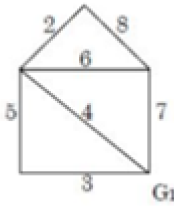
US N										

CELEBRATING 25 YEARS CMRIT CMR INSTITUTE OF TECHNOLOGY, BENGALURU. ACCREDITED WITH A+ GRADE BY NAAC

Solution of Internal Assessment Test 2 – Jun. 2021

Sub:	Design & Analysis of Algorithms				Sub Code:	18CS42	Branch:	CSE	
Date:	23/06/2021	Duration:	60 min's	Max Marks: 50	Sem/Sec:	4/A, B, C & D		OBE	
							MARKS	CO	RBT

	Section I MCQ **10 X 1 = 10**			
1	**The reason why binary search is O(lg n) while Quick Sort and Merge Sort are O(n lg n) is because:** **(a) Binary search only recurses on half the array** (b) Quick sort and Merge sort recurse on the full array (c) Binary search does not use recursion (d) Sorting techniques use recursion but not searching techniques	1	CO1	L2
2	**If the pivot in Quick Sort is always taken as the least value of the array, then the complexity of Quick Sort would be the same as:** (a) Merge sort (b) Heap sort (c) Binary search **(d) Selection sort**	1	CO1	L2
3	**The number of recursive calls made is always the same for:** (a) Quick sort **(b) Merge sort** (c) Heap sort (d) Binary search	2	CO1	L2
4	**Strassen's matrix multiplication cannot be applied to matrices of order ________:** (a) 4 X 4 **(b) 6 X 6** (c) 8 X 8 (d) 16 X 16	1	CO1	L2
5	**If there are no cross edges in a DFS forest, then the number of trees in the forest are:** **(a) 0** (b) 1 (c) 2 (d) Infinite	2	CO3	L2

6	**When solving fractional knapsack using the greedy algorithm, if the weights of items are {5, 10, 15} and values are {25, 50, 75}, then the first item to be selected is the item with weight:** (a) 5 (b) 10 (c) 15 **(d) any**	1	CO3	L3
7	The weight of minimum spanning tree in graph G is:  Graph G (a) 12 (b) 14 (c) 16 **(d) 15**	2	CO3	L3
8	**One of the following sets of codes can be generated by the Huffman Coding algorithm:** (a) { 0, 11, 10, 010 } (b) { 01, 000, 010 } (c) { 00, 01, 111 } **(d) { 0, 10, 11 }**	1	CO3	L2
9	**If five jobs have the same profit of 10, but the deadlines are {1, 1, 1, 2, 3}, then the maximum profit that can be obtained by scheduling them is:** (a) 5 (b) 10 (c) 15 **(d) 30**	1	CO3	L3
10	**Consider the values** { 10, 5, 15, 30, 25, 40, 55, 50, 35, 45 } **to be sorted using Heap Sort. After the largest element among the values is added to the result, the number of swaps required to reheap the remaining values is:** (a) 2 **(b) 3** (c) 4 (d) 5	1	CO1	L3

Section II Short Answer any 5 Questions **5 X 5 = 25**

<table>
<tr>
<td>1</td>
<td>

Find the solution to the following recurrence equations using the Master's theorem, laying out the detailed steps. [5M]

a) T (n) = 2T (n/2) + n log n……………………..[2.5 M]

b) T (n) = 2^nT (n/2) + n^n………………...………..[2.5 M]

Solution:

(a) $T(n) = 2T(n/2) + n\log n$

$\theta(n\log n)$

$a = 2 \quad b = 2$

$\log_b a = 1 \qquad n^k = n \Rightarrow k = 1$

$\therefore \log_b a = K \longrightarrow case(2)$

here $p = 1 \Rightarrow p > -1$

$\theta(n^k \log^{p+1} n) = \theta(n \log^2 n)$

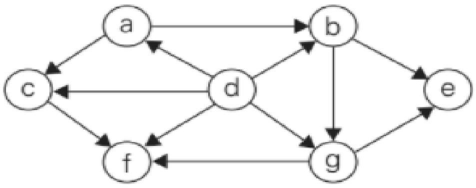
b) No Solution since "a" is not constant

