III - Internal Assessment Test
Strength of Materials
Scheme & Solution

1. Draw Shear Force Diagram (SFD) and Bending Moment diagram for given beam.

i) Determination of Reactions

$\Sigma V = 0 \Rightarrow R_A - 1.5 \times 2 - 2 = 0$

$\boxed{R_A = 5\ kN}$

$\Sigma M_A = 0 \Rightarrow -M_A + (1.5 \times 2)(\frac{2}{2}) + 2 \times 1.5 = 0$

$\boxed{M_A = 6\ kN\text{-}m}$

ii) Determination of Shear Force calculation

S.F. Just before A = 0

S.F. Just after A = 5 kN

S.F. Just before B = $5 - 1.5 \times 1.5 = 2.75$ kN

S.F. Just after B = $5 - 1.5 \times 1.5 - 2 = 0.75$ kN

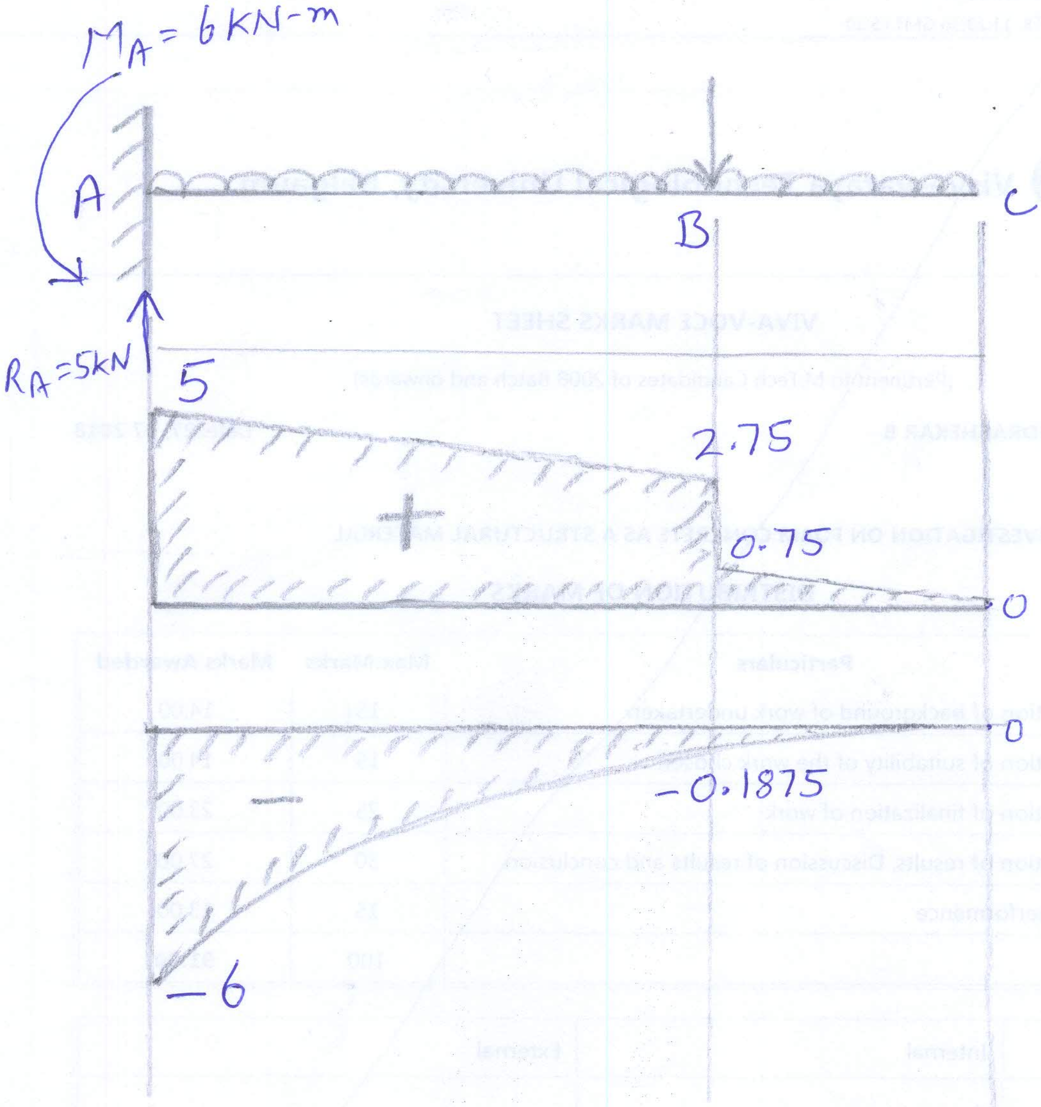
S.F. at C = $5 - 1.5 \times 1.5 - 2 - 1.5 \times 0.5 = 0$ kN

