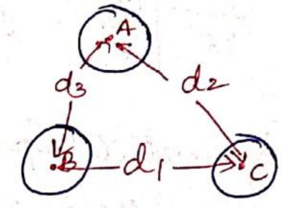
1. With neat diagram, obtain an expression for inductance of a 3 phase overhead line with unsymmetrical spacing

Induct[illegible] line:

Three conductors A, B & C of a 3 φ line carrying currents I_A, I_B & I_C respectively as shown in figure. let d_1, d_2 & d_3 be the spacings between the conductors as shown. let us assume that the loads are balanced

i.e $I_A + I_B + I_C = 0$.



Consider the flux linkages with conductor A. There will be flux linkages with conductor A due to its own current & also due to mutual inductance effects of I_B & I_C,

Flux linkages with conductor A due to its own current,

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right] \quad -(1)$$

Flux linkages with conductor A due to current I_B,

$$= \frac{\mu_0 I_B}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x} \quad -(2)$$

Flux linkages with conductor A due to current I_C,

$$= \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x} \quad -(3)$$

Total flux linkages with conductor A, (adding the 3eq)

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right] + \frac{\mu_0 I_B}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x} + \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right) I_A + I_B\int_{d_3}^{\infty}\frac{dx}{x} + I_C\int_{d_2}^{\infty}\frac{dx}{x}\right]$$

(2) unsymmetrical spacing :-

when Three phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. under such conditions, the flux linkages & inductance of each phase are not the same.

A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced. Hence the vlg recieved at the end will not be the same for all phases.

The vlg drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that the conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as Transposition

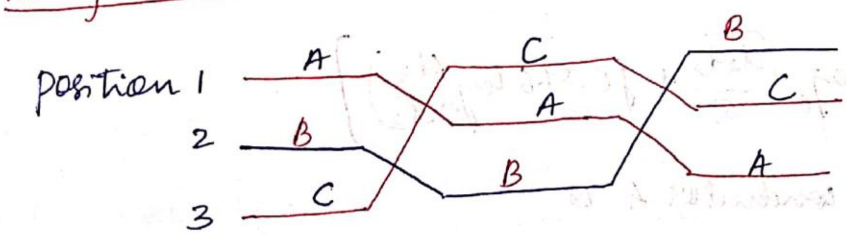


Fig shows the 3-ϕ transposed line having unsymmetrical spacing. let us assume that each line of the three sections is 1m in length.

Assume the balanced conditions i.e $I_A + I_B + I_C = 0$.

$$I_A = 1(1 + j0)$$

$$I_B = 1(-0.5 - j0.866)$$

$$I_C = 1(-0.5 + j0.866)$$

The total flux linkages / meter of conductor A is.

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

Substituting the values of I_A, I_B & I_C in the above eq'n,

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I - I(-0.5 - j0.866)\log_e d_3 - I(-0.5 + j0.866)\log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - \log_e r\, I + 0.5 I \log_e d_3 + j0.866 I \log_e d_3 + I\,0.5 - j0.866 \log_e\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + 0.5 I\left(\log_e d_3 + \log_e d_2\right) + j0.866 I\left(\log_e d_3 - \log_e d_2\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I - I\log_e r + I \log_e (d_3 d_2)^{1/2} + j0.866 I\left(\log \frac{d_3}{d_2}\right)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I + I\left[\log_e \frac{\sqrt{d_3 d_2}}{r}\right] + j0.866 I \log_e\left(\frac{d_3}{d_2}\right)\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_3 d_2}}{r} + j0.866 \log_e\left(\frac{d_3}{d_2}\right)\right]$$

Inductance of conductor A is,

$$L_A = \frac{\psi_A}{I_A} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j0.866 \log_e\left(\frac{d_3}{d_2}\right)\right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j0.866 \log_e \frac{d_3}{d_2}\right] \text{H/m}$$

$$L_A = 10^{-7}\left[\frac{1}{2} + 2\log \frac{\sqrt{d_2 d_3}}{r} + j1.732 \log_e \frac{d_3}{d_2}\right] \text{H/m}$$

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log \frac{\sqrt{d_3 d_1}}{r} + j1.732 \log_e \frac{d_1}{d_3}\right] \text{H/m}$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_1 d_2}}{r} + j1.732\log_e\frac{d_2}{d_1}\right] H/m.$$

