CBCS SCHEME

USN [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] **15CS36**

Third Semester B.E. Degree Examination, July/August 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions.***

1 a. Prove that for any propositions p, q, r the compound proposition $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a Tautology. **(05 Marks)**

b. Consider the following open statements with set of all real numbers as the universe. $p(x): x \geq 0$, $q(x): x^2 \geq 0$, $r(x): x^2 - 3x - 4 = 0$.
Determine the truth values of the following statements.
(i) $\exists x, p(x) \wedge q(x)$
(ii) $\forall x, p(x) \to q(x)$
(iii) $\exists x, p(x) \wedge r(x)$ **(05 Marks)**

c. Write down the negation of each of the following statements:
(i) For all integers n, if n is not divisible by 2, then n is odd.
(ii) If k, m, n are any integers where (k-m) and (m-n) are odd, then (k-n) is even. **(06 Marks)**

2 a. Prove the following logical equivalences without using truth tables:
(i) $[(\sim p \vee \sim q) \to (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$.
(ii) $\sim [\sim \{(p \vee q) \wedge r\} \vee \sim q] \Leftrightarrow q \wedge r$ **(05 Marks)**

b. Test the validity of the following argument.
If Gopi goes out with friends, he will not study.
If Gopi does not study, his father becomes angry.
His father is not angry.
∴ Gopi has not gone out with friends. **(05 Marks)**

c. Give (i) a direct proof (ii) an indirect proof and (iii) Proof by contradiction for the following statement.
"If n is an odd integer, then n+9 is an even integer". **(06 Marks)**

3 a. Using mathematical induction, prove that for each $n \in z^+$,
$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{1}{6} n(n+1)(2n+1)$. **(05 Marks)**

b. A bit is either 0 or 1. A byte is a sequence of 8 bits. Find :
(i) the number of bytes,
(ii) the number of bytes that being with 11 and end with 11
(iii) the number of bytes that begin with 11 and do not end with 11
(iv) number of bytes that begin with 11 end with 11. **(06 Marks)**

c. Find the co-efficient of $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$. **(05 Marks)**

4 a. Suppose U is a universal set and
$A, B_1, B_2 --- B_n \subseteq U$, prove that
$A \cap (B_1 \cup B_2 \cup \ldots\ldots \cup B_n) = (A \cap B_1) \cup (A \cap B_2) \ldots\ldots \cup (A \cap B_n)$. **(05 Marks)**

b. How many positive integers n can be formed using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? **(06 Marks)**

c. A total amount of Rs.1500 is to be distributed to 3 students A, B, C of a class. In how many ways the distribution can be made in multiples of Rs.100.
(i) If everyone of these must get atleast Rs.300?
(ii) If A must get atleast Rs.500, and B and C must get atleast Rs.400 each? **(05 Marks)**

5 a. Show that if any n+1 numbers from 1 to 2n are chosen, then 2 of them will have their sum equal to 2n+1. **(05 Marks)**

b. If $f : A \to B$ and $g : B \to C$ are invertible functions then prove that $gof : A \to C$ is an invertible function and $(gof)^{-1} : f^{-1} o\, g^{-1}$. **(06 Marks)**

c. On the set of all positive divisors of $36 - D'_{36}$ a relation R is defined by aRb if and only if "a divides b". Prove that R is a partial order and draw its Hasse diagram. **(05 Marks)**

6 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if a is a multiple of b. Represent the relation R as a matrix and draw its digraph. **(05 Marks)**

b. Let $f : A \to B$, $g : B \to C$ be 2 functions then prove that

(i) If f and g are one-to-one so is gof.
(ii) If gof is one-to-one then f is one-to-one.
(iii) If f and g are onto, so is gof. **(06 Marks)**

c. On the set Z of all integers, a relation R is defined by aRb if and only if $a^2 = b^2$. Verify that R is an equivalence relation. Determine the partition induced by this relation. **(05 Marks)**

7 a. There are eight letters to eight different people to be placed in eight different addressed envelopes. Find the number of ways of doing this so that atleast one letter gets to the right person. **(05 Marks)**

b. In how many ways can the 26 letters of English alphabet be permuted so that none of the patterns CAR, DOG, PUN or BYTE occurs? **(06 Marks)**

c. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$, given that $a_0 = 2$. **(05 Marks)**

8 a. Find the number of derangements of a, b, c, d. **(05 Marks)**

b. Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party find that only chair at each of the five tables T_1, T_2, T_3, T_4 and T_5 is vacant. P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chairs. **(06 Marks)**

c. Solve the recurrence relation $a_{n+2}^2 - 5a_{n+1}^2 + 4a_n^2 = 0$ for $n \geq 0$, given $a_0 = 4$ and $a_1 = 13$. **(05 Marks)**

9 a. Prove that in every graph, the number of odd degree vertices is even. **(05 Marks)**

b. Discuss the Konigsberg bridge problem. **(06 Marks)**

c. Construct an optimal prefix code for the letters of the word "SWACH BHARAT": Indicate the code. **(05 Marks)**

10 a. Verify whether the following graphs are isomorphic: **(05 Marks)**

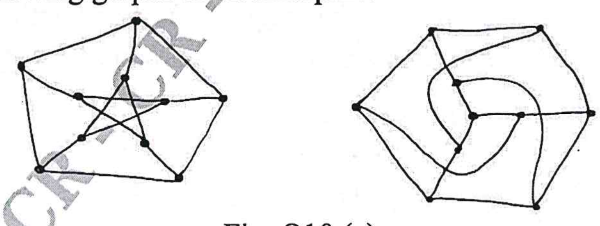

Fig. Q10 (a)

b. Prove that a tree with n vertices has exactly n-1 edges. **(05 Marks)**

c. Define the following:

(i) Regular graph (ii) Complete graph. (iii) Bipartite graph.
(iv) Path (v) Cycle. **(06 Marks)**

* * * * *