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CMR Institute of Technology, Bangalore

DEPARTMENT OF MECHANICAL ENGINEERING

III - INTERNAL ASSESSMENT

Semester: 4-CBCS 2018 Date: 02 Aug 2021

Subject: KINEMATICS OF MACHINES (18ME44)

Faculty: Mr Vinay M N

Time: 09:00 AM - 10:30 AM Max Marks: 50

Instructions to Students:

Answer all five questions

ANSWER_ALL_QUESTIONS

Question	Marks	CO	PO	BT/CL
1. Define instantaneous centre and state, explain the types of instantaneous centres	**[10.0]**	**1**	**[2]**	**[2]**
2. Explain Klein's construction for slider crank mechanism	**[10.0]**	**1**	**[2]**	**[3]**
3. Derive an expression for Freudenstein's equation for slider crank mechanism	**[10.0]**	**1**	**[2, 3]**	**[3]**
4. Derive an expression for Freudenstein's equation for four bar mechanism	**[10.0]**	**1**	**[2, 3]**	**[3]**
5. Explain function generation for four bar mechanism	**[10.0]**	**1**	**[2, 3]**	**[3]**

SOLUTIONS

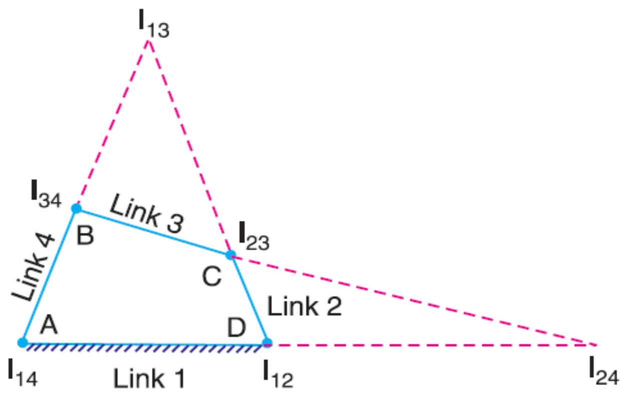
1. The instantaneous centres for a mechanism are of the following three types:

   1. Fixed instantaneous centres,

   2. Permanent instantaneous centres, and

   3. Neither fixed nor permanent instantaneous centres.

   The first two types i.e. fixed and permanent instantaneous centres are together known as primary instantaneous centres and the third type is known as secondary instantaneous centres.



The instantaneous centres I12 and I14 are called the fixed instantaneous centres as they remain in the same place for all configurations of the mechanism. The instantaneous centres I23 and I34 are the permanent instantaneous centres as they move when the mechanism moves, but the joints are of permanent nature. The instantaneous centres I13 and I24 are neither fixed nor permanent instantaneous centres as they vary with the configuration of the mechanism.

Question 2: Klien's Construction

Let *OC* be the crank and *PC* the connecting rod of a reciprocating steam engine, as shown in Fig. 6 (*a*). Let the crank makes an angle θ with the line of stroke *PO* and rotates with uniform angular velocity ω rad/s in a clockwise direction. The Klien's velocity and acceleration diagrams are drawn as discussed below:

Klien's velocity diagram

First of all, draw *OM* perpendicular to *OP*; such that it intersects the line *PC* produced at *M*. The triangle *OCM* is known as ***Klien's velocity diagram***. In this triangle *OCM*, *OM* may be regarded as a line perpendicular to *PO*,

CM may be regarded as a line parallel to *PC*, and ...(It is the same line.)

CO may be regarded as a line parallel to *CO*.

