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CMR Institute of Technology, Bangalore
DEPARTMENT OF MASTER OF BUSINESS ADMINISTRATION
I - INTERNAL ASSESSMENT

Semester: 2-CBCS 2020

Date: 11 Jun 2021

Subject: OPERATIONS RESEARCH (20MBA24)

Faculty: Ms Namita P Konnur

Time: 02:00 PM - 03:30 PM

Max Marks: 50

PART HYPHEN A

ANSWER ANY 2 Question(s)

Marks CO BT/CL

1a. Define Operations Research

the application of scientific and especially mathematical methods to the study and analysis of problems involving complex systems.

[3.0] 1 [1]

1b. A firm manufactures two products A and b on which the profits earned per unit are Rs 3 and Rs 4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2 while B requires one minute on M1 and one minute on M2. Machine M1 is available for not more than 7 hours, while machine M2 is available for 10 hours during any working day. Formulate the number of units of products A and B to be manufactured to get maximum profit

Formulation of LPP:

$$Z_{\max} = 3x_1 + 4x_2$$

subject to constraints

$$x_1 + x_2 \leq 420$$
$$2x_1 + x_2 \leq 600$$

&

$$x_1, x_2 \geq 0.$$

[7.0] 1 [3]

1c. Explain the models in OR in detail

Model: A model is defined as a representation of a system for the purpose of studying the system. It is necessary to consider only those aspects of a system that affect the plan under investigation for studying them.

Types: Mathematical Model: It uses symbolic notation and mathematical equation to represent a system. It represents a system in terms of logical and quantitative relationships.

Example: Area of circle = πr^2

Physical Model: It is based on some analogy between systems such as mechanical and electrical, electrical and hydraulic systems attributes are represented by measurements such as voltage or the position of a motor shaft. E.g. Scale models, prototype plants.

Static model: It sometimes called as Monte Carlo simulation model, that represents a system at a particular point in time. E.g. Map, photo

Dynamic model: It represents systems as they change over time e.g. simulation of cafeteria from 2 pm to 4 pm.

Deterministic model: It is one which contains no random variables. it has a known set of i/p's which will result in a unique set of o/p's. e.g. Arrival of patient as per appointment time.

Stochastic model: It is one which contains one or more random variables as i/p's. Random o/p's are generated which can be considered only as estimates of the true characteristics of a model. e.g. Simulation of a bank

Discrete model: It is one in which the state variables change only at a discrete set of points in time. e.g. no. of customers waiting in line in bank.

Continuous model: It is one in which the state variables change continuous over time e.g. Head of water behind the dam.

[10.0] 1 [2]

2a. Let us assume that you have inherited Rs. 1, 00,000 from your father-in-law that can be invested in a combination of only two stock portfolios, with the maximum investment allowed in either portfolio at Rs. 75,000. The first portfolio has an average rate of return of 10%, whereas the second has 20%. In terms of risk factors associated with these portfolios, the first has a risk rating 4 (on a scale from 1 to 10), and the second has 9. Since you wish to maximize your return, you will not accept an average rate of return below 12% or a risk factor above 6. Hence, you then face the important question. How much should you invest in each portfolio?

[10.0] 1 [4]

2b. Solve graphically the following linear programming problem.

Minimize	$Z=3x_1 + 5x_2$
Subject to	$-3x_1 + 4x_2 \leq 12.$

$2x_1 + 3x_2 \geq 12$
$2x_1 - x_2 \geq -2.$
$x_1 \leq 4$
$x_2 \geq 2.$
$x_1, x_2 \geq 0.$

[10.0] 1 [3]

3a. Define LPP in the mathematical form

LPP in mathematical form:

A linear programming model seeks to maximize or minimize a linear function, subject to a set of linear constraints.

It consists of the following components:

- a set of decision variables
- an objective function
- a set of constraints.

Mathematical form of LPP:

Max/Min $Z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

subject to

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n (\leq, =, \geq) b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n (\leq, =, \geq) b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n (\leq, =, \geq) b_m \end{cases}$$

x_i = decision variables
 b_i = constraint levels
 c_i = objective function co-effs
 a_{ij} = constraint co-effs

[3.0] 1 [1]

3b. A farmer has a 100-acre farm. He can sell all tomatoes, carrots, and Radish and can raise the price to obtain Rs. 1 kg for tomatoes, Rs. 0.75 for carrot and Rs. 2 per kg for Radish. The average yield per acre is 2000 kg of tomato, 3,000 kg of carrot, and 1000 kg of radish. Fertilizers are available at Rs. 0.50 per kg and the amount required per acre is 100 kg for each tomatoes and carrot and 50 kg for radish. Labour required for harvesting per acre is 5 man days for tomatoes and radish and 6 man days for carrot. A total of 400 man days of labour are available at Rs. 20 per day. Formulate LPP in order to maximize the profit.