iii) Determination of Bending Moment

B.M at A = -6 kN-m

B.M at B = $-6 - (1.5 \times 1.5)(\frac{1.5}{2}) + 5 \times 1.5 = -0.1875$ ~~$= -4.3125$ kN-m~~

B.M. at C = $-6 - (1.5 \times 2)(\frac{2}{2}) - 2 \times 0.5 + 5 \times 2 = 0$


M_A = 6KN-m
A
B
C
R_A=5kN
5
2.75
+
0.75
0
0
-0.1875
-
-6

2) Draw the shear force diagram (SFD) and bending moment diagram (BMD) for the given beam. (3)

A) Determination of Reactions

$\Sigma V = 0 \Rightarrow R_B + R_D - 800 - 2000 - 1000 = 0$

$R_B + R_D = 3800$ —— (1)

$\Sigma M_B = 0 \Rightarrow -800 \times 3 + 2000 \times 5 - R_D \times 8 + 1000 \times 10 = 0$

$-2400 + 10{,}000 - 8R_D + 10{,}000 = 0$

$$\boxed{\begin{array}{l} R_D = 2200\ N \\ \Rightarrow R_B = 1600\ N \end{array}}$$

Calculation of S.F. Values

S.F. at Just before A = 0

S.F. at Just after A = −800 N

S.F. at Just before B = −800 N

S.F. at Just after B = −800 + 1600 = +800 N

S.F. at Just before C = −800 + 1600 = 800 ~~= −600 N~~

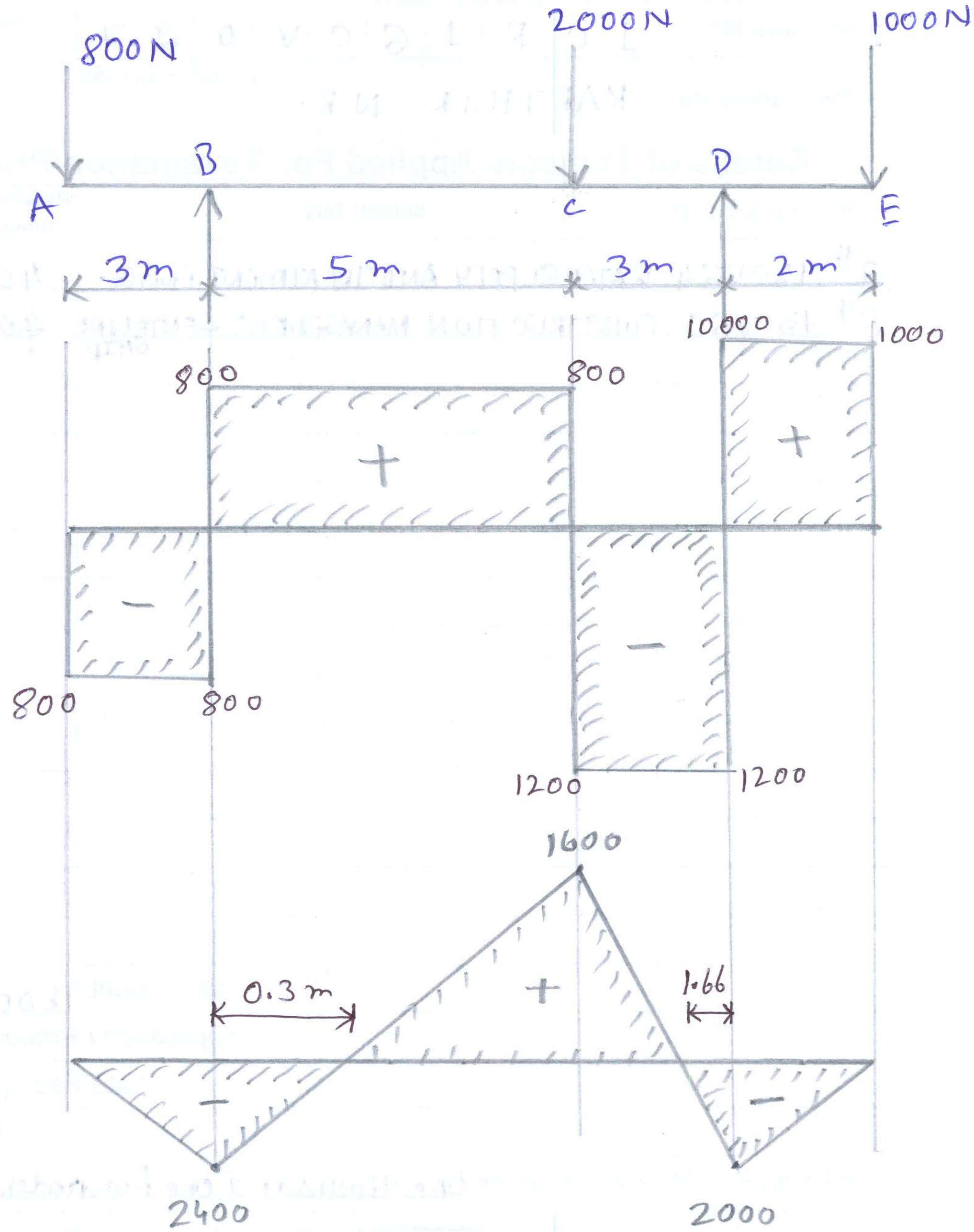
S.F. at Just after C = −800 + 1600 − 2000 = −1200 N