Inductance of each line conductor,

$$= \frac{1}{3}(L_A + L_B + L_C)$$

$$= \frac{10^{-7}}{3}\left\{\left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_2 d_3}}{r} + j1.732\log_e\frac{d_3}{d_2}\right] + \left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_3 d_1}}{r} + j1.732\log_e\frac{d_1}{d_3}\right] + \left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_1 d_2}}{r} + j1.732\log_e\frac{d_2}{d_1}\right]\right\}$$

$$= \frac{10^{-7}}{3}\left[\frac{3}{2} + 2\log_e\frac{d_3 d_2 d_1}{r} + j1.732\left(\log_e d_3 - \log_e d_2 + \log_e d_1 - \log_e d_3 + \log_e d_2 - \log_e d_1\right)\right]$$

$$= \frac{10^{-7}}{3}\left[\frac{3}{2} + 2\log_e\frac{d_3 d_2 d_1}{r}\right]$$

$$= \left[\frac{1}{2} + \frac{1}{3}\,2\log_e\frac{d_3 d_2 d_1}{r}\right]\times 10^{-7}\ H/m.$$

$$= \left[\frac{1}{2} + 2\log_e\sqrt[3]{\frac{d_3 d_2 d_1}{r}}\right]\times 10^{-7}\ H/m.$$

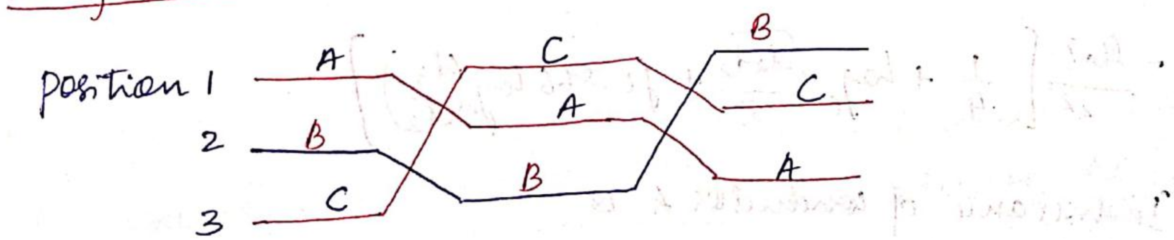
2. A 3-phase 50Hz line consists of three conductors each of diameter 21mm. the spacing between the conductors is as follows. AB=2.5m, BC= 4.5m, CA = 3.5m Find the capacitance and capacitive reactance per phase per km of the line. The line is transposed at regular intervals

3. a.Calculate inductance of each conductor in a 3-phase, 3 wire system. Conductors are arranged in a horizontal plane with spacing d31=8m, d12=d23=4m. the conductors are transposed and have a diameter of 2cm

3.b.

end will not

The vlg drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that the conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as Transposition



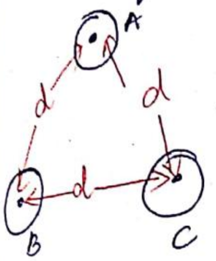
Advantages:

- When conductors are not transposed at regular intervals, the inductance and capacitance of the conductors will not be equal.
- When conductors such as telephone lines are run in parallel to transmission lines, there is a possibility of high voltages induced in the telephone lines. This can result in acoustic shock or noise. Transposition greatly reduces this undesired phenomenon.
- In practice, however, conductors are not transposed in the transmission lines. The transposition is done in the switching stations and the substations.

4. Derive an expression for capacitance of a 3 phase single circuit line with Equilateral spacing

Capacitance of a 3-ϕ overhead line :-

In a 3ϕ TL, the capacitance of each conductor is considered instead of capacitance from conductor to conductor.
Capacitance of conductor can be found at
1. Equal spacing   2. Unsymmetrical spacing.