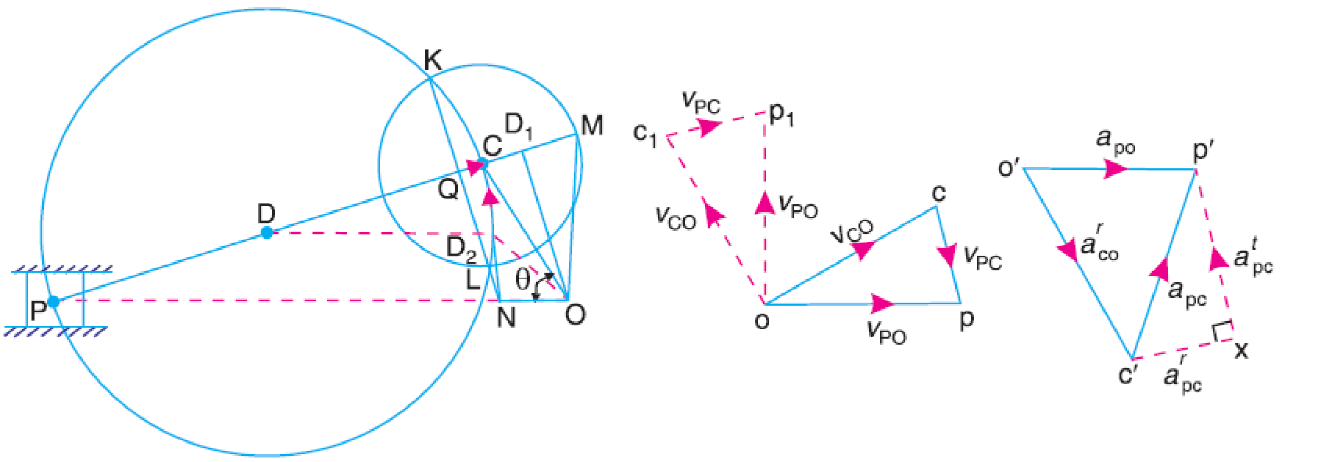
We have already discussed that the velocity diagram for given configuration is a triangle *ocp* as shown in Fig. 6 (*b*). If this triangle is revolved through 90°, it will be a triangle oc_1 p_1, in which oc_1 represents v_{CO} (*i.e.* velocity of *C* with respect to *O* or velocity of crank pin *C*) and is paralel to *OC*, op_1 represents v_{PO} (*i.e.* velocity of *P* with respect to *O* or velocity of cross-head or piston *P*) and is perpendicular to *OP*, and c_1p_1 represents v_{PC} (*i.e.* velocity of *P* with respect to *C*) and is parallel to *CP*. A little consideration will show that the triangles oc_1p_1 and *OCM* are similar. Therefore,

$$\frac{oc_1}{OC} = \frac{op_1}{OM} = \frac{c_1p_1}{CM} = \omega \ (a\ constant)$$

or
$$\frac{v_{CO}}{OC} = \frac{v_{PO}}{OM} = \frac{v_{PC}}{CM} = \omega$$

Therefore, $v_{co} = \omega \times OC$; $v_{PO} = \omega \times OM$ and $v_{PC} = \omega \times CM$

Thus, we see that by drawing the Klien's velocity diagram, the velocities of various points may be obtained without drawing a separate velocity diagram.



(*a*) Klien's acceleration diagram. (*b*) Velocity diagram. (*c*) Acceleration diagram.

Fig.6: Klein's construction

Klien's acceleration diagram

The Klien's acceleration diagram is drawn as discussed below:

1. First of all, draw a circle with *C* as centre and *CM* as radius.
2. Draw another circle with *PC* as diameter. Let this circle intersect the previous circle at *K* and *L*.

3. Join *KL* and produce it to intersect *PO* at *N*. Let *KL* intersect *PC* at *Q*. This forms the quadrilateral *CQNO*, which is known as ***Klien's acceleration diagram*.**

We have already discussed that the acceleration diagram for the given configuration is as shown in Fig. 6 (*c*). We know that

i) *o'c'* represents a^r_{CO} (*i.e.* radial component of the acceleration of crank pin *C* with respect to *O*) and is parallel to *CO*;

ii) *c'x* represents a^r_{PC} (*i.e.* radial component of the acceleration of crosshead or piston *P* with respect to crank pin *C*) and is parallel to *CP* or *CQ*;

iii) *xp'* represents a^t_{PC} (*i.e.* tangential component of the acceleration of *P* with respect to *C*) and is parallel to *QN* (because *QN* is perpendicular to *CQ*); and

iv) *o'p'* represents a_{PO} (*i.e.* acceleration of *P* with respect to *O* or the acceleration of piston *P*) and is parallel to *PO* or *NO*.