Total sales = price \times quantity

tomato $\rightarrow 1 \times 2000 = 2000x_1$

carrot $\rightarrow 0.75 \times 3000 = 2250x_2$

radish $\rightarrow 2 \times 1000 = 2000x_3$

	Tomato	Carrot	Radish
Sales	$2000x_1$	$2250x_2$	$2000x_3$
Fertilizer (0.5/kg)	100×0.5 $50x_1$	100×0.5 $50x_2$	50×0.5 $25x_3$
Labour 20/day	5×20 $100x_1$	6×20 $120x_2$	5×20 $100x_3$
Profit	$1850x_1$	$2080x_2$	$1875x_3$

Step 2: determining objective fun

$$Z_{max} = 1850x_1 + 2080x_2 + 1875x_3$$

Step 3: Constraints

$$x_1 + x_2 + x_3 \leq 100$$

$$5x_1 + 6x_2 + 5x_3 \leq 400$$

Step 4: Identifying non -ve restriction

$$x_1, x_2 \geq 0$$

Formulation of LPP:

$$Z_{max} = 1850x_1 + 2080x_2 + 1875x_3$$

subject to

$$x_1 + x_2 + x_3 \leq 100$$

$$5x_1 + 6x_2 + 5x_3 \leq 400$$

$$\& x_1, x_2 \geq 0$$

[7.0] 1 [5]

3c. Solve the LPP (Graphically)

Minimise	$Z=6x+8y$
Subject to constraints:	$2x+3y \geq 16$ $x+2y \geq 16$ $x \leq 8$, $y \geq 0$

3c) $Z_{min} = 6x + 8y$

Step 1: Convert inequalities to equations.

$C_1 \Rightarrow 2x + 3y = 16$

$C_2 \Rightarrow 4x + 2y = 16$

$C_3 \Rightarrow y = 8$

Step 2: determine points to plot

$C_1: 2x + 3y = 16$

$x=0 \quad y=5.33 \quad (0, 5.33)$

$y=0 \quad x=8 \quad (8, 0)$

$C_2 \Rightarrow 4x + 2y = 16$

$x=0 \quad y=8 \quad (0, 8)$

$y=0 \quad x=4 \quad (4, 0)$

$C_3 \Rightarrow y=8 \quad (0, 8)$

Step 3: find feasible region

Step 4:

$Z_{min} = 6x + 8y$

$A(0, 8)$

$6(0) + 8(8)$

64

$B(2, 4)$

$6(2) + 8(4)$

44

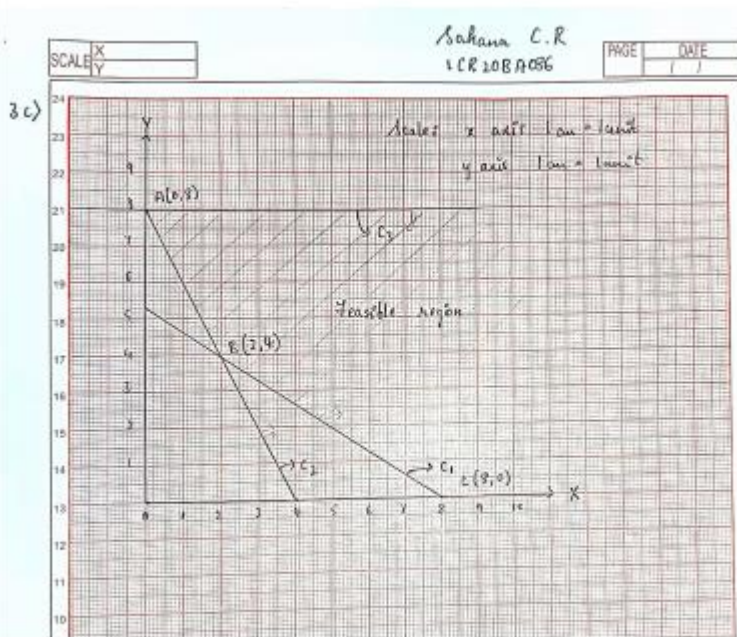
$C(8, 0)$

$6(8) + 8(0)$

48

\therefore The minimum feasible soln exists at $B(2, 4)$ at $Z_{min} = 44$.

[10.0] 1



[4]

PART HYPHEN B
ANSWER ALL QUESTIONS

Marks CO BT/CL

4. A company purchasing scrap material has two types of scrap materials available. The first type has 30% of material X, 20% of material Y, and 50% of material Z by weight. The second type has 40% of material X, 10% of material Y, and 30% of material Z. The cost of

the two scraps is Rs. 120 and Rs. 160 per Kg. respectively. The company requires at least 240 Kg of material X, 100 Kg of material Y and 290 Kg of material Z. Find the optimum quantities of the two scraps to be purchased so that the company requirements of the three materials are satisfied at a minimum cost.

