CMR Institute of Technology, Bengaluru
DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

Solutons of Internal Assesment Test – II
Subject: OPERATIONAL AMPLIFIERS AND LINEAR ICS (18EE46)
Semester: 4A

1. Derive the expression for the gain and phase angle of 1st order low pass Butterworth filter and draw its frequency response. Define the term "cut-off frequency".

Solution:

The first order low pass butterworth filter is realised by R-C circuit used along with an op-amp, used in the noninverting configuration. The circuit diagram is shown in Fig. 2.5.1.

This also called **one pole low pass Butterworth filter.**

The resistances R_f and R_1 decide the gain of the filter in the pass band.

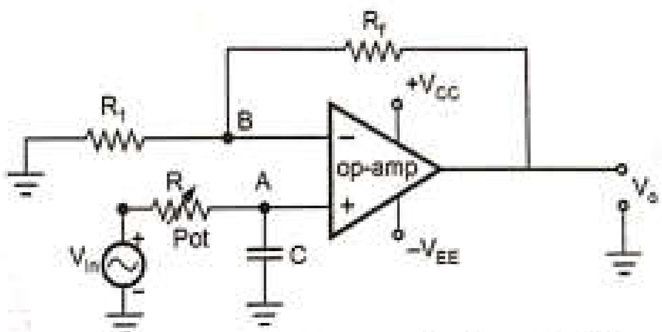


Fig. 2.5.1 First order low pass Butterworth filter

2.5.1 Analysis of the Filter Circuit

The impedance of the capacitor C is $-jX_C$ where X_C is the capacitive reactance given by $X_C = \frac{1}{2\pi f C}$.

By the potential divider rule, the voltage at the non-inverting input terminal A which is the voltage across capacitor C is given by,

$$V_A = \frac{-jX_C}{R - jX_C} \cdot V_{in} \qquad \text{... (2.5.1)}$$

$$\therefore \quad V_A = \frac{-j\left(\frac{1}{2\pi f C}\right)}{R - j\left(\frac{1}{2\pi f C}\right)} \cdot V_{in} = \frac{-j}{2\pi f R C - j} \cdot V_{in} = \frac{V_{in}}{1 - \frac{2\pi f R C}{j}}$$

but $\quad -j = \frac{1}{j} \quad$ and $\quad -\frac{1}{j} = j$

$$\therefore \quad V_A = \frac{V_{in}}{1 + j2\pi f R C} \qquad \text{... (2.5.2)}$$

As the op-amp is in the non-inverting configuration,

$$V_o = \left(1 + \frac{R_f}{R_1}\right)V_A = \left(1 + \frac{R_f}{R_1}\right)\frac{V_{in}}{(1 + j\,2\,\pi\,f\,R\,C)} \qquad (2.5.3)$$

$$\therefore \quad \frac{V_o}{V_{in}} = \frac{A_F}{1 + j\left(\frac{f}{f_H}\right)} \qquad \dots (2.5.4)$$

where $\quad A_F = \left(1 + \frac{R_f}{R_1}\right)$ = Gain of filter in pass band ... (2.5.5)

and $\quad f_H = \frac{1}{2\,\pi\,R\,C}$ = High cut-off frequency of filter ... (2.5.6)

and $\quad f$ = Operating frequency

The $\frac{V_o}{V_{in}}$ is the transfer function of the filter and can be expressed in the polar form as,

$$\frac{V_o}{V_{in}} = \left|\frac{V_o}{V_{in}}\right| \angle \phi \text{ where } \left|\frac{V_o}{V_{in}}\right| = \frac{A_F}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}} \text{ and } \phi = -\tan^{-1}\left(\frac{f}{f_H}\right) \dots (2.5.7)$$

The phase angle ϕ is in degrees.

The equation (2.5.7) describes the behaviour of the low pass filter.

1. At very low frequencies, $f < f_H$, $\left|\frac{V_o}{V_{in}}\right| \cong A_F$ i.e. constant

2. At $f = f_H$, $\left|\frac{V_o}{V_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707\,A_F$ i.e. 3 dB down to the level of A_F.

