FLUID MECHANICS JULY 2021

IAT-3 SCHEME AND SOLUTION

SOLUTION

1	State Pascal's law and derive an expression for the same	10	CO1	PO1	L2

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as :

The fluid element is of very small dimensions *i.e.*, *dx*, *dy* and *ds*.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest as shown in Fig. 2.1. Let the width of the element perpendicular to the plane of paper is unity and p_x, p_y and p_z are the pressures or intensity of pressure acting on the face *AB*, *AC* and *BC* respectively. Let $\angle ABC = \theta$. Then the forces acting on the element are :

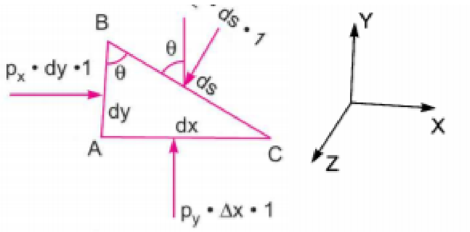


Fig. 2.1 *Forces on a fluid element.*

1. Pressure forces normal to the surfaces, and
2. Weight of element in the vertical direction.

The forces on the faces are :

Force on the face *AB* $= p_x \times$ Area of face *AB*

$= p_x \times dy \times 1$

Similarly force on the face $AC = p_y \times dx \times 1$

Force on the face *BC* $= p_z \times ds \times 1$

Weight of element = (Mass of element) $\times g$

$$= (\text{Volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g,$$

where ρ = density of fluid.

Resolving the forces in *x*-direction, we have

$$p_x \times dy \times 1 - p\,(ds \times 1)\sin(90° - \theta) = 0$$

or

$$p_x \times dy \times 1 - p_z\, ds \times 1 \cos\theta = 0.$$

But from Fig. 2.1, $ds \cos\theta = AB = dy$

$\therefore$

$$p_x \times dy \times 1 - p_z \times dy \times 1 = 0$$

or

$$p_x = p_z \qquad \text{...(2.1)}$$

Similarly, resolving the forces in *y*-direction, we get

$$p_y \times dx \times 1 - p_z \times ds \times 1 \cos(90° - \theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

or

$$p_y \times dx - p_z\, ds \sin\theta - \frac{dxdy}{2} \times \rho \times g = 0.$$

But $ds \sin\theta = dx$ and also the element is very small and hence weight is negligible.

$\therefore$

$$p_y dx - p_z \times dx = 0$$

or

$$p_y = p_z \qquad \text{...(2.2)}$$

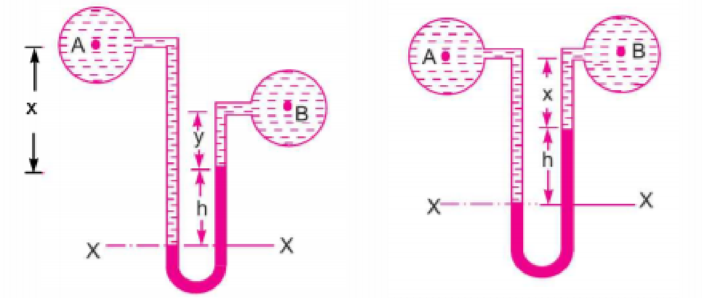
From equations (2.1) and (2.2), we have

$$p_x = p_y = p_z \qquad \text{...(2.3)}$$

The above equation shows that the pressure at any point in *x*, *y* and *z* directions is equal.

Since the choice of fluid element was completely arbitrary, which means the pressure at any point is the same in all directions.

2	Derive an expression to find the difference in pressure between 2 points using U-tube manometer	10	CO1	PO1	L2



(a)Two pipes at different levels (b) A and B are at the same level

Fig. 2.18 *U-tube differential manometers.*

In Fig. 2.18 (*a*), the two points *A* and *B* are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at *A* and *B* are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of *B*, from the mercury level in the right limb.

x = Distance of the centre of *A*, from the mercury level in the right limb.

ρ_1 = Density of liquid at *A*.

ρ_2 = Density of liquid at *B*.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at *X-X*.