A little consideration will show that the quadrilateral *o'c'x p'* [Fig. 6 (*c*)] is similar to quadrilateral *CQNO* [Fig. 6 (*a*)]. Therefore,

$$\frac{o'c'}{OC} = \frac{c'x}{CQ} = \frac{xp'}{QN} = \frac{o'p'}{NO} = \omega^2\ (a\ constant)$$

$$\frac{a^r_{CO}}{OC} = \frac{a^r_{PC}}{CQ} = \frac{a^t_{PC}}{QN} = \frac{a_{PO}}{NO} = \omega^2$$

Therefore, $a^r_{CO} = \omega^2 \times OC$; $a^r_{PC} = \omega^2 \times CQ$

$a^t_{PC} = \omega^2 \times QN$; and $a_{PO} = \omega^2 \times NO$

Thus we see that by drawing the Klien's acceleration diagram, the acceleration of various points may be obtained without drawing the separate acceleration diagram.

Question 3:

Freudenstein's equation for slider crank mechanism.

A design problem where the link lengths of a crank-slider mechanism must be determined so that the translation r and the rotation ϕ are functionally related.

The desired relation is represented by $f(\phi, r) = 0$.

The vector loop equation is

$$\bar{r}_2 + \bar{r}_3 - \bar{r}_4 - \bar{r}_1 = 0 \qquad -(i)$$

Considering the links to be vectors, displacement along X-axis is,

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 = 0$$

$$\therefore \; r_3 \cos\theta_3 = -r_2 \cos\theta_2 + r_4 \qquad -(ii)$$

Squaring equation (ii)

$$r_3^2 \cos^2\theta_3 = r_2^2 \cos^2\theta_2 + r_4^2 - 2 r_2 r_4 \cos\theta_2 \qquad -(iii)$$

Displacement along Y-axis is,

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_1 = 0$$

$$\therefore \; r_3 \sin\theta_3 = -r_2 \sin\theta_2 + r_1 \qquad (-iv)$$

Squaring equation (iv)

$$r_3^2 \sin^2\theta_3 = r_2^2 \sin^2\theta_2 + r_1^2 - 2 r_1 r_2 \sin\theta_2 \qquad (v)$$

To eliminate θ_3, add both sides of the equation (iii) and (v)

$$\therefore \; r_3^2 \cos^2\theta_3 + r_3^2 \sin^2\theta_3 = r_2^2 \cos^2\theta_2 + r_4^2 - 2 r_2 r_4 \cos\theta_2 + r_2^2 \sin^2\theta_2 + r_1^2 - 2 r_1 r_2 \sin\theta_2$$

i.e. $$r_3^2(\cos^2\theta_3 + \sin^2\theta_3) = r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_4^4 + r_1^2 - 2 r_2 r_4 \cos\theta_2 - 2 r_1 r_2 \sin\theta_2$$

$$r_3^2 = r_2^2 + r_4^2 + r_1^2 - 2 r_2 r_4 \cos\theta_2 - 2 r_1 r_2 \sin\theta_2$$

$$r_3^2 - r_2^2 - r_1^2 + 2 r_2 r_4 \cos\theta_2 + 2 r_1 r_2 \sin\theta_2 = r_4^2$$