</td>
<td>2</td>
<td>CO2</td>
<td>L3</td>
</tr>
</table>

2	**Sort the following elements using Quick Sort with the average of the elements as the pivot:** {9, 7, 5, 11, 12, 2, 14, 3, 10, 6}. **Give details of the steps. [5M]** **Solved by taking mid element as pivot with complete steps……………..[5M]** **Solution:** 9 7 5 11 12 2 14 3 10 6 i i i i P j Here 12 is pivot left elements are lesser than pivot 9 7 5 6 12 P 9 7 5 6 12 2 14 3 10 11 i i i j swap 9 7 5 6 12 2 11 3 10 14 i i j i swap 9 7 5 6 10 2 11 3 12 14 i i i i Pi i j j swap 9 7 5 6 10 2 3 11 12 14 j i swap 9 7 5 6 3 2 [10 11 12 14 i P j 2 7 5 6 3 9 [10 11 12 14 i i P j swap 2 3 5 6 7 9 [10 11 12 14] i i Pi j j swap 2 3 5 6 7 9 10 11 12 14 i P i 2 3 5 6 7 9 10 11 12 14	1	CO1	L3

3

Apply the DFS-based algorithm to solve the topological sorting problem for the following digraph:



(a)

Solving with complete steps…………………[5M]

Solution:

let source vertex be a.

	visited	popped
a → b → e	a, b, e.	
pop e		e
a → b → g → f	a, b, e, g, f	
pop f	"	f
a → b → g		
pop g	"	g
a → b		b
pop b		
a → c	a, b, e, g, f, c	
pop c		c
a	"	
pop a		a
d	a, b, e, g, f, c, d	
pop d		d

The topological sorting is (by reversing the popped list)

d a c b g f e.

1 | CO1 | L3

4	**Apply greedy method to obtain an optimal solution to the knapsack problem given the capacity of the knapsack M = 60, Weights { w1, w2, w3, w4, w5 } = {5,10,20,30,40}, and Values {v1, v2, v3, v4, v5} = { 30, 20, 100, 90, 160 }. Find the total profit earned.**	;2	CO3	L3

Solving using fractional method …………….[5M]

Solution:

Knapsack capacity $M = 60$

Weights $\{w_1, w_2, w_3, w_4, w_5\} = \{5, 10, 20, 30, 40\}$

Values $\{v_1, v_2, v_3, v_4, v_5\} = \{30, 20, 100, 90, 160\}$

Values	Weights	v_i/w_i → profit/weight ratio
30	5	6
20	10	2
100	20	5
90	30	3
160	40	4

arranging in increasing order of v_i/w_i ratio

	6	5	4	3	2	
w_i →	5	20	40	30	10	
v_i →	30	100	160	90	20	
	P_1	P_2	P_3	P_4	P_5	→ process/job ID's.

$w[1] < M$

profit = 30

$M = M - W[1] = 60 - 5 = 55$

$w[2] < M$

Profit = 30 + 100

$M = 55 - 20 = 35$

$w[3] \not< M$ so we take fractional value

$$\frac{M}{W[3]} = \frac{35}{40} = \frac{7}{8}$$

$$\text{profit} = 30 + 100 + \frac{7}{\cancel{8}} \times \overset{20}{\cancel{160}}$$

$$= 30 + 100 + 140 = \underline{\underline{270}}$$

$$M = 35 - \frac{7}{8} \times 40 = 0$$

The knapsack is full with maximum profit of $\underline{\underline{270}}$.

5	**Find the solution generated by the "**Job Sequencing With Deadlines**" problem for 7 jobs with profits: 3, 5, 20, 18, 1, 6, 30 and deadlines: 1, 3, 4, 3, 2, 1, 2 respectively.** Solving problem with complete steps…………[5M] Solution: Jobs:- J_1 J_2 J_3 J_4 J_5 J_6 J_7 Profit:- 3 5 20 18 1 6 30 deadline:- 1 3 4 3 2 1 2 Initially $J = \phi$ & $\sum_{i \in J} P_i = 0$ 1. Select b/w J_6 & J_1 for 1st deadline. J_6 profit > J_1 $J = \{J_6\}$ $\sum P_i = 6$ 2. Select b/w J_7 & J_5 for 2nd deadline J_7 profit > J_5 $J = \{J_6, J_7\}$ $\sum P_i = 36$  3. Select b/w J_4 & J_2 for 3rd deadline J_4 profit > J_2 $J = \{J_6, J_7, J_4\}$ $\sum P_i = 54$ 4. For last deadline we have only Job J_3. select J_3 $J = \{J_6, J_7, J_4, J_3\}$ $\sum P_i = 74$ optimal solution = $\{J_6, J_7, J_4, J_3\}$ profit = 74	5	CO3	L3
6	**Suppose that each of the following English letters are known to appear in the messages of a certain organization with the probability indicated:**	5	CO3	L3

U - 10%, **V -** 25%, **X -** 10%, **Y -** 40%, **Z -** 15%,

Find a Huffman coding for this set of words. Show your work.