Pressure above *X-X* in the left limb = $\rho_1 g(h + x) + p_A$

where p_A = pressure at *A*.

Pressure above *X-X* in the right limb = $\rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at *B*.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx \quad \text{...(2.12)}$$

$\therefore$ Difference of pressure at *A* and *B* = $h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

In Fig. 2.18 (*b*), the two points *A* and *B* are at the same level and contains the same liquid of density ρ_1. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \quad \text{...(2.13)}$$

3	The efficiency η of a fan depends on density ρ , dynamic viscosity μ, angular velocity ω, diameter D and discharge Q. Express η in terms of dimensionless parameters	10	CO5	PO1	L3

Solution.

Given : η is a function of ρ, μ, ω, D and Q

$$\therefore \quad \eta = f(\rho, \mu, \omega, D, Q) \quad \text{or} \quad f_1(\eta, \rho, \mu, \omega, D, Q) = 0 \quad \text{...}(i)$$

Hence total number of variables, $n = 6$.

The value of m, *i.e.*, number of fundamental dimensions for the problem is obtained by writing dimensions of each variable. Dimensions of each variable are

η = Dimensionless, $\rho = ML^{-3}$, $\mu = ML^{-1}T^{-1}$, $\omega = T^{-1}$, $D = L$ and $Q = L^3T^{-1}$

$\therefore \quad m = 3$

Number of π-terms $= n - m = 6 - 3 = 3$

Equation (*i*) is written as $f_1(\pi_1, \pi_2, \pi_3) = 0$ ...(*ii*)

Each π-term contains $m + 1$ variables, where m is equal to three and is also repeating variable.

Choosing D, ω and ρ as repeating variables, we have

$$\pi_1 = D^{a_1} . \omega^{b_1} . \rho^{c_1} . \eta$$
$$\pi_2 = D^{a_2} . \omega^{b_2} . \rho^{c_2} . \mu$$
$$\pi_3 = D^{a_3} . \omega^{b_3} . \rho^{c_3} . Q$$

First π-term $\quad \pi_1 = D^{a_1} . \omega^{b_1} . \rho^{c_1} . \eta$

Substituting dimensions on both sides of π_1,

$$M^0L^0T^0 = L^{a_1} . (T^{-1})^{b_1} . (ML^{-3})^{c_1} . M^0L^0T^0$$

Equating the powers of M, L, T on both sides

Power of M, $\quad 0 = c_1 + 0, \quad \therefore \quad c_1 = 0$

Power of L, $\quad 0 = a_1 + 0, \quad \therefore \quad a_1 = 0$

Power of T, $\quad 0 = -b_1 + 0, \quad \therefore \quad b_1 = 0$

Substituting the values of a_1, b_1 and c_1 in π_1, we get

$$\pi_1 = D^0\omega^0\rho^0 . \eta = \eta$$

[If a variable is dimensionless, it itself is a π-term. Here the variable η is a dimensionless and hence η is a π-term. As it exists in first π-term and hence $\pi_1 = \eta$. Then there is no need of equating the powers. Directly the value can be obtained.]

Second π-term $\quad \pi_2 = D^{a_2} . \omega^{b_2} . \rho^{c_2} . \mu$

Substituting the dimensions on both sides

$$M^0L^0T^0 = L^{a_2} . (T^{-1})^{b_2} . (ML^{-3})^{c_2} . ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides

Power of M, $\quad 0 = c_2 + 1, \quad \therefore \quad c_2 = -1$

Power of L, $\quad 0 = a_2 - 3c_2 - 1, \quad \therefore \quad a_2 = 3c_2 + 1 = -3 + 1 = -2$

Power of T, $\quad 0 = -b_2 - 1, \quad \therefore \quad b_2 = -1$

Substituting the values of a_2, b_2 and c_2 in π_2,

$$\pi_2 = D^{-2} . \omega^{-1} . \rho^{-1} . \mu = \frac{\mu}{D^2\omega\rho}$$

Third π-term $\quad \pi_3 = D^{a_3} . \omega^{b_3} . \rho^{c_3} . Q$

Substituting the dimensions on both sides

$$M^0L^0T^0 = L^{a_3} . (T^{-1})^{b_3} . (ML^{-3})^{c_3} . L^3T^{-1}$$

Equating the powers of M, L and T on both sides

Power of M, $\quad 0 = c_3, \quad \therefore \quad c_3 = 0$

Power of L, $\quad 0 = a_3 - 3c_3 + 3, \quad \therefore \quad a_3 = 3c_3 - 3 = -3$