S.F. at Just before D = −800 + 1600 − 2000 = −1200 N

S.F. at Just after D = −800 + 1600 − 2000 + 2200 = 10,00 N

S.F at Just before E = −800 + 1600 − 2000 + 2200 = 10,00 N

S.F. at Just after E = −800 + 1600 − 2000 + 2200 − 10,00 = 0



800N
2000N
1000N
A
B
C
D
E
3m
5m
3m
2m
800
800
10000
1000
+
+
-
-
800
800
1200
1200
1600
0.3m
1.66
+
-
-
2400
2000

(5)

B.M. at A = 0

B.M. at B $= -800 \times 3 = -2400$ N-m

B.M at C $= -800 \times 8 + 1600 \times 5 = 1600$ N-m

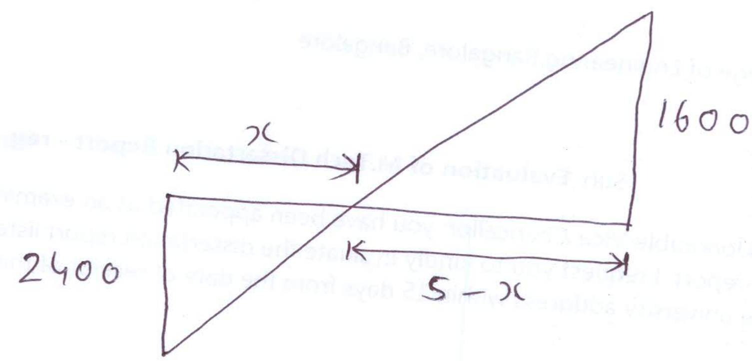
B.M at D $= -800 \times 11 + 1600 \times 8 - 2000 \times 3 = -2000$ N-m

B.M. at E $= -800 \times 13 + 1600 \times 10 - 2000 \times 5 + 2200 \times 2$

$= 0$

Point of contraflexure

i) Span BC :