3. At $f > f_H$, $\left|\frac{V_o}{V_{in}}\right| < A_F$

Thus, for the range of frequencies, $0 < f < f_H$, the gain is almost constant equal to f_H which is high cut-off frequency. At $f = f_H$, gain reduces to $0.707\,A_F$ i.e. 3 dB down from A_F. And as the frequency

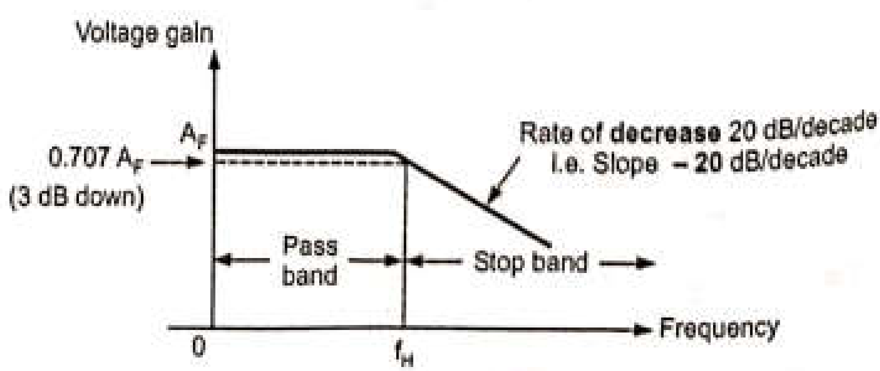


Fig. 2.5.2 Frequency response

increases than f_H, the gain decreases at a rate of 20dB/decade. The rate 20 dB/decade means decrease of 20 dB in gain per 10 times change in frequency. The same rate can be expressed as 6 dB/octave i.e. decrease of 6 dB per two times change in the frequency. The frequency f_H is called **cut off frequency, break frequency, – 3dB frequency** or corner frequency. The frequency response is shown in the Fig. 2.5.1.

2. Design an active high pass filter to meet the following specification: cut-off frequency= 4 kHz, Decay rate in the stop band=40 dB/decade

Solution:

$f_1 = 4 KHz$

$R_2 = R_3$, $\quad C_2 = C_3 = C = 0.1 \mu F$

To find R.

