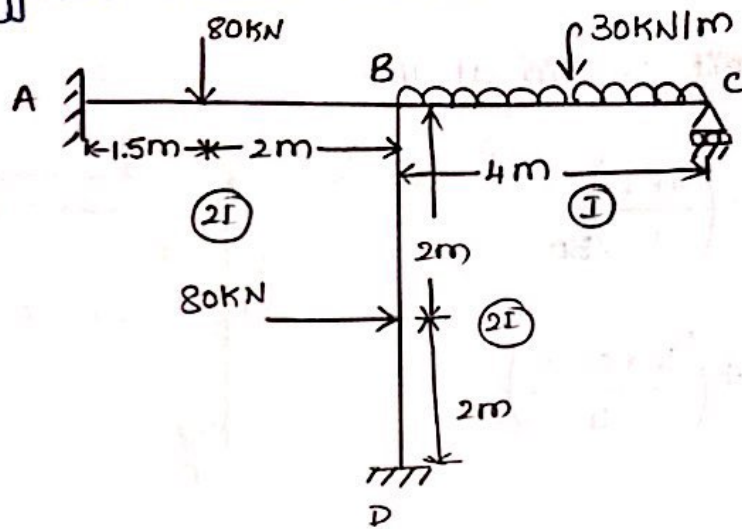


Date: 14/12/2020

Q1 Analyse the portal frame shown in fig 1. Using the Stiffness matrix method Sketch BMD and SFD



Fixed End moments:

$$M_{FAB} = -\frac{wab^2}{l^2} = -\frac{80 \times 1.5 \times 2^2}{(3.5)^2} = -39.18 \text{ kNm}$$

$$M_{FBA} = \frac{wa^2b}{l^2} = \frac{80 \times (1.5)^2 \times 2}{(3.5)^2} = 29.38 \text{ kNm}$$

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{30 \times 4^2}{12} = -40 \text{ kNm}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{30 \times 4^2}{12} = 40 \text{ kNm}$$

$$M_{FDB} = -\frac{wL}{8} = -\frac{80 \times 4}{8} = -40 \text{ kNm}$$

$$M_{FBD} = \frac{wL}{8} = \frac{80 \times 4}{8} = 40 \text{ kNm}$$

Step (2) $[\Delta] = \begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = P = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$[P_L] = \begin{bmatrix} M_{FBA} + M_{FBC} + M_{FBD} \\ M_{FCB} \end{bmatrix} = \begin{bmatrix} 29.38 + (-40) + 40 \\ 40 \end{bmatrix} = \begin{bmatrix} 29.38 \\ 40 \end{bmatrix}$$

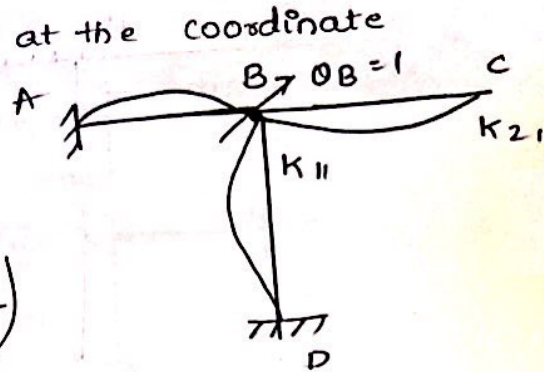
Step 3: Apply the Unit rotation at the coordinate

$$K_{11} = \left(\frac{4EI}{L}\right)_{BA} + \left(\frac{4EI}{L}\right)_{BC} + \left(\frac{4EI}{L}\right)_{BD}$$

$$= \left(\frac{4 \times 2EI}{3.5}\right) + \left(\frac{4 \times EI}{4}\right) + \left(\frac{4 \times 2EI}{4}\right)$$