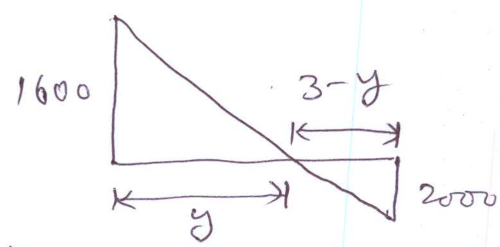


$$\frac{x}{2400} = \frac{5-x}{1600} \Rightarrow 1600x = 2400(5-x)$$

$$1600x = 2400 \times 5 - 2400x$$

$$\Rightarrow \boxed{x = 0.3\ m}$$

ii) Span CD:

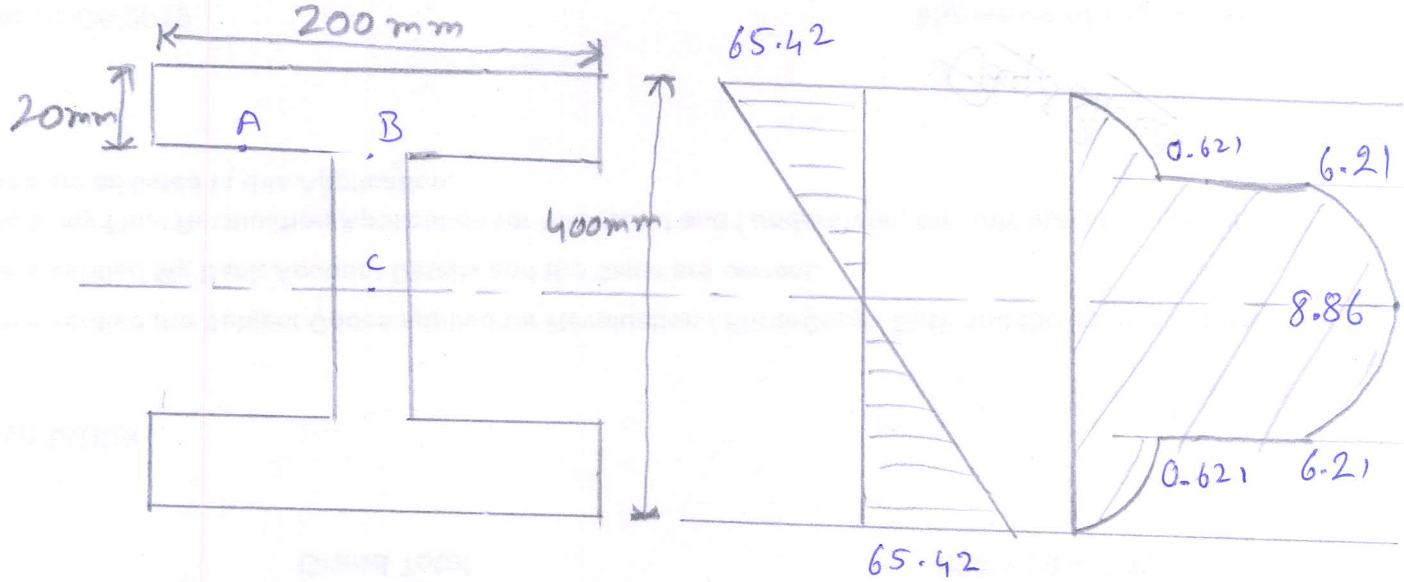


$$\frac{y}{1600} = \frac{3-y}{2000} \Rightarrow 2000y = 1600(3-y)$$

$$2000y = 1600 \times 3 - 1600y$$

$$y = 1.33\ m$$

Q3) Draw the shear stress diagram and bending stress diagram at point in a beam subjected to shear force of 60 KN and bending moment of 120 KN-m. The cross-section of the beam is shown below. (6)



~~As~~ the section is symetric ~~about~~ the N.A. will be in centre

$$I = \frac{200 \times 400^3}{12} - \frac{180\,(360)^3}{12} = 366.826 \times 10^6 \ mm^4$$

~~The~~ Bending stress calculation

$$\sigma_b = \frac{M}{I} \times y \qquad \text{at } y = 200 \ mm$$

$$\sigma_b = \frac{120 \times 10^6}{366.826 \times 10^6} \times 200 = \underline{65.42 \ N/mm^2}$$