$$f_L = \frac{1}{2\pi RC}$$

$$R = \frac{1}{2 \times 3.14 \times 4 \times 10^3 \times 0.1 \times 10^{-6}}$$

$$R_2 = R_3 = 397 \Omega$$

to find $\frac{R_f}{R_1}$ and R_1

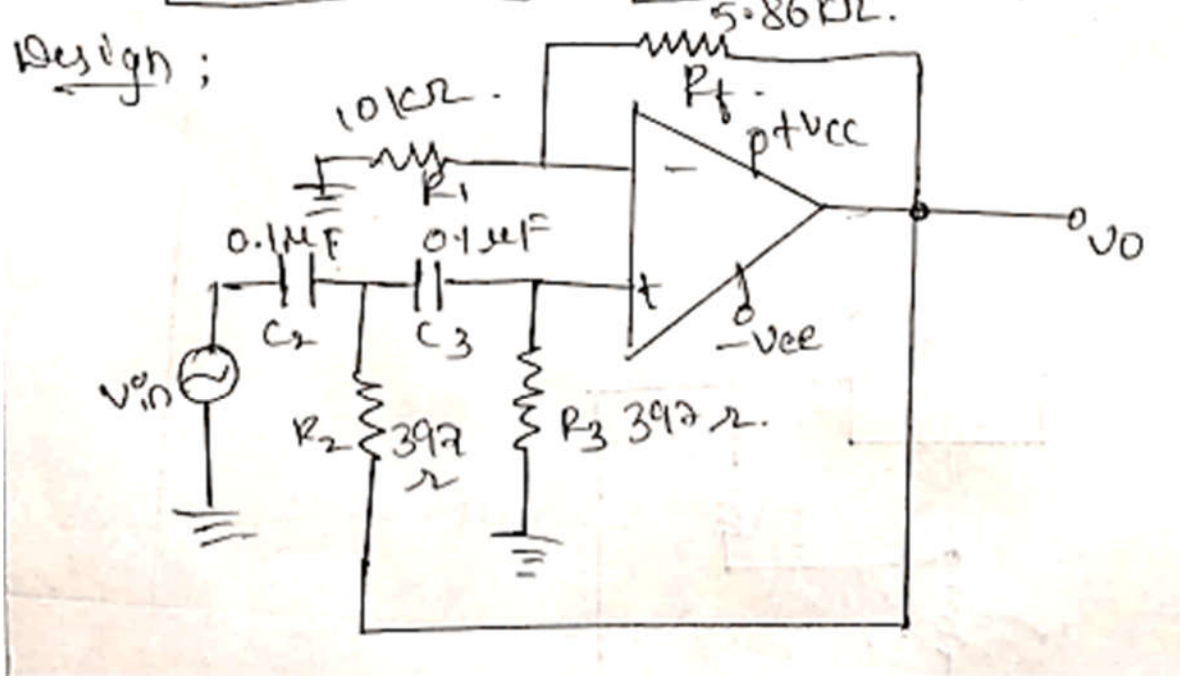
$$A_F = 1 + \frac{R_f}{R_1}$$

$$1.586 - 1 = \frac{R_f}{R_1}$$

$$0.586 R_1 = R_f$$

if $R_f = 10\ k\Omega$, $R_f = 5.86\ k\Omega$.

Design ;



3. Design a wide band pass filter having f_L=200 Hz and f_H=1 kHz and pass band gain=4. Assume the capacitor value of high pass & low pass filter as 0.01 μF & 0.02 μF respectively. Draw the frequency response of the filter and also calculate the Q value of the filter.

Solution:

Given,

$f_L = 200\text{Hz}$, $f_H = 1\text{kHz}$,

pass band gain = 4.

for HPF Capacitor = 0.01 μF = C'

LPF Capacitor = 0.02 μF. = C.

$$f_H = \frac{1}{2\pi R'C'} \Rightarrow 1\times10^3 = \frac{1}{2\times3.14\times R'\times0.01\times10^{-6}}$$

$$\boxed{R' = 15.91\ k\Omega}$$

$$f_L = \frac{1}{2\pi RC} \Rightarrow 200 = \frac{1}{2\times3.14\times R\times0.02\times10^{-6}}$$

$$\boxed{R = 39.7\ k\Omega}$$

$A_{FT} = A_1 A_2$

$4 = A_{FT}$

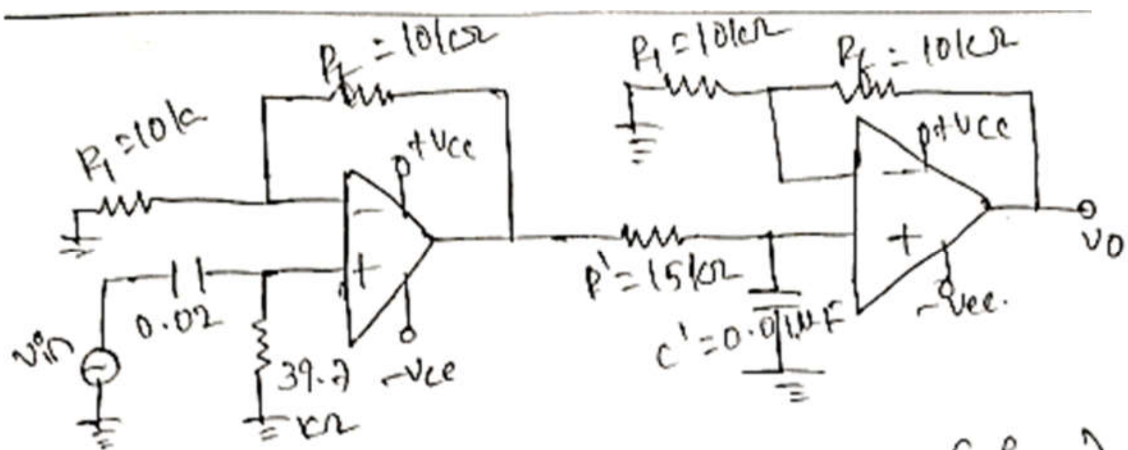
$A_1 = A_2 = 2.$

$A_1 = A_2 = 1 + \frac{R_f}{R_1} = 2$

$R_f = R_1$ then only A_1 or $A_2 = 2$.