$$= 5.28EI$$

$$K_{21} = \frac{2EI}{L} = \frac{2EI}{4} = 0.5EI$$



Apply the Unit rotation at the coordinate 2 - joint C

$$K_{12} = \frac{2EI}{4} = 0.5EI$$

$$K_{22} = \frac{4EI}{4} = 1EI$$

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = EI \begin{bmatrix} 5.28 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$\theta = [K]^{-1} [P - P_L]$$

$$\begin{bmatrix} \theta_B \\ \theta_c \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} 5.28 & 0.5 \\ 0.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -29.38 \\ -40 \end{bmatrix}$$

$$\theta_B = -1.86 / EI$$

$$\theta_C = -39.06 / EI$$

Substitute the θ_B and θ_C in Slope deflection Equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l} \left\{ 2\theta_A + \theta_B - \frac{3\delta}{l} \right\}$$

$$= -39.18 + \frac{2 \times 2EI}{3.5} \left\{ \theta_B \right\} = -39.18 + \frac{4EI}{3.5} \left\{ \frac{-1.86}{EI} \right\}$$

$$= -41.305 \text{ kNm}$$

$$M_{BA} = M_{FBA} + \frac{2 \times 2EI}{3.5} \left\{ 2\theta_B \right\}$$

$$= 29.38 + \frac{4EI}{3.5} \left\{ 2(-1.86 / EI) \right\}$$

$$= 25.12 \text{ kNm}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l} \left\{ 2\theta_B + \theta_C - \frac{3\delta}{l} \right\}$$

$$= -40 + \frac{2 \times EI}{4} \left\{ 2 \left(\frac{-1.86}{EI} \right) + \left(\frac{-39.06}{EI} \right) \right\}$$

$$= -61.39 \text{ kNm}$$

$$M_{CB} = 40 + \frac{2EI}{4} \left\{ 2 \left(\frac{-39.06}{EI} \right) + \left(\frac{-1.86}{EI} \right) \right\}$$

$$= 0 \text{ kNm}$$

$$M_{BD} = M_{FBD} + \frac{2 \times 2EI}{l} \left\{ 2\theta_B + \theta_D - \frac{3\delta}{l} \right\}$$

$$= 40 + \frac{2 \times 2EI}{4} \left\{ 2 \left(\frac{-1.86}{EI} \right) \right\}$$

$$= 36.28 \text{ kNm}$$

$$M_{DB} = M_{FDB} + \frac{2 \times 2EI}{4} \left\{ \theta_B + 2\theta_D - \frac{3\delta}{L} \right\}$$

$$= -40 + \frac{4EI}{4} \left\{ \left(-\frac{1.86}{EI} \right) \right\}$$

$$= -41.86 \text{ kNm}$$

Shear force diagram

$$V_A + V_B + V_C = 80 + 30 \times 4$$

$$EM_A = 0$$

$$V_A \times 0 + 80 \times 1.5 - V_B \times 3.5 + 30 \times 4 \times \{2 + 3.5\}$$

$$+ (-41.305) + 25.12 - 61.39 + 0 - V_C \times 7.5$$

$$= V_B = 105.36 \text{ kN}$$

$$EM_B = 0 \text{ (LHS)}$$

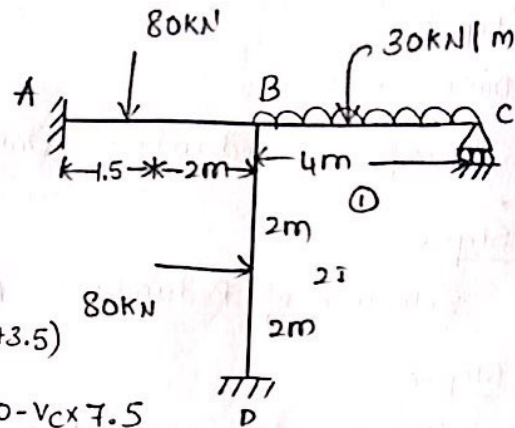
$$V_A \times 3.5 - 80 \times 2 - 41.30 + 25.12$$