1. Symmetrical spacing :-

fig shows the three conductors A, B & C of the 3-ϕ overhead T.L having charges Q_A, Q_B & Q_C per meter respectively. let the conductors be equidistant at 'd' meter apart from each other.

we shall find the capacitance from the line conductor to neutral line in this symmetrical spaced line.

overall p.d b/w the conductor A & infinite neutral plane is given by,

$$V_A = \int_r^\infty \frac{Q_A}{2\pi x\epsilon_0} dx + \int_d^\infty \frac{Q_B}{2\pi x\epsilon_0} dx + \int_d^\infty \frac{Q_C}{2\pi x\epsilon_0} dx.$$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d} + (Q_A + Q_B + Q_C) \log_e \infty\right]$$

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d} + 0 \cdot \log_e \infty\right]$$

Assuming balanced supply, $Q_A + Q_B + Q_C = 0$.

$$= \frac{1}{2\pi\epsilon_0}\left[Q_A\left(\log_e \frac{1}{r} - \log_e \frac{1}{d}\right)\right]$$

$$= \frac{Q_A}{2\pi\epsilon_0}\left(\log_e \frac{d}{r}\right)$$

$$\boxed{V_A = \frac{Q}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ volts}}$$

Capacitance of conductor A w.r.t neutral;

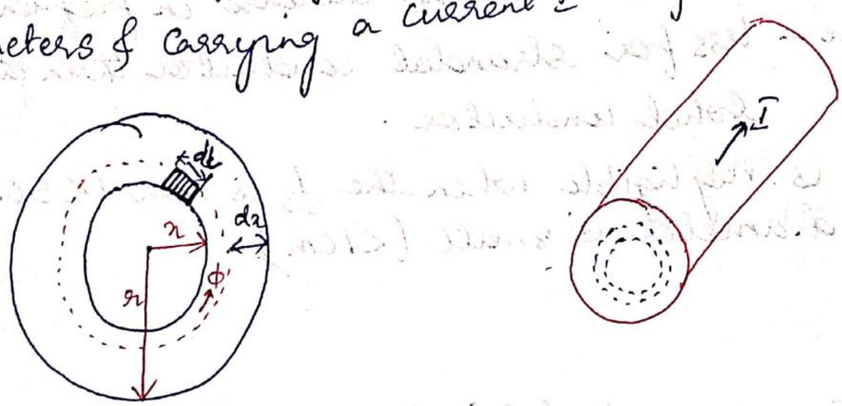
$$C_A = \frac{Q}{V_A} = \frac{Q}{Q/2\pi\epsilon_0 \log_e d/r}$$

$$\boxed{C_A = \frac{2\pi\epsilon_0}{\log_e d/r} \text{ F/m}} \rightarrow \text{Capacitance to two-wire line}$$

5. Obtain an expression of an inductance of conductor due to internal flux and external flux

Inductance of a conductor due to the Internal flux

Consider a long, straight conductor with radius r meters & carrying a current I amperes as shown in the fig



→ The value of inductance due to internal flux is given by the ratio of flux linkages to current.

→ The exact value of ~~flux~~ inductance of transmission line is obtained by considering the flux inside each conductor as well as external flux. The lines of flux are concentric.

From, Ampere's law,

mmf in ampere turns around the any closed loop is equal to the current enclosed by the path.

$$mmf = \oint H.dL = I$$

Let the magnetic field intensity at a point x meters from the centre of the conductor be H_x (This is constant at all points as field is symmetrical).

$$\therefore \oint dL = 2\pi x$$

$$mmf = \oint H_x \cdot dL = I_x.$$

$$H_x \cdot 2\pi x = I_x.$$

$$\boxed{H_x = \frac{I_x}{2\pi x}} \quad - (1)$$

W.K.T I flows is $\propto$ cross area of conductor.

$$\frac{I_x}{I} = \frac{\cancel{\pi} x^2}{\cancel{\pi} R^2}$$

$$\boxed{I_x = \frac{x^2}{R^2} \cdot I} \quad - (2)$$

Substitute the value of I_x in eq (1),

$$H_x = \frac{x^{\cancel{2}}}{R^2} \cdot I \cdot \frac{1}{2\pi \cancel{x}}$$

$$H_x = \frac{x}{2\pi R^2} \cdot I \quad AT/m.$$

$$B_x = \mu H_x = \mu_0 \mu_R H_x$$

$$= \mu_0 \cdot \frac{x}{2\pi R^2} \cdot I \quad (\mu_R = 1 \text{ for non-magnetic material})$$

$$\boxed{B_x = \frac{\mu_0 x}{2\pi R^2} \cdot I}$$

For the element of thickness dL, the flux will be product of B_x & the cross-sectional area of the element normal to the flux lines. This area is dx times axial length. If the axial length is 1 m,