Consider,

$R_f = R_1 = 10\ k\Omega$,



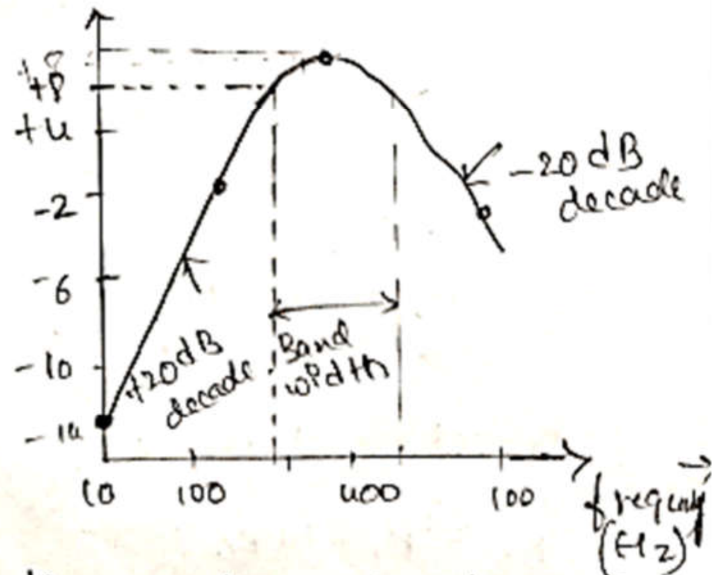
$$\frac{v_o}{v_{in}} = \frac{A_H\left(f/f_L\right)}{\sqrt{\left[1+\left(\frac{f}{f_L}\right)^2\right]\left[\left(1+\frac{f}{f_L}\right)^2\right]}} = \frac{4\left(\frac{f}{200}\right)}{\sqrt{1+\left(\frac{f}{200}\right)^2 \cdot \left(1+\left(f/1\times10^3\right)^2\right)}}$$

f in Hz	v_o/v_{in}	v_o/v_{in} in dB
10	0.19	-14.42
100	1.1236	1.01
400	2.247	7.03
1000	0.706	-3.023

$$f_c = \sqrt{f_L f_H} = \sqrt{200 \times 1000} = 44721$$

$$BW = f_H - f_L = 1000 - 200 = 800$$

$$Q = \frac{f_c}{BW} = \frac{44721}{800} = 0.559$$



∴ The Q is always less than 10 for a band pass filter.

4. With a neat circuit diagram explain the working of All-pass filter and derive the expression for the gain and phase shift as produced by an All-pass filter.

Solution:

The filters which are discussed uptil now are used to adjust the magnitude of the transfer function of the circuit. But, this also alters the phase angle characteristics of the circuit. It is many times required to control the phase response of the filter. The filter which is used to control the phase response by adding a phase shift between input and output signals is called as all pass filter. Its gain is one for all the frequencies. Thus, as the name suggests, it passes all the frequencies of the input signal. It does not produce any attenuation but provide the required phase shift for the different frequencies of the input signal.

For example, when signals are transmitted over the transmission lines, there is change in their phase. To compensate for such phase change, all pass filters are used. Hence, all pass filters are used. Hence, all pass filters are also called as **delay equalizers** or phase **correctors.**

The Fig. 2.12.1 shows the simple first order all pass filter.