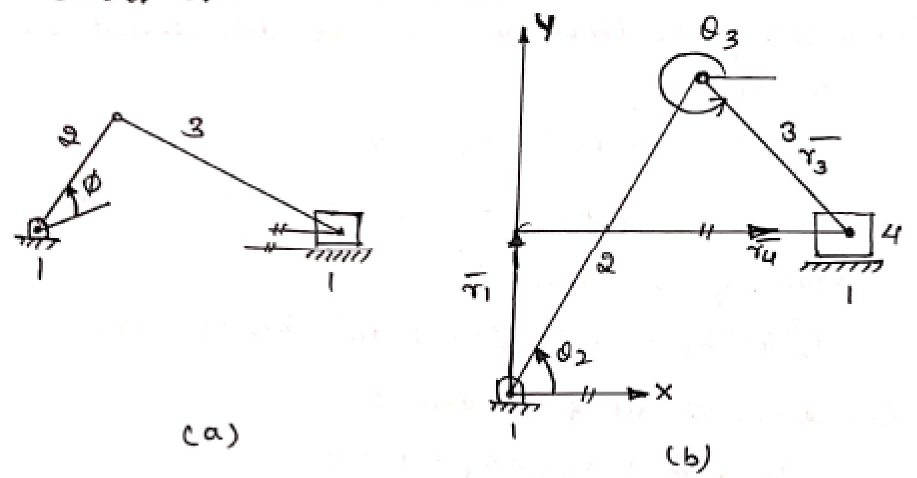
$-(vi)$

Let $S_1 = 2r_2$; $S_2 = 2r_1 r_2$; $S_3 = r_3^2 - r_2^2 - r_1^2$

Substituting these values in equation (vi)

$$\boxed{S_1 r_4 \cos\theta_2 + S_2 \sin\theta_2 + S_3 = r_4^2} \quad \text{— (vii)}$$

Above Eqn is the Freudenstein's equation for a slider crank mechanism.

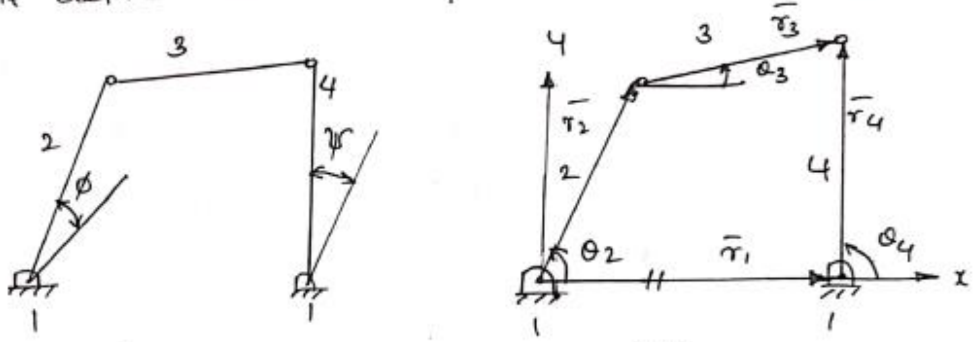


Question 4:

(1) Freudenstein's Equation for four bar mechanism

A design problem where the link lengths of a four bar mechanism must be determined so that the rotations of the two levers within the mechanism, ϕ and ψ, are functionally related.

The desired relation is represented by $f(\phi, \psi) = 0$.



(a) (b)

Fig.(b) shows the four bar mechanism and the vector loop necessary for the mechanism's analysis. The vector loop equation is,

$$\bar{r}_2 + \bar{r}_3 - \bar{r}_4 - \bar{r}_1 = 0 \qquad -(1)$$

Considering the links to be vectors, displacement along the x-axis is,

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 \cos\theta_4 - r_1 = 0$$

$$\therefore \ r_3 \cos\theta_3 = -r_2 \cos\theta_2 + r_4 \cos\theta_4 + r_1 \qquad -(2)$$

Squaring equation (2)

$$r_3^2 \cos\theta_3^2 = r_2^2 \cos\theta_2^2 + r_4^2 \cos\theta_4^2 + r_1^2 - 2r_2 r_4 \cos\theta_2 \cos\theta_4 - 2r_2 r_1 \cos\theta_2 + 2r_4 r_1 \cos\theta_4 \qquad -(3)$$

Displacement along y-axis is,

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_4 \sin\theta_4 = 0$$

$$\therefore \ r_3 \sin\theta_3 = -r_2 \sin\theta_2 + r_4 \sin\theta_4 \qquad -(4)$$

Squaring equation(4)

$$r_3^2 \sin\theta_3^2 = r_2^2 \sin\theta_2^2 + r_4^2 \sin\theta_4^2 - 2r_2 r_4 \sin\theta_2 \sin\theta_4 \qquad -(5)$$

Equation (3) and (5) can be reduced to a single equation relating θ_2, θ_4 and the four link lengths by eliminating θ_3.