Generating code for each character………………[5M]

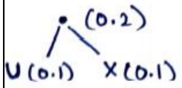
Solution:

U → 0.1 (10%) V → 0.25 (25%) X → 0.1 (10%) Y → 0.4 (40%) Z → 0.15 (15%)

Arrange in increasing probability order

• U (0.1) • X (0.1) • Z (0.15) • V (0.25) • Y (0.4)

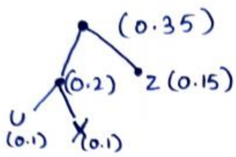
Add to tree the least valued nodes. & rearrange and add.



• Z (0.15) • V (0.25) • Y (0.4)

Add to tree the least valued nodes. & rearrange and add.

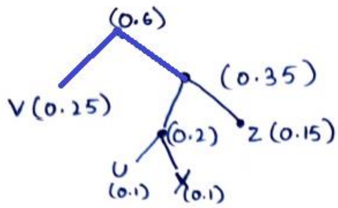
• V (0.25)

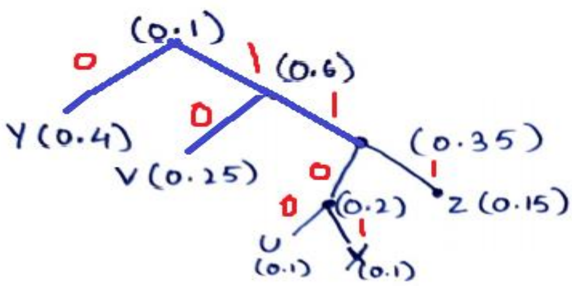


• Y (0.4)

Add to tree the least valued nodes. & rearrange and add.

• Y (0.4)



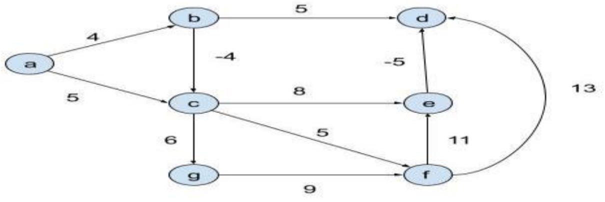


Y=0, V=10, U=1100, X=1101, Z=111

Section III Long Answer Questions 1 X 15= 15

1

Trace Dijkstra's algorithm for finding shortest paths on the graph below and explain how it fails to print the values correctly.



Finding shortest path using algorithm with complete steps……….[15M]

Solution:

We start with vertex 'a' for this problem because there are no incoming edges.

[Bold indicates that we are **setting** the distance]

Dijkstra's algorithm:

Select a starting vertex and set its final distance as 0.

Repeat

select the closest vertex to the starting vertex (say v) and set its distance, if the distance is not already set.

update the value of each vertex if its distance from v is less.

Until there is no vertex left

a	b	c	d	e	f	g
0	4	5	-	-	-	-
0	**4**	-4	5	-	-	-
0	**4**	**-4**	5	8	5	6
0	**4**	**-4**	**5**	8	5	6
0	**4**	**-4**	**5**	8	**5**	6
0	**4**	**-4**	**5**	8	**5**	**6**
0	**4**	**-4**	**5**	**8**	**5**	**6**

We can see that we could have reached vertex 'd' in -5 but in the table it is set as 5 and cannot be changed after it is set. It can be seen that this happens a lot throughout the problem. That is why it fails to print the distances correctly.

15 CO3 L3

2	**1. For each of the following two statements, decide whether it is true or false. If it is true, give a short explanation. If it is false, give a counterexample.** (a) Suppose we are given an instance of the Minimum Spanning Tree Problem on a graph G, with edge costs that are all positive and distinct. Let T be a minimum spanning tree for this instance. Now suppose we replace each edge cost c_e by its square, c_e^2, thereby creating a new instance of the problem with the same graph but different costs. True or false? T must still be a minimum spanning tree for this new instance……………………..[2.5M] (b) Suppose we are given an instance of the shortest s-t Path Problem on a directed graph G. We assume that all edge costs are positive and distinct. Let P be a minimum-cost s-t path for this instance. Now suppose we replace each edge cost c_e by its square, c_e^2, thereby creating a new instance of the problem with the same graph but different costs. True or false? P must still be a minimum-cost s-t path for this new instance…………………….[2.5M] Solution: a. is **True** because, for any two edges, if c_1 is less than c_2 , then c^2_1 must be less than c^2_2 as well. Since Kruskal's algorithm orders edges by their weights, their ordering remains unaltered when the edge weights are squared. b. is **False** because, for any two edges, if c_1 is less than c_2 + c_3 it is not necessary that c^2_1 must be less than c^2_2 + c^2_3. E.g. 5 is less than 2 + 4, but 25 is not less than 2^2 + 4^2	15	CO2	L4