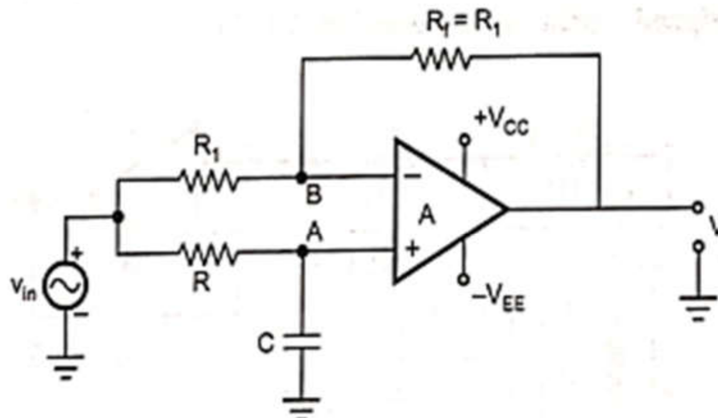


Fig. 2.12.1 All pass filter

2.12.1 Analysis of the Circuit

Let us use the superposition principle to obtain the expression for the output voltage V_O.

Assume input to the non-inverting terminal zero. The circuit acts as an inverting amplifier.

$$\therefore \quad V_{O1} = -\frac{R_f}{R_1} V_{in}$$

$$\therefore \quad V_{O1} = -V_{in} \quad \text{as } R_f = R_1 \qquad \text{... (2.12.1)}$$

Now, assume input to the inverting terminal zero. The circuit acts as a non-inverting amplifier.

$$\therefore \quad V_{O2} = \left(1 + \frac{R_f}{R_1}\right) V_A$$

$$\therefore \quad V_{O2} = 2\, V_A \quad \text{as } R_f = R_1 \qquad \text{... (2.12.2)}$$

and $\quad V_A$ = Voltage at node A

By the potential divider rule, the voltage V_A can be obtained as

$$V_A = V_{in}\left[\frac{-jX_C}{R - jX_C}\right]$$

where $\quad -jX_C = -j\left(\frac{1}{2\pi f C}\right) = \left(\frac{1}{j2\pi f C}\right) \quad \text{as } -j = \frac{1}{j}$

$$\therefore \quad V_A = V_{in}\left[\frac{\frac{1}{j2\pi fC}}{R+\frac{1}{j2\pi fC}}\right] = V_{in}\left[\frac{1}{1+j2\pi fRC}\right] \quad \text{... (2.12.3)}$$

Substituting in (2.12.2),

$$V_{O2} = 2\,V_{in}\left[\frac{1}{1+j2\pi fRC}\right] \quad \text{...(2.12.4)}$$

Hence, the total output voltage is

$$V_O = V_{O1} + V_{O2} = -V_{in} + 2\,V_{in}\left[\frac{1}{1+j2\pi fRC}\right]$$

$$\therefore \quad V_O = V_{in}\left[-1+\frac{2}{1+j2\pi fRC}\right] \quad \text{... (2.12.5)}$$

$$\therefore \quad \boxed{\frac{V_O}{V_{in}} = \frac{1-j2\pi fRC}{1+j2\pi fRC}} \quad \text{... (2.12.6)}$$

The magnitude of the transfer function is

$$\boxed{\left|\frac{V_O}{V_{in}}\right| = \frac{\sqrt{1+(2\pi fRC)^2}}{\sqrt{1+(2\pi fRC)^2}} = 1} \quad \text{... (2.12.7)}$$

It is mentioned earlier that the magnitude is always 1 for all pass filter and it can pass the entire range of frequency. But the phase angle is given by

$$\boxed{\phi = -2\tan^{-1}\left(\frac{2\pi fRC}{1}\right)} \quad \text{... (2.12.8)}$$

This is the phase angle in degrees which indicates that there is a phase shift of ϕ degrees between input and output signal. If the positions of R and C are interchanged, we get the positive phase shift. The negative phase shift indicates that the output V_O lags input V_{in} by angle ϕ, while positive phase shift indicates that V_O leads input V_{in} by angle ϕ.

The Fig. 2.12.2 shows the phase shift produced by all pass filter between input and output.