Power of T, $\quad 0 = -b_{3-1}, \quad \therefore \quad b_3 = -1$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = D^{-3} . \omega^{-1} . \rho^0 . Q = \frac{Q}{D^3\omega}$$

Substituting the values of π_1, π_2 and π_3 in equation (*ii*)

$$f_1\left(\eta, \frac{\mu}{D^2\omega\rho}, \frac{Q}{D^2\omega}\right) = 0 \text{ or } \eta = \phi\left[\frac{\mu}{\mathbf{D^2\omega\rho}}, \frac{\mathbf{Q}}{\mathbf{D^2\omega}}\right]. \textbf{ Ans.}$$

4	A pressure guage consists of two cylindrical bulbs B and C each of 10 sq.cm cross section area, which are connected by a U-tube vertical limbs each of 0.25 sq.cm area. A red liquid of sp.gr 0.9 is filled into B, the surface of seperation being in the limb attached to C. Find the displacement of the surface of seperation when the pressure on the surface in C is greater than that in B by an amount equal to 1cm of water	10	CO1	PO1	L3

Solution. Given :

Area of each bulb B and C, $A = 10 \text{ cm}^2$

Area of each vertical limb, $a = 0.25 \text{ cm}^2$

Sp. gr. of red liquid $= 0.9$ $\therefore$ Its density $= 900 \text{ kg/m}^3$

Let $X\text{-}X$ = Initial separation level

h_C = Height of red liquid above $X\text{-}X$

h_B = Height of water above $X\text{-}X$

Pressure above $X\text{-}X$ in the left limb $= 1000 \times 9.81 \times h_B$

Pressure above $X\text{-}X$ in the right limb $= 900 \times 9.81 \times h_C$

Equating the two pressure, we get

$$1000 \times 9.81 \times h_B = 900 \times 9.81 \times h_C$$

$$\therefore \quad h_B = 0.9\, h_C \quad \text{...}(i)$$

When the pressure head over the surface in C is increased by 1 cm of water, let the separation level falls by an amount equal to Z. Then $Y\text{-}Y$ becomes the final separation level.

Now fall in surface level of C multiplied by cross-sectional area of bulb C must be equal to the fall in separation level multiplied by cross-sectional area of limb.

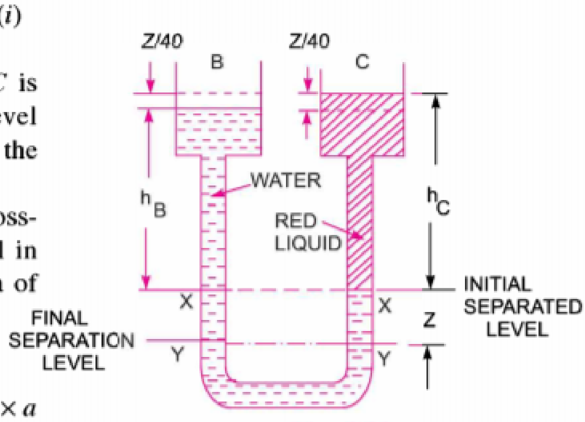


Fig. 2.14

$\therefore$ Fall in surface level of C

$$= \frac{\text{Fall in separation level} \times a}{A}$$

$$= \frac{Z \times a}{\text{A}} = \frac{Z \times 0.25}{10} = \frac{Z}{40}.$$

Also fall in surface level of C

$$= \text{Rise in surface level of } B$$

$$= \frac{Z}{40}$$

The pressure of 1 cm (or 0.01 m) of water $= \rho g h = 1000 \times 9.81 \times 0.01 = 98.1 \text{ N/m}^2$