$$V_A = 50.3 \text{ kN}$$

$$EM_B = 0 \text{ (RHS)}$$

$$= V_C \times 4 + 30 \times 4 \times 2 + 0 - 61.39$$

$$V_C = 44.65 \text{ kN}$$

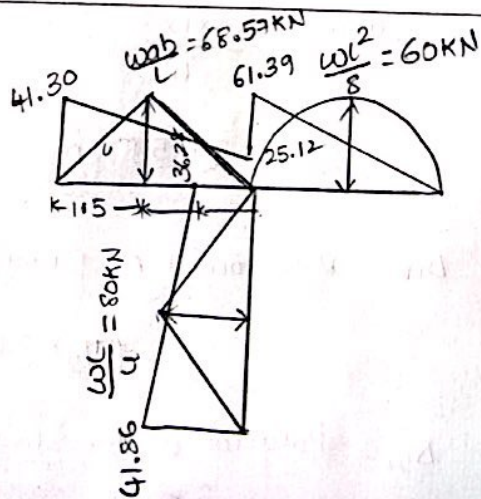
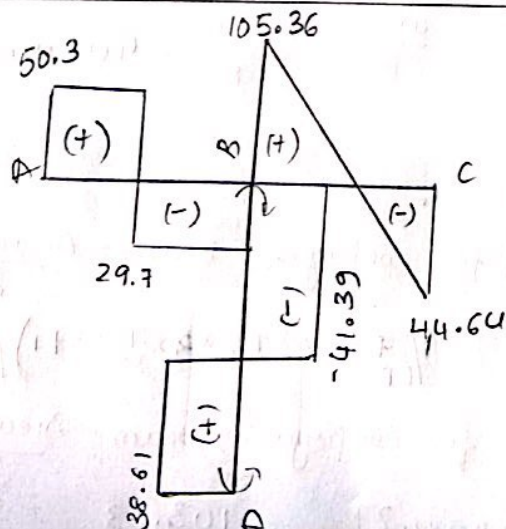


$$H_D + H_B = 80$$

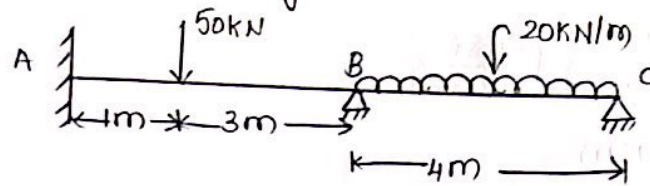
$$EM_D = 0 - 80 \times 2 + H_B \times 4 + 36.28 - 41.86$$

$$H_B = 41.38 \text{ kN}$$

$$H_D = 38.61 \text{ kN}$$



02 Analyze the Continuous beam shown in fig 2
Using the Flexibility matrix method. Sketch BMD & SFD



Step-1

$$DOF = 5 - 3 = 2$$

Therefore 2 Redundant = Unknown M_A and M_B

Step-02

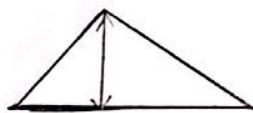
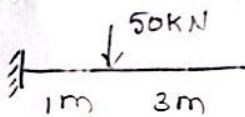
Selection of Redundant = A and B are coordinates 1 and 2

Step-03

Applying Conjugate beam method

Split the beam as AB and BC

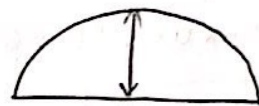
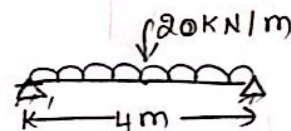
AB Span



$$\frac{w_{ab}}{L} = \frac{50 \times 1 \times 3}{4}$$

$$= 37.5 \text{ kNm}$$

BC Span



$$\frac{wL^2}{8} = \frac{20 \times 4^2}{8} = 40 \text{ kNm}$$

$$\Delta_{1L} = \text{Rotation at A - Shear at A for conjugate beam} = \frac{\text{Area of BMD}}{EI} = \frac{50}{EI}$$