Flux / meter $\Rightarrow$ $d\phi = B_x \times 1 \times dx = \frac{\mu_0 x I}{2\pi R^2} \cdot dx$

This flux links with current I_x. Hence flux linkage/metre length of conductor,

$$d\psi = \frac{\cancel{\pi} x^2}{\cancel{\pi} r^2} \cdot d\phi \quad wb/m.$$

$$= \frac{x^2}{r^2} \cdot d\phi = \frac{x^2}{r^2} \cdot \frac{\mu_0 I x}{2\pi r^2} \cdot dx.$$

$$d\psi = \frac{\mu_0 I x^3}{2\pi r^4} dx.$$

The total flux linkage inside the conductor, ~~we~~ ~~have~~ current flows from the centre of conductor to its outside edge.

$$\psi_{int} = \int_0^r d\psi = \int_0^r \frac{\mu_0 x^3}{2\pi r^4} \cdot I \cdot dx.$$

$$= \frac{\mu_0 I}{2\pi r^4} \cdot \frac{x^4}{4}\Big|_0^r = \frac{\mu_0 I}{2\pi \cancel{r^4}} \times \frac{\cancel{r^4}}{4}$$

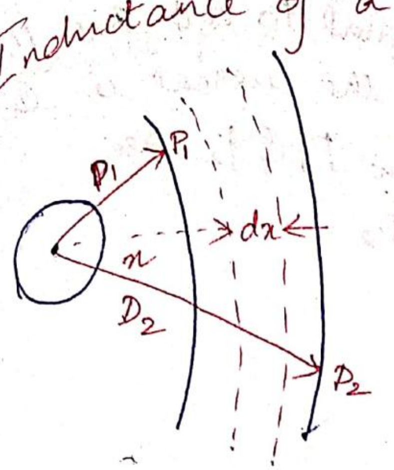
$$= \frac{\mu_0 I}{8\pi} \; wb\text{-turn}/m$$

$$\psi_{int} = \frac{\overset{2}{\cancel{4\pi}} \times 10^{-7} \times I}{\cancel{8\pi}} \quad [\mu_0 = 4\pi \times 10^{-7} H/m]$$

$$\psi_{int} = \frac{I}{2} \times 10^{-7} \; wb\text{-}t/m.$$

$$L_{int} = \frac{\psi_{int}}{I} = \frac{10^{-7}}{2} \; H/m.$$

Inductance of a conductor due to External Flux



To find the flux linkages of the conductor due to the external flux.

We consider the flux linkages of an isolated conductor due to that portion of the external flux which lies between two points distant D_1 & D_2 meters from the centre of conductor at P_1 & P_2 distances respectively.

The conductor carries a current I, consider a tubular element which is x meter from the centre of conductor. The field Intensity at this point is H_x.
The mmf around the element is,

$$2\pi x H_x = I$$

$$H_x = \frac{I}{2\pi x}$$

The flux density B_x at this point is given by,

$$B_x = \mu . H_x$$

$$= \mu . \frac{I}{2\pi x} \ wb/m^2$$

The flux $d\phi$ in the tubular element is given by,

$$d\phi = B_x \times dx \times 1m$$ (Axial length is considered as 1m)

$$d\phi = \frac{\mu I}{2\pi x} \times dx$$

The flux linkages $d\psi$ / meter are equal to $d\phi$. Since flux external to the conductor links all the current in the conductor. The total flux linkage b/w P_1 & P_2 are obtained by integrating $d\psi$ from D_1 to D_2

$$\psi_{12} = \int_{D_1}^{D_2} \frac{\mu I}{2\pi x} dx$$

$$= \frac{\mu I}{2\pi} \int_{D_1}^{D_2} \frac{dx}{x}$$

$$= \frac{\mu I}{2\pi} \ln x \Big|_{D_1}^{D_2} = \frac{\mu I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)$$