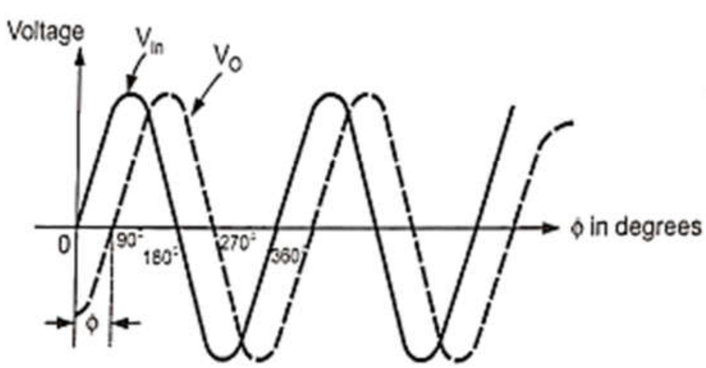


Fig. 2.12.2 Phase shift between V_O and V_{in}

5. Define the terms: Line regulation & Load regulation (Write the equation for each).
An unregulated dc power supply output changes from 20 V to 18.8 V when the load is increased from zero to maximum. Calculate the load effect and Load regulation. The voltage also increases to 20.8 V when the ac supply increases by 10 %. Calculate the source effect and Line/source regulation.
Solution:

- The purpose of **Line Regulation** is to maintain a nearly constant output voltage when the **input voltage** varies.

- The Line regulation defines the variation in output voltage (ΔV_o) that occurs when the supply voltage (V_s) increases or decreases by a specified amount, usually 10 %. The output voltage change is expressed as a percentage of the normal dc output voltage (V_o). Thus, **Line Regulation** can be mathematically expressed as:

$$\text{Line regulation} = \frac{(\Delta V_o \text{ for a } 10\% \text{ change in } V_S) \times 100\%}{V_o}$$

- The purpose of **Load Regulation** is to maintain a nearly constant output voltage when the **Load current** varies.

- The Load regulation defines the regulator performance in relation to load current variation. When the load current changes from zero to full load, then the output voltage also changes by an amount of (ΔV_o). It is expressed as a percentage of the normal dc output voltage (V_o). The **Load Regulation** can be mathematically expressed as:

$$\text{Load regulation} = \frac{(\Delta V_o \text{ for } \Delta I_{L(max)}) \times 100\%}{V_o}$$

Load regulation = $\frac{(\Delta V_0 \text{ for } \Delta I_{L(max)})}{V_0} \times 100\%$

V_0 -> initial voltage.

load effect (ΔV_0 for $\Delta I_{L(max)}$)

I_L => 0 -> max

=> $I_L = 0$ -> $I_L = max$

$V_0 = 20V$ -> $V_0 = 18.8V$

load effect = (20 - 18.8)

load effect => 1.2V

Load effect = $\frac{1.2}{20} \times 100$

Load effect = 6%

line regulation = $\left[\frac{\Delta V_0 \text{ for } 10\% \text{ change in } V_S}{V_0}\right] \times 100\%$

Source effect => 20.8V - 20V

Source effect = 0.8V

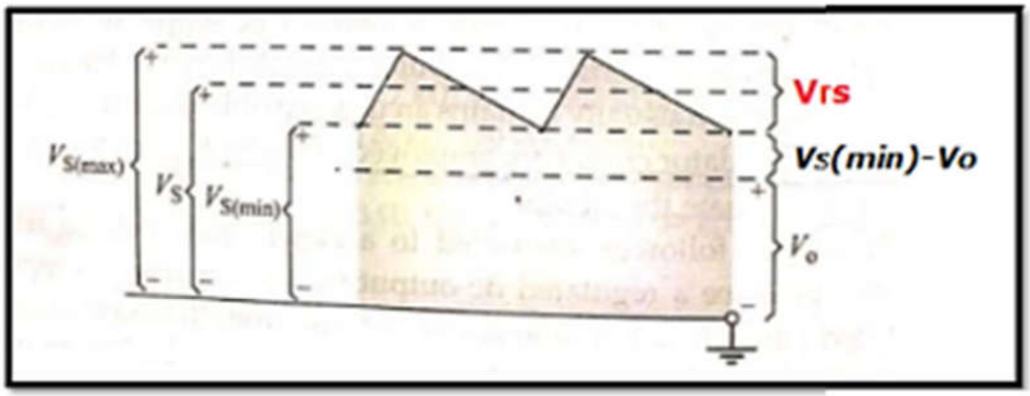
line regulation => $\frac{0.8}{20} \times 100\%$

4%

line regulation = 4%

6. Design an Adjustable Voltage Regulator to produce a 12 V output with a 50 mA maximum load current (use 1N756 Zener diode, Vrs=2V peak-to-peak, Vs(min)-Vo=3V).