Shear stress distribution

$$\tau = \frac{F\,(a\bar{y})}{I\,b}$$

i) Shear stress at top fibre $= 0$

ii) Shear stress at 'A'

$F = 60\,kN = 60 \times 10^3\,N$

$b = 200\,mm$

$a = 200 \times 20 = 4000\,mm^2$

$\bar{y} = 200 - 10 = 190\,mm$

$$\tau_A = \frac{60 \times 10^3 \times 4000 \times 190}{366.82 \times 10^6 \times 200} = \underline{\underline{0.621\,N/mm^2}}$$

$$\tau_B = \frac{60 \times 10^3 \times 4000 \times 190}{366.82 \times 10^6 \times 20} = \underline{\underline{6.21\,N/mm^2}}$$

iii) Shear stress at C

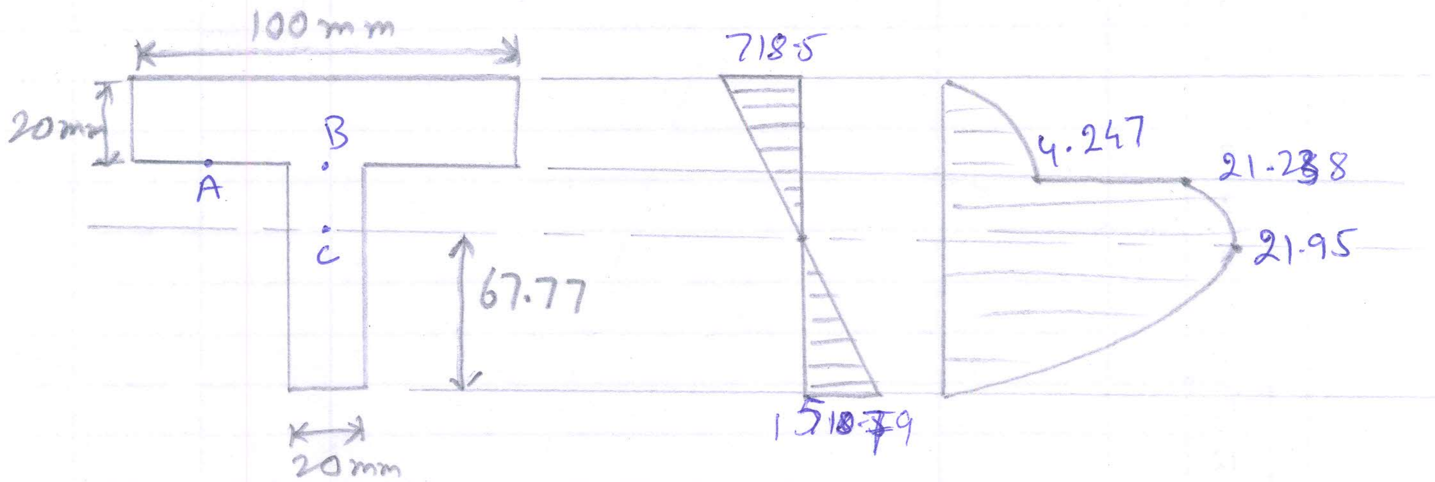
$b = 20\,mm$, $(a\bar{y}) = a_1 y_1 + a_2 y_2$

$= (200 \times 20)(190) + (180 \times 20)(90)$

$=$

$$\tau_c = \frac{60 \times 10^3 \times \cancel{4000 \times 190 +} 1084000}{366.82 \times 10^6 \times 20} = \underline{\underline{8.86\,N/mm^2}}$$

Q4) Draw the shear stress diagram and bending stress diagram at point in a beam subjected to shear force of 30KN and bending moment of 70KN-m. The cross-section of the beam is shown below. (8)



Location of Neutral axis

$$\bar{Y} = \frac{(100\times20)_{\times 90} + (20\times80)\times40}{100\times20 + 20\times80} = 67.77\ mm$$