To eliminate θ_3, add both sides of the eqn (3) and (5)

$$\therefore r_3^2 \cos^2\theta_3 + r_3^2 \sin^2\theta_3 = r_2^2\cos^2\theta_2 + r_4^2\cos^2_4 + r_1^2 - 2r_2 r_4 \cos\theta_2 \cdot \cos\theta_4$$
$$- 2r_2 r_1 \cos\theta_2 + 2r_4 r_1 \cos\theta_4 + r_2^2 \sin\theta_2^2$$
$$+ r_4^2 \sin\theta_4^2 - 2r_2 r_4 \sin\theta_2 \sin\theta_4$$

i.e., $$r_3^2(\cos^2\theta_3 + \sin^2\theta_3) = r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_4^2(\cos^2\theta_4 + \sin^2\theta_4)$$
$$+ r_1^2 - 2r_2 r_4 \cos\theta_2 \cos\theta_4 \cancel{+ r_4 r_1} - 2r_2 r_1 \cos\theta_2$$
$$+ 2r_4 r_1 \cos\theta_4 - 2r_2 r_4 \sin\theta_2 \sin\theta_4$$

i.e., $$r_2^2 - r_3^2 + r_4^2 + r_1^2 + 2r_4 r_1 \cos\theta_4 - 2r_2 r_1 \cos\theta_2 = 2r_2 r_4 \cos\theta_2 \cos\theta_4$$
$$+ 2r_2 r_4 \sin\theta_2 \sin\theta_4$$

Dividing both sides by $2r_2 r_4$ we get,

$$\frac{r_2^2 - r_3^2 + r_4^2 + r_1^2}{2r_2 r_4} + \frac{r_1}{r_2}\cos\theta_4 - \frac{r_1}{r_4}\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4 \quad -(6)$$

Let $\frac{r_1}{r_4} = R_1$; $\frac{r_1}{r_2} = R_2$ and $\frac{r_2^2 - r_3^2 + r_4^2 + r_1^2}{2r_2 r_4} = R_3$

Substituting these values in equation (6) we get,

$$R_3 + R_2\cos\theta_4 - R_1\cos\theta_2 = \cos\theta_2\cos\theta_4 + \sin\theta_2\sin\theta_4$$

$$\therefore \boxed{R_3 + R_2\cos\theta_4 - R_1\cos\theta_2 = \cos(\theta_2 - \theta_4)} \quad -(7)$$

Eqn (7) is called Freudenstein's equation.

* It is the relationship between input rotation θ_2 and output rotation θ_4 as determined by the link lengths r_1 through r_4.

In function generation via Freudenstein's equation, the idea is to use equation (7) to determine a set of link lengths that will result in a $(\theta_2 - \theta_4)$ relationship that matches a desired function.

Question 5:

Function generation - Function generation is similar to curve fitting. There are two basic methods:

(i) Point matching method

(ii) Derivative matching method.

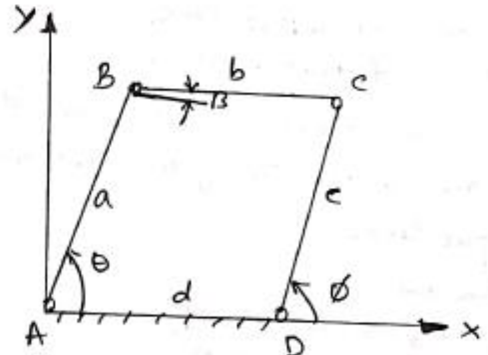
Function generation for four bar mechanism

A four bar mechanism shown in fig. is in equillibrium.