Graphically solve the LPP

	120 Rs Scrap I	160 Rs Scrap II	availability/requirement
X	30%	40%	240
Y	20%	10%	100
Z	50%	30%	290

Step 1: Identification of decision variables
Scrap type I - x_1
Scrap type II - x_2

Step 2: Identification of objective function.
 $Z_{min} = 120x_1 + 160x_2$

Step 3: Identification of constraints
 $0.3x_1 + 0.4x_2 \geq 240$
 $0.2x_1 + 0.1x_2 \geq 100$
 $0.5x_1 + 0.3x_2 \geq 290$

Step 4: Identification of non-neg. restrictions
 $x_1, x_2 \geq 0$

Normal form of LPP:
 $Z_{min} = 120x_1 + 160x_2$
Subject to constraints
 $0.3x_1 + 0.4x_2 \geq 240$
 $0.2x_1 + 0.1x_2 \geq 100$
 $0.5x_1 + 0.3x_2 \geq 290$
 $x_1, x_2 \geq 0$

Step 1: Convert the inequalities of constraints to equalities.

$$C_1 \Rightarrow 0.3x_1 + 0.4x_2 = 240$$

$$C_2 \Rightarrow 0.2x_1 + 0.1x_2 = 100$$

$$C_3 \Rightarrow 0.5x_1 + 0.3x_2 = 290$$

Step 2: Determine the points to plot graph.

$$C_1: 0.3x_1 + 0.4x_2 = 240$$

$$x_1 = 0, x_2 = 600 (0, 600)$$

$$x_2 = 0, x_1 = 800 (800, 0)$$

$$C_2: 0.2x_1 + 0.1x_2 = 100$$

$$x_1 = 0, x_2 = 1000 (0, 1000)$$

$$x_2 = 0, x_1 = 500 (500, 0)$$

$$C_3: 0.5x_1 + 0.3x_2 = 290$$

$$x_1 = 0, x_2 = 966.66 (0, 966.66)$$

$$x_2 = 0, x_1 = 580 (580, 0)$$

Step 3: Identify the feasible region

< towards origin

> away from origin

Step 4: A(0, 1000)

B(100, 300)

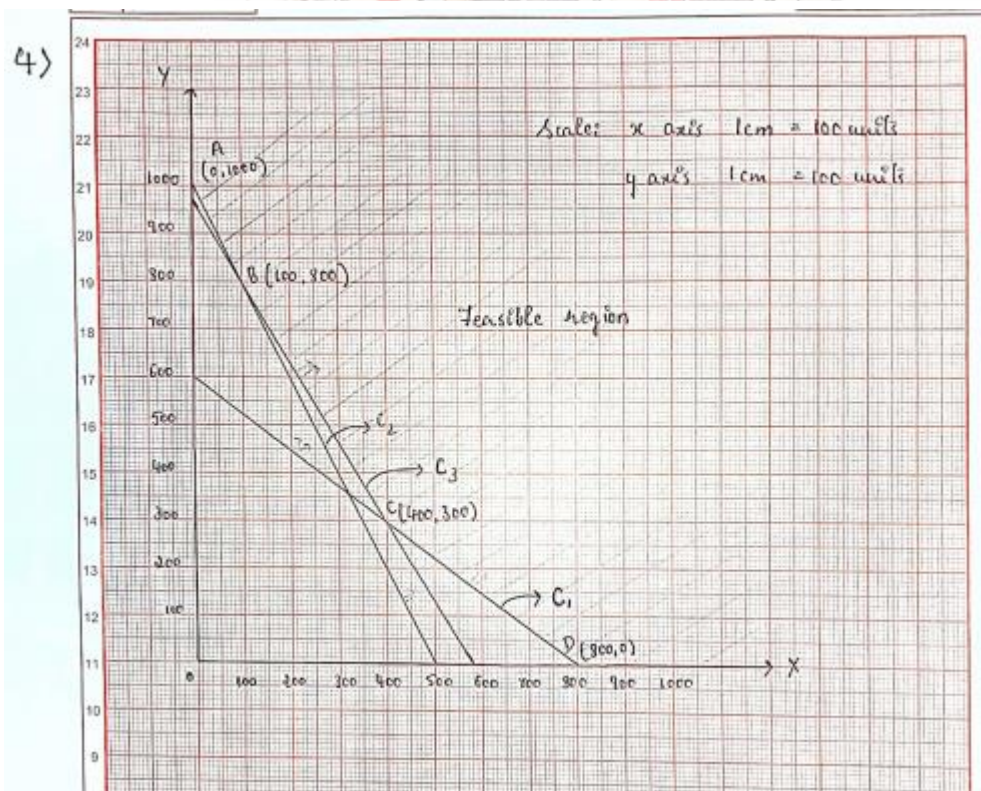
C(400, 300)

D(800, 0)

$$Z_{\min} = 120x_1 + 160x_2$$

A(0, 1000)	$120(0) + 160(1000)$	1,60,000
B(100, 800)	$120(100) + 160(800)$	1,40,000
C(400, 300)	$120(400) + 160(300)$	<u>96,000</u>
D(800, 0)	$120(800) + 160(0)$	<u>96,000</u>

∴ The minimum solution exists at 2 points C(400, 300) & D(800, 0) where $Z_{\min} = 96,000$



[10.0] 1 [5]