

$\rho = \frac{\lambda}{\mu} = \frac{8}{10} = 0.8$

$P_0 = 0.2$

$\boxed{P_0 = 20\%}$

b) $L = L_q + \frac{\lambda}{\mu}$

$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{8^2}{10(10-8)} = \frac{64}{20} = 3.2$ customers

$L = 3.2 + \frac{8}{10} = \boxed{4\ \text{customers} = L}$

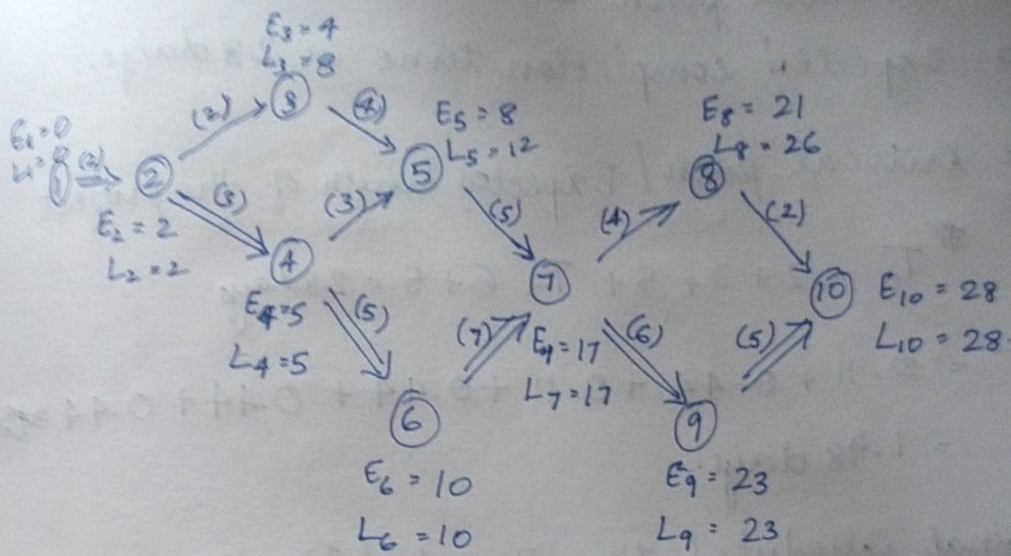
c) $\rho = 0.8 = \boxed{80\% = \rho}$

d) $W = W_q + \frac{1}{\mu}$

$W_q = \frac{L_q}{\mu} = \frac{3.2}{10} = 0.32$ hours

$W = 0.32 + \frac{1}{10} = \boxed{0.42\,hrs = W}$

6A) The network :



Completion time, $t_e = \dfrac{(t_o + 4t_m + t_p)}{6}$

variance, $\sigma^2 = \left(\dfrac{t_p - t_o}{6}\right)^2$

Activity	t_o	t_m	t_p	t_e	σ^2
1-2	1	2	3	2	0.11
2-3	1	2	3	2	0.11
2-4	1	3	5	3	0.44
3-5	3	4	5	4	0.11
4-5	2	3	4	3	0.11
4-6	3	5	7	5	0.44
5-7	4	5	6	5	0.11
6-7	6	7	8	7	0.11
7-8	2	4	6	4	0.44
8-10	1	2	3	2	0.11
9-10	3	5	7	5	0.44

The critical path is 1-2-4-6-7-9-10 and expected completion time is 28 days.

The critical path / Expected length of the project

$T_e = 2+3+5+7+6+5 = 28$ days

$\sigma^2 = 0.11 + 0.44 + 0.11 + 0.44 + 0.44 + 0.44$

$= 1.98$ days.

Original scheduled time of completing the project $= T_s = 30$ days.

$$Z = \left(\frac{T_s - T_e}{\sigma}\right) = \frac{30-28}{1.41} = 1.42$$

From the normal distribution table,

At $Z = 1.42$, the probability of completing the projects is $0.9222 \approx 92.22\%$.

If the probability is 90% i.e., 0.9,

$p = 0.9, \; Z = 1.29$

$$Z = \frac{T_s - T_e}{\sigma} \Rightarrow 1.29 = \frac{T_s - 28}{1.41}$$

$T_s = 29.82$ days - due date.

3A) Order 'B – A'

Job	Machine B	Machine A
1	4	7
2	8	3
3	9	11
4	10	5
5	6	12

Optimal sequence, 1 5 3 4 2

B → ← A

Job	Machine B In time	Machine B Out time	Machine A In time	Machine A Out time.
1	0	4	4	11
5	4	10	11	23
3	10	19	23	34
4	19	29	34	39
2	29	37	39	42

Total eloped time = 42 hours

Idle time of machine B = 4 hours

Idle time of machine A = 5 hours.

4A).

Processing time

Job	Machine G	Machine H
1	9	12
2	12	16
3	9	14
4	7	12
5	9	16
6	12	11
7	21	15
8	13	15

Optimal sequence

4 1 3 5 2 8 7 6

$G \longrightarrow$ $\longleftarrow H$

Job	Machine C		Machine A		Machine B	
	In time	Out time	In time	Out time	In time	Out time.
4	0	3	3	7	7	15
1	3	8	8	12	15	23
3	8	10	12	19	23	30
5	10	14	19	24	30	41
2	14	20	24	30	41	51
8	20	31	31	33	51	64
7	31	46	46	52	64	73
6	46	55	55	58	73	81

Total time elapsed = 81 hours

Idle time of machine C = 26 hours

Idle time of machine A = 44 hours

Idle time of machine B = 7 hours.

2A) Given data $k = 2$ $\lambda = 10/\text{hrs}$ $\mu = \frac{1}{5} \times 60 = 12$

$n = 0, 1$

$$W_q = \frac{L_q}{\lambda}$$

$$L_q = \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)^k}{(k-1)(k\mu-\lambda)^2} \times P_0$$

$$P_0 = \frac{1}{\sum_{n=0}^{k-1} \frac{1}{\lfloor n} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{\lfloor k} \left(\frac{\lambda}{\mu}\right)^k \times \frac{k\mu}{(k\mu-\lambda)}}$$

$$= \frac{1}{\frac{1}{\lfloor 0} \left(\frac{10}{12}\right)^0 + \frac{1}{\lfloor 1} \left(\frac{10}{12}\right)^1 + \frac{1}{\lfloor 2} \left(\frac{10}{12}\right)^2 \times \frac{2 \times 12}{(2\times 12) - 6}}$$

$$= \frac{1}{1 + \left(\frac{10}{12}\right) + \frac{1}{2}\left(\frac{10}{12}\right)^2 \times \frac{24}{12}} = 0.3956$$

$P_0 = 39.56\%$

$$L_q = \frac{10 \times 12 \left(\frac{10}{12}\right)^2}{\lfloor 1 (12 \times 2 - 10)^2} \times 0.3956 = 0.16819$$

$$W_q = \frac{0.16819}{10} = 0.01681$$

$P_0 = 39.56\%$

$$W_s = W_q + \frac{1}{\mu} = 0.0168 + \frac{1}{12} = 0.10013$$

$$\rho = \frac{\lambda}{k\mu} = \frac{10}{24} = 0.4166 \approx 41.66\%$$