Let a, b, c and d be the magnitudes of the links AB, BC, CD and DA respectively. θ, β and φ are the angles of AB, BC and DC respectively with the x- axis.

AD is the fixed link.

AB and DC are the input and output links respectively.



Considering the links to be vectors, displacement along the x-axis is, $a\cos\theta + b\cos\beta - c\cos\phi - d = 0$

$$\therefore\ b\cos\beta = -a\cos\theta + c\cos\phi + d$$

Squaring on both sides

$$(b\cos\beta)^2 = (-a\cos\theta + c\cos\phi + d)^2$$

$$b^2\cos^2\beta = a^2\cos^2\theta + c^2\cos^2\phi + d^2 - 2ac\cos\theta\cos\phi - 2ad\cos\theta + 2cd\cos\phi \quad -(1)$$

Displacement along y- axis

$$a\sin\theta + b\sin\beta - c\sin\phi = 0$$

$$\therefore\ b\sin\beta = -a\sin\theta + c\sin\phi$$

$$b^2\sin^2\beta = a^2\sin^2\theta + c^2\sin\phi - 2ac\sin\phi\sin\theta \quad -(2)$$

Adding equations ① and ②

$$b^2 = c^2 + a^2 + d^2 - 2ac\cos\theta\cos\phi - 2ad\cos\theta + 2cd\cos\phi - 2ac\sin\theta\sin\phi$$

$$a^2 - b^2 + c^2 + d^2 + 2cd\cos\phi - 2ad\cos\theta = 2ac(\cos\theta\cos\phi - \sin\theta\sin\phi)$$

Dividing both sides by $2ac$

$$\frac{a^2-b^2+c^2+d^2}{2ac} + \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta = \cos(\theta-\phi) \text{ (or) } \cos(\phi-\theta) \quad -③$$

Equation ③ is Known as Freudenstein's equation.

It can be written as

$$K_3 + K_1\cos\phi + K_2\cos\theta = \cos(\theta-\phi) \quad -④$$

Where $K_3 = \frac{a^2-b^2+c^2+d^2}{2ac}$; $K_1 = \frac{d}{a}$; $K_2 = -\frac{d}{c}$

Let the input and the output are related by some function such as $y = f(x)$. For the given positions.

$\theta_1, \theta_2, \theta_3$ = three positions of input link.

ϕ_1, ϕ_2, ϕ_3 : Three positions of output link.

It is required to find the values of a, b, c and d to form a four-link mechanism giving the prescribed motions of the input and output links.

Eqn ④ can be written as

$$K_1\cos\phi_1 + K_2\cos\theta_1 + K_3 = \cos(\theta_1-\phi_1)$$

$$K_1\cos\phi_2 + K_2\cos\theta_2 + K_3 = \cos(\theta_2-\phi_2)$$

$$K_1\cos\phi_3 + K_2\cos\theta_3 + K_3 = \cos(\theta_3-\phi_3)$$

K_1, K_2 and K_3 can be evaluated by Gaussian elimination method or by cramer's rule.

$$\Delta = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_3 & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} \cos(\theta_1-\phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2-\phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3-\phi_3) & \cos\theta_3 & 1 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \cos\phi_1 & \cos(\theta_1-\phi_1) & 1 \\ \cos\phi_2 & \cos(\theta_2-\phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3-\phi_3) & 1 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1-\phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2-\phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3-\phi_3) \end{vmatrix}$$

K_1, K_2 and K_3 are given by

$$K_1 = \frac{\Delta_1}{\Delta}\ ;\ K_2 = \frac{\Delta_2}{\Delta}\ ;\ K_3 = \frac{\Delta_3}{\Delta}$$

Knowing K_1, K_2, and K_3, the values of a, b, c and d can be computed from the relations,

$$K_1 = \frac{d}{a}\ ;\ K_2 = -\frac{d}{c}\ ;\ K_3 = \frac{a^2-b^2+c^2+d^2}{2ac}$$

Value of either a or d can be assumed to be unity to get the proportionate values of other parameters.