Solution:



$$V_{s(min)} = V_o + 3\text{ V} = 12\text{ V} + 3\text{ V}$$
$$= 15\text{ V}$$

allowing $V_{rs} = 2$ V peak-to-peak,

$$V_s = V_{s(min)} + \frac{V_{rs}}{2} = 15\text{ V} + 1\text{ V}$$
$$= 16\text{ V}$$

Supply voltage, $V_s = 16$ V with a 2 V (max) ripple superimposed

Let $V_z \approx \frac{V_s}{2} = \frac{16\text{ V}}{2}$

≈ 8 V (use a 1N756 Zener diode which has $V_z = 8.2$ V)

$I_z \approx 20$ mA

$$R_1 = \frac{V_s - V_z}{I_z} = \frac{16\text{ V} - 8.2\text{ V}}{20\text{ mA}}$$

$= 390\ \Omega$ (standard value)

The output of Adjustable Voltage Regulator can be expressed as:

$$V_0 = V_z\left(1+\frac{R_1}{R_2}\right)$$

Here, we are taking 1N756 Zener diode with Vz=8.2 V

$$12 = 8.2\left(1+\frac{R_1}{R_2}\right)$$

$$\frac{12}{8.2} = \left(1+\frac{R_1}{R_2}\right)$$

$$0.4634 = \frac{R_1}{R_2}$$

$$R_1 = 0.4634 \times R_2$$

Let R2=10 kΩ,

So, R1=0.4634*R1

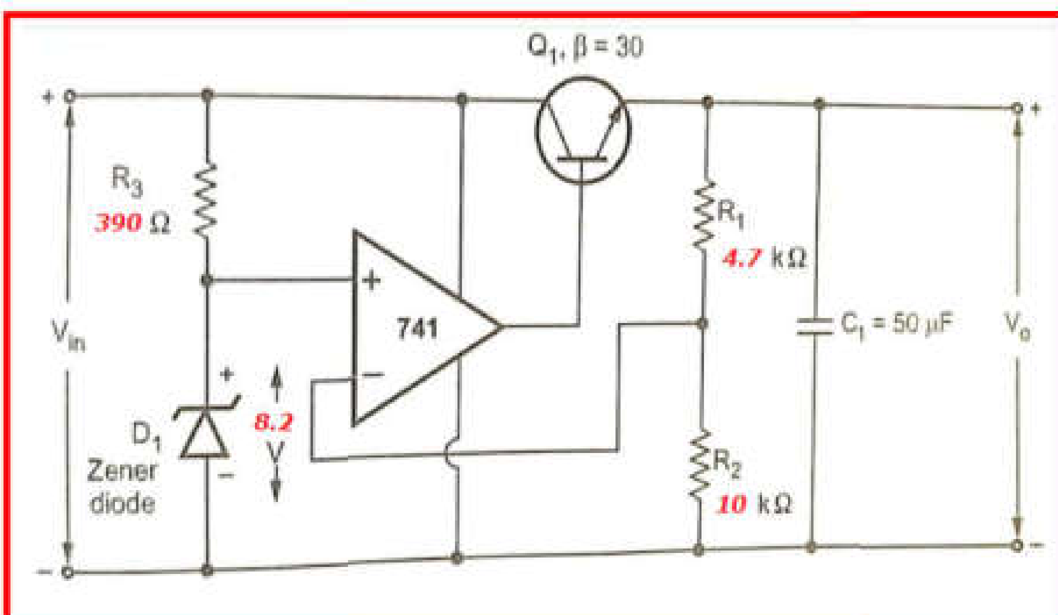
=4.63 kΩ

~ 4.7 kΩ (standard value)