Bending stress distribution

$$\sigma_b = \frac{M}{I}\times y$$

$$I = \left[\frac{100(20)^3}{12} + (100\times20)(67.77-90)^2\right] + \left[\frac{20(80)^3}{12} + (20\times80)(67.77-40)^2\right]$$

$$I = [66666.66 + 988345.8] + [853333.33 + 1233876.64]$$

$$I = 3.14\times10^6 \ mm^4$$

Bending stress at top fibre

$$\sigma_b = \frac{70\times10^6 \ N\text{-}mm}{3.14\times10^6 \ mm^4} \times (100-67.77)$$

$$\sigma_b = 718.5 \ N/mm^2$$

Bending stress at bottom fibre

$$\sigma_b = \frac{70\times10^6}{3.14\times10^6} \times (67.77) = 1510.79 \ N/mm^2$$

Shear stress at A

$$\tau_A = \frac{F(a\bar{y})}{Ib} = \frac{30\times10^3 (100\times20)(90-67.77)}{3.14\times10^6 \times 100} = 4.247 \ N/mm^2$$

Shear stress at B

$$\tau_B = \frac{30\times10^3(100\times20)(90-67.77)}{3.14\times10^6 \times 20} = 21.238 \ N/mm^2$$

Shear stress at C

$$\tau_C = \frac{30\times10^3\left[(100\times20)(90-67.77) + (20\times12.23)\left(\frac{12.23}{2}\right)\right]}{3.14\times10^6 \times 20} = 21.95 \ N/mm^2$$

Q5) A shaft has to transmit 105 kW at 160 rpm. If the shear stress is not to exceed 65 N/mm² and the twist in a length of 3.5 m must not exceed 1°, find a suitable diameter. Take rigidity modulus as 8×10^4 N/mm².

A) $P = 105\ \text{KW} = 105 \times 10^3\ \text{W}$

$N = 160$ rpm

$\sigma_s = 65\ \text{N/mm}^2$

$\theta = 1^\circ = \frac{\pi}{180}$ radians

$l = 3.5\ \text{m} = 3500\ \text{mm}$

$C = 8 \times 10^4\ \text{N/mm}^2$ $\quad d = ?$

$$P = \frac{2\pi N T}{60} \Rightarrow \frac{2 \cdot \pi \cdot 160 \cdot T}{60} = 105 \times 10^3$$

$$\boxed{T = 6266.72\ \text{N-m}}$$

Condition ① : Maximum shear stress = 65 N/mm²

$$\frac{T}{I_p} = \frac{\sigma_s}{R} \Rightarrow \frac{6266.72 \times 10^3}{\frac{\pi D^3}{32}} = \frac{65}{D/2}$$

$$\Rightarrow D^3 = 491017.58 \Rightarrow D = 78.89\ \text{mm}$$

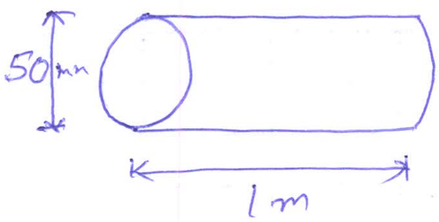
Condition ② Maximum angle of twist = 1°

$$\frac{T}{I_p} = \frac{C\theta}{l} \Rightarrow \frac{6266.72 \times 10^3}{\frac{\pi D^4}{32}} = \frac{8 \times 10^4 \times \pi/180}{3500}$$

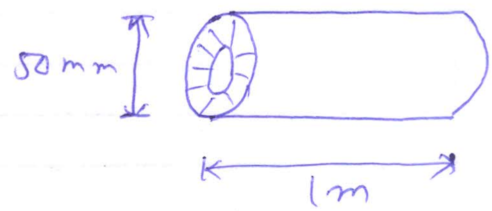
$$D^4 = \cancel{5026779250.3}\ 160007775 \Rightarrow D = 112.46\ \text{mm}$$

Adopt $\boxed{D = 112.46\ \text{mm}}$

6) A solid aluminium shaft 1m long and 50 mm diameter is to be replaced by a tubular steel shaft of the same length and the same outside diameter, such that each of the two shafts could have the same angle of twist per unit torsional moment over total length. What must be the inner diameter of the tubular steel shaft? Modulus of rigidity of steel is three times that of aluminium.