For relative permeability, $\mu_r = 1$,

$$\psi_{12} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{D_2}{D_1}\right)$$

$$= \frac{\overset{2}{\cancel{4\pi}} \times 10^{-7} I}{\cancel{2\pi}} \ln\left(\frac{D_2}{D_1}\right)$$

$$\psi_{12} = 2 \times 10^{-7} I \ln\left(\frac{D_2}{D_1}\right)$$

The inductance due to flux ~~linkage~~ included b/w P_1 & P_2

$$\boxed{L_{12} = \frac{\psi_{12}}{I} = \frac{2\times 10^{-7} \cancel{I} \ln\left(\frac{D_2}{D_1}\right)}{\cancel{I}} = 2\times 10^{-7} \ln\left(\frac{D_2}{D_1}\right)}$$

In the external flux is considered to be extended from the surface of conductor to ∞, then the total flux linkages is given by,

$$\boxed{\psi_{12} = \int_{r}^{\infty} \frac{\mu_0 I}{2\pi x} dx \ \text{wb-T}}$$

overall flux linkages is given by,

$$\text{Total flux linkage} = \frac{\mu_0 I}{8\pi} + \int_r^{\infty} \frac{\mu_0 I}{2\pi x}\, dx$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right]$$

6.

$D_{12} = 2m$, $D_{23} = 2.5m$ $D_{31} = 4.5m$. $r = \frac{1.24}{2} = 0.62cm$

$$D_{eq} = \sqrt[3]{D_{12} \times D_{23} \times D_{31}} = \sqrt[3]{2 \times 2.5 \times 4.5} = 2.82 = \underline{\underline{282cm}}$$

$$\text{Inductance/phase/m} = \cancel{\sqrt[3]{D_{12} \times D_{23} \times D_{31}}} = \cancel{\sqrt[3]{2 \times 2.5 \times 4.5}}$$

$$= 10^{-7}\left(0.5 + 2\log_e \frac{D_{eq}}{r}\right) H$$

$$= 10^{-7}\left(0.5 + 2\log_e \frac{282}{0.62}\right) = \underline{12.74 \times 10^{-7} H}$$

$$\text{Inductance/phase/km} = 12.74 \times 10^{-7} \times 1000 = 1.274 \times 10^{-3} H$$

$$= \underline{1.27 mH}$$

7.b.

(ii) Mutual-GMD :-

The mutual-GMD is the geometrical mean of the distan from one conductor to the other. It is represents the equivalent geometrical spacing.

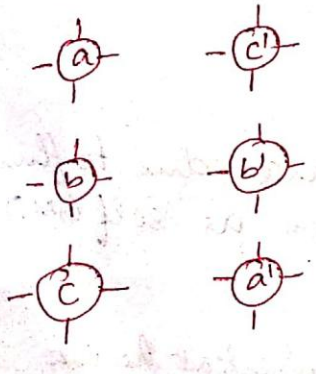
a) Mutual GMD between two conductors is equal to the distance b/w their centres

$$D_m = \text{spacing b/w conductors} = d$$

(b) For a single ckt 3ϕ line, the mutual GMD is equal to the equivalent equilateral spacing i.e $(d_1 d_2 d_3)^{1/3}$

$$D_m = (d_1 d_2 d_3)^{1/3}$$

(c) consider the conductor arrangement of the double ckt shown.



Self GMD of conductor $= 0.7788\, r$

self GMD of combination aa'

$$D_{S1} = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$$

$$D_{S2} = \sqrt[4]{D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b}}$$

$$D_{S3} = \sqrt[4]{D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{cc'}}$$

$$D_S = \sqrt[3]{D_{S1} \times D_{S2} \times D_{S3}}$$

The value of D_S is same for all the phases as each conductor has the same radius.

mutual - GMD b/w phases A & B is,

$$D_{AB} = \sqrt[4]{D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'}}$$

mutual - GMD b/w phases B & C is,

$$D_{BC} = \sqrt[4]{D_{bc} \times D_{bc'} \times D_{b'c} \times D_{b'c'}}$$

mutual - GMD b/w phases C & A is

$$D_{CA} = \sqrt[4]{D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'}}$$

Eq mutual - GMD, $D_m = \sqrt[3]{D_{AB} \times D_{BC} \times D_{CA}}$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2 + \log_e \infty \left(I_A + I_B + I_C \right)\right]$$

As $I_A + I_B + I_C = 0$.

$$\phi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - \log_e d_3 I_B - I_C \log_e d_2\right]$$