Transistor (Q1) specification: The Power dissipation of thetransistor can be defined as:

$$\begin{aligned} P &= V_{CE} \times I_L = (V_s - V_o) \times I_L \\ &= (16\ \text{V} - 12\ \text{V}) \times 50\ \text{mA} \\ &= 200\ \text{mW} \end{aligned}$$

Circuit Diagram:



7. Design an adjustable positive voltage regulator using LM317 for output voltage varying from 2 V to 10 V and output current of 1.5 A

Solution:

Design Sol^n:

$I_{adj} = 100\ \mu A$

$I_{R1} = 1\ mA$

for $V_o = 2V$

$R_1 = \frac{1.25}{I_{R1}}$

$= \frac{1.25}{1 \times 10^{-3}}$

$R_1 = 1.25 K\Omega$

$V_o = 1.25\left(1 + \frac{R_2}{R_1}\right)$

$2 = 1.25\left(1 + \frac{R_2}{1.25}\right)$

$2 - 1.25 = 1 + \frac{R_2}{1.25}$

$0.75 = 1 + \frac{R_2}{1.25}$

$-0.25 = \frac{R_2}{1.25}$

$R_2 = -0.31\Omega$

for $V_o = 10V$

$R_1 = \frac{1.25}{I_{R_1}}$

$= \frac{1.25}{1 \times 10^{-3}}$

$R_1 = 1.25 K\Omega$

$V_0 = 1.25\left(1 + \frac{R_2}{R_1}\right)$

$10 = 1.25\left(1 + \frac{R_2}{R_1}\right)$

$8.75 = 1 + \frac{R_2}{1.25}$

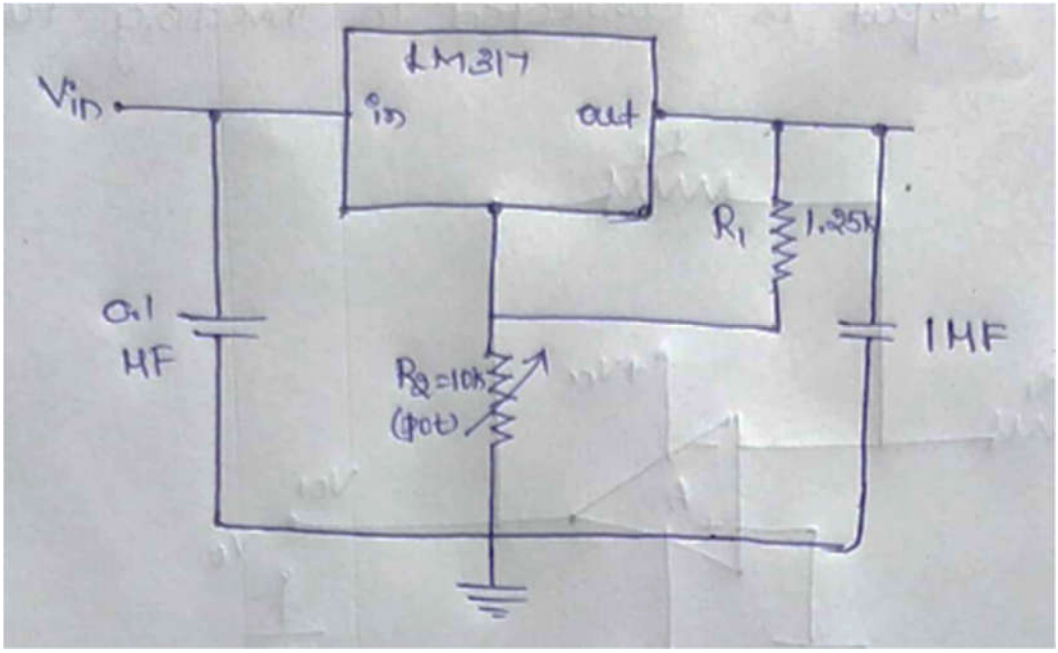
$7.75 = \frac{R_2}{1.25}$

$R_2 = 9.68$

$P_o = (V_{in} - V_{max})\, I_L$

$= (15 - 10)\, 1.5$

$P_o = 7.5 W$


LM317
Vin
in
out
R1
1.25k
0.1
MF
1MF
R2 =10k
(pot)