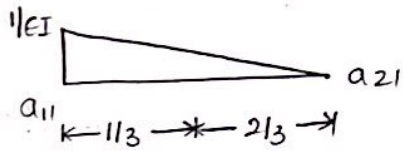
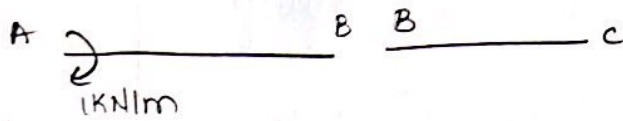
$$= \left(\frac{1}{2} \times 37.5 \times 4 \times \frac{2}{3} \right) / EI + \left(\frac{1}{2} \times 37.5 \times 3 \times \left(\frac{1}{3} \times 3 + 1 \right) \right) / EI$$

$$\Delta_{2L} = \text{Rotation at B - Shear at B for conjugate beam} = \frac{\text{Area of BMD}}{EI}$$

$$= \frac{50}{EI} + \frac{1}{2} \left\{ \frac{2}{3} \times 40 \times 4 \right\} / EI = \frac{103.33}{EI}$$

Step 4 - Getting Flexibility matrix:

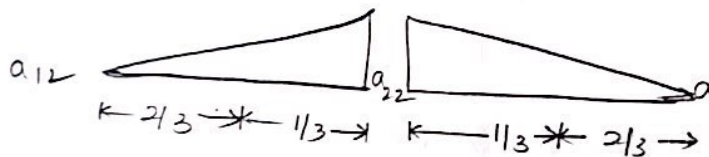
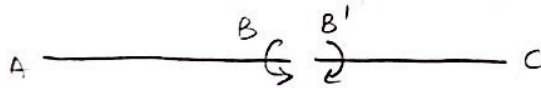
Apply unit moment at coordinate -1



$$a_{11} = \left(\frac{1}{2} \times 4 \times 1 \times 1 \times \frac{2}{3} \right) = 1.33 EI$$

$$a_{21} = \left(\frac{1}{2} \times 4 \times 1 \times 1 \times \frac{1}{3} \right) = 0.67 EI$$

Apply Unit moment at coordinate -2



$$a_{12} = \left(\frac{1}{2} \times 4 \times 1 \times 1 \right) \times \frac{1}{3} = 0.67 EI$$

$$a_{22} = \left(\frac{1}{2} \times 4 \times 1 \times 1 \times \frac{2}{3} \right) + \left(\frac{1}{2} \times 4 \times 1 \times 1 \times \frac{2}{3} \right) = 2.66 EI$$

$$P = [a]^{-1} [\Delta - \Delta_{1L}]$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta - \Delta_{1L} \\ \Delta - \Delta_{2L} \end{bmatrix}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \left[EI \begin{bmatrix} 1.33 & 0.67 \\ 0.67 & 2.66 \end{bmatrix} \right]^{-1} \frac{1}{EI} \begin{Bmatrix} 0 - 50 \\ 0 - 103.33 \end{Bmatrix}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} -20.64 \\ -33.64 \end{bmatrix}$$

$$M_{AB} = -20.64$$

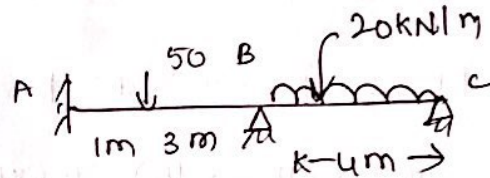
$$M_{BA} = 33.64$$

$$M_{BC} = -33.64$$

$$M_{CB} = 0$$

Shear force diagram

$$V_A + V_B + V_C = 50 + 20 \times 4$$



$$\sum M_B = 0 \text{ (LHS)}$$

$$V_A \times 4 - 50 \times 3 - 20.64 + 33.64$$

$$V_A = +34.25$$

$$\sum M_B = 0 \text{ (RHS)}$$

$$-V_C \times 4 + 20 \times 4 \times 2 + 0 - 33.64$$

$$V_C = 31.58 \text{ kN}$$

$$V_B = 64.17$$