Consider final separation level $Y\text{-}Y$

Pressure above $Y\text{-}Y$ in the left limb $= 1000 \times 9.81 \left(Z + h_B + \dfrac{Z}{40} \right)$

Pressure above $Y\text{-}Y$ in the right limb $= 900 \times 9.81 \left(Z + h_C - \dfrac{Z}{40} \right) + 98.1$

Equating the two pressure, we get

$$1000 \times 9.81 \left(Z + h_B + \frac{Z}{40} \right) = \left(Z + h_C - \frac{Z}{40} \right) 900 \times 9.81 + 98.1$$

Dividing by 9.81, we get

$$1000 \left(Z + h_B + \frac{Z}{40} \right) = 900 \left(Z + h_C - \frac{Z}{40} \right) + 10$$

Dividing by 1000, we get $Z + h_B + \dfrac{Z}{40} = 0.9 \left(Z + h_C - \dfrac{Z}{40} \right) + 0.01$

But from equation (i), $h_B = 0.9\, h_C$

$$\therefore \quad Z + 0.9\, h_C + \frac{Z}{40} = \frac{39Z}{40} \times 0.9 + 0.9\, h_C + 0.01$$

or $$\frac{41Z}{40} = \frac{39}{40} \times .9Z + .01$$

or $$Z\left(\frac{41}{40} - \frac{39 \times .9}{40}\right) = .01 \quad \text{or} \quad Z\left(\frac{41 - 35.1}{40}\right) = .01$$

∴ $$Z = \frac{40 \times 0.01}{5.9} = \mathbf{0.0678\ m = 6.78\ cm.\ Ans.}$$

5	The diameters of a small piston and a large piston of hydraulic jack are 3cm and 10cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when: a) the psitons are at same level b) small piston is 40 cm above the large piston The density of liquid in the jack is given as 1000 kg/m3	10	CO1	PO1	L3

Solution. Given :

Dia. of small piston, $d = 3$ cm

∴ Area of small piston, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (3)^2 = 7.068\ \text{cm}^2$

Dia. of large piston, $D = 10$ cm

∴ Area of larger piston, $A = \frac{p}{4} \times (10)^2 = 78.54\ \text{cm}^2$

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(*a*) When the pistons are at the same level

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068}\ \text{N/cm}^2$$

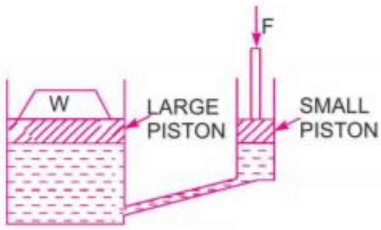


Fig. 2.5

This is transmitted equally on the large piston.

∴ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

∴ Force on the large piston = Pressure × Area

$$= \frac{80}{7.068} \times 78.54\ \text{N} = \mathbf{888.96\ N.\ Ans.}$$

(*b*) When the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068}\ \frac{\text{N}}{\text{cm}^2}$$

∴ Pressure intensity at section *A-A*

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

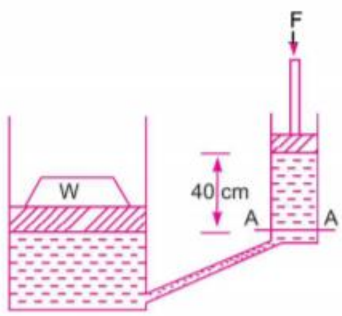


Fig. 2.6

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4\ \text{N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4}\ \text{N/cm}^2 = 0.3924\ \text{N/cm}^2$$

∴ Pressure intensity at section *A-A*

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71\ \text{N/cm}^2$$

∴ Pressure intensity transmitted to the large piston = 11.71 N/cm^2

∴ Force on the large piston = Pressure × Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7\ \text{N.}$$

SCHEME OF EVALUATION

Question Number	*Max Marks*	*Split-Up*	*Marks Distribution*
1	10	Diagram	2
		Derivation	8
2	10	Diagram	2
		Derivation	8
3	10	Solution	10
4	10	Steps	2
		Answer	8
5	10	Steps	2
		Answer	8