Aluminium



Steel

$d_1 = 50\text{ mm}$

$d_1 = 50\text{ mm}$
$d_2 = ?$

$$C_s = 3C_a$$

Condition :- Same angle of twist per unit torsional moment (Same flexibility)

$$\frac{\theta_s}{T_s} = \frac{\theta_a}{T_a}$$

but $\frac{\theta_s}{T_s} = \frac{l}{I_p C}$

$$\frac{l_s}{I_{p_s} C_s} = \frac{l_a}{I_{p_a} C_a}$$

$$\frac{1}{\frac{\pi(d_1^4 - d_2^4)}{32} 3C_a} = \frac{1}{\frac{\pi d_1^4}{32} \times C_a}$$

$$\Rightarrow \frac{\pi d_1^4}{\cancel{32}} = \frac{3\pi d_1^4}{\cancel{32}} - \frac{3\pi d_2^4}{\cancel{32}}$$

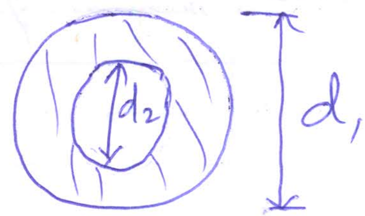
$$\Rightarrow 3d_2^4 = 2d_1^4$$

$$d_2^4 = \frac{2(50)^4}{3}$$

$$d_2 = 45.18\text{ mm}$$

7) a) A hollow shaft of external diameter 120 mm transmits 300 KW power at 200 rpm. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm².

given data



$d_1 = 120$ mm

$P = 300$ KW

$N = 200$ rpm

$\sigma_{max} = 60$ N/mm²

$$P = \frac{2\pi N T}{60} \Rightarrow 300\times10^3 = \frac{2\pi\, 200\, T}{60}$$

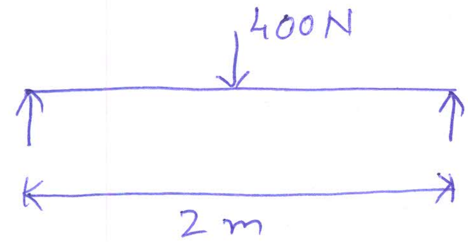
$$\Rightarrow T = 14323.94 \text{ N-m}$$

$$\frac{T}{I_p} = \frac{\sigma_s}{R} \Rightarrow \frac{14323.94\times10^3}{\frac{\pi}{32}(120^4 - d_2^4)} = \frac{60}{\frac{120}{2}}$$

$$\Rightarrow d_2 = 88.54 \text{ mm}$$

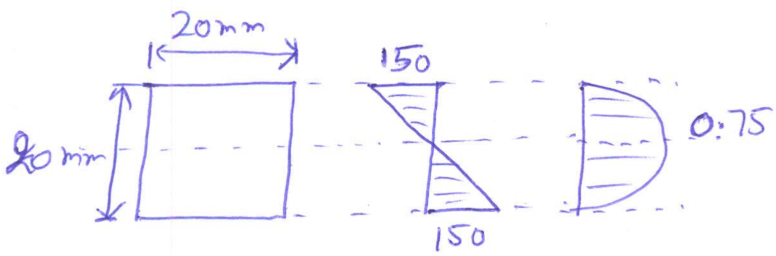
7)b) A square beam 20mm x 20mm in section and 2m is simply supported at the ends. The beam is loaded with 400N load at the centre. Draw the shear stress and bending stress distribution for maximum bending moment and maximum shear stress.

A)



$$F_{max} = \frac{W}{2} = \frac{400}{2} = 200\ N$$

$$M_{max} = \frac{Wl}{4} = \frac{400}{4} \times 2 = 200\ N\text{-}m$$



$$I = \frac{bd^3}{12} = \frac{20(20)^3}{12}$$

$$I = 13333.33\ mm^4$$

Bending stress values

$$\sigma = \frac{M}{I} \times y = \frac{200\times10^3}{13333.33} \times 10 = 150\ N/mm^2$$

Shear stress values

$$\tau = \frac{F(a\bar{y})}{Ib} = \frac{200\left[(20\times10)\left(\frac{10}{5}\right)\right]}{\frac{20(20)^3}{12} \times 20} = 0.75\ N/mm^2$$