$\therefore$ Required answer $= 360 + 180 + 180 = 720$

9) (i) Since there is no restriction.

answer $= {}^{11}C_5 = 462$

(ii) Since two particular persons will not attend separately,

(a) they should both be invited

This can be done in ${}^{9}C_3 = 84$ ways

(b) both shouldn't be invited

this can be done in ${}^{9}C_5 = 126$ ways.

Required answer $= 84 + 126 = 210$

(iii) Since two particular persons $(P_1 \,\&\, P_2)$ will not attend together,

(a) invite P_1, not P_2

no of ways $= {}^{9}C_4 = 126$

(b) invite P_2, not P_1

no of ways $= {}^{9}C_4 = 126$

(c) both P_1 & P_2 are not invited

no of ways $= {}^{9}C_5 = 126.$

Required answer $= 378$.

Induction step:

Assume $S(k)$ is true for $k \geq 6$.

$$4k < (k^2 - 7) \quad — ①$$

Consider $4(k+1) = 4k + 4$

$$< (k^2 - 7) + 4$$

$$< (k^2 - 7) + (2k+1) \quad \Big| \; 2k+1 > 4 \text{ for } k \geq 6$$

$$= (k+1)^2 - 7$$

$$\therefore 4(k+1) < (k+1)^2 - 7 \quad — ②$$

Comparing ② with ①, $S(k+1)$ is also true.

Hence by M.I., $S(n)$ is true for all +ve int $n \geq 6$.

8. n must ~~be greater some~~ begin with 5, 6 or 7.

(i) Suppose n starts with 5.

No of such arrangements $= \dfrac{6!}{2!\,1!\,1!\,1!\,1!} = 360$

(ii) suppose n starts with 6.

No of such arrangements $= \dfrac{6!}{2!\,2!} = 180$

(iii) Suppose n starts with 7,

No of such arrangements $= \dfrac{6!}{2!\,2!} = 180$

For $n=1$,

$LHS = L_1 = 1$

$RHS = \left(\frac{1+\sqrt5}{2}\right) + \left(\frac{1-\sqrt5}{2}\right) = \frac{1}{2} + \frac{1}{2} = 1$

$\therefore S(1)$ is true.

Assume $S(k)$ is true for $n=1,2,\dots k$

Consider $L_{k+1} = L_k + L_{k-1}$

$$= \left[\left(\frac{1+\sqrt5}{2}\right)^k + \left(\frac{1-\sqrt5}{2}\right)^k\right] + \left[\left(\frac{1+\sqrt5}{2}\right)^{k-1} + \left(\frac{1-\sqrt5}{2}\right)^{k-1}\right]$$

$$= \left(\frac{1+\sqrt5}{2}\right)^{k-1}\left[\frac{1+\sqrt5}{2}+1\right] + \left(\frac{1-\sqrt5}{2}\right)^{k-1}\left[\frac{1-\sqrt5}{2}+1\right]$$

$$= \left(\frac{1+\sqrt5}{2}\right)^{k-1}\left(\frac{1+\sqrt5}{2}\right)^2 + \left(\frac{1-\sqrt5}{2}\right)^{k-1}\left(\frac{1-\sqrt5}{2}\right)^2$$

$$= \left(\frac{1+\sqrt5}{2}\right)^{k+1} + \left(\frac{1-\sqrt5}{2}\right)^{k+1}$$

Hence, $S(k+1)$ is also true.

$\therefore S(n)$ is true $\forall n$.

7. Let $S(n)$: $4n < n^2 - 7$

Basis step: For $n=6$

$4(6) < 6^2 - 7$

$\Rightarrow 24 < 29$ which is true. $\therefore S(6)$ is true.

5. (i) The general term in the given expansion is

$$\binom{6}{n_1\ n_2\ n_3}(2x^3)^{n_1}(-3xy^2)^{n_2}(z^2)^{n_3}$$

For $n_3=0,\ n_2=2,\ n_1=3$, this becomes

$$\binom{6}{3\ 2\ 0}(2^3x^9)(3^2x^2y^4)$$

∴ the required coeff^t is

$$\binom{6}{3\ 2\ 0}2^3\,3^2 = 4320.$$

(ii) By the binomial theorem, we have

$$(1-2x)^{10} = \sum_{r=0}^{10}\binom{10}{r}1^{10-r}(-2x)^r$$

$$\therefore\ x^3(1-2x)^{10} = \sum_{r=0}^{10}{}^{10}C_r(-2)^r x^{r+3}$$

For $r=9$, coeff^t of x^{12} is $\binom{10}{9}(-2)^9 = -5120.$

6) ~~For~~

Let S_n: $L_n = \left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$

For $n=0$, LHS $= L_0 = 2$

RHS $= 1+1 = 2$

LHS = RHS

$\neg q(x)$ is true But 2 not ≤ 2.

4. (i) $S = \{4, 6, 8, \ldots\ldots 28\}$ is the universe.

$4 = 2+2$	$12 = 7+5$	$20 = 13+7$	$28 = 23+5$
$6 = 3+3$	$14 = 7+7$	$22 = 17+5$	
$8 = 5+3$	$16 = 11+5$	$24 = 19+5$	
$10 = 7+3$	$18 = 11+7$	$26 = 23+3$	

Given statement is true for every element of the universe. $\therefore$ the statement is true.

(ii) If n is odd integer then n^2 is even integer.

Let p: n is odd int

q: n^2 is even int.

Given $p \rightarrow q$

Assume that the given statement is true.

$\Rightarrow$ p is true and q is true.

$\Rightarrow$ n is odd $\Rightarrow n^2$ is even

$\Rightarrow n = 2k+1$ for $k \in Z$

$\Rightarrow n^2 = (2k+1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2+2k) + 1$

which is odd

This is a contradiction. This contradiction is because of our wrong assumption. $\therefore$ Given statement is false.

ie Some integers are not rational numbers or

all rational numbers are integers.

3. $p(x): x^2-8x+15=0$ $\quad x=3, 5$

$q(x)$: x is odd

$r(x)$: $x>0$

(i) $\forall x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$

False.

$x=-1$ which is odd no

$\therefore p(x) \vee q(x)$ is true.

But $x=-1 \ngtr 0$.

$\therefore r(x)$ is false.

(ii) $\forall x, [q(x) \rightarrow p(x)]$

False

$x=7$ is odd.

$\therefore q(x)$ is true,

But $p(x)$ is false.

(iii) $\exists x, [p(x) \rightarrow \{q(x) \wedge r(x)\}]$

True

(iv) $\forall x, [\neg q(x) \rightarrow \neg r(x)]$

False.

Eg:- $x=2$ which is even

Method of Contradiction:

Assume $p \to q$ is false

$\Rightarrow$ p is true and q is false

$\Rightarrow$ n is even and $n+3$ is even

$\Rightarrow n+3 = 2k$ for some $k \in \mathbb{Z}$

$\Rightarrow n = 2(k-1) - 1$

$\Rightarrow$ n is odd

which is a contradiction.

This contradiction is because of our wrong assumption.

$\therefore$ $p \to q$ must be true.

2. "All integers are rational numbers and some rational numbers are not integers."

Let Z: set of all integers   Q: set of all rational nos

$p(x)$: x is a rational number $q(x)$: x is an integer.

Given statement: $\{\forall x \in Z, p(x)\} \wedge \{\exists x \in Q, \neg q(x)\}$

Negation: $\neg [\{\forall x \in Z, p(x)\} \wedge \{\exists x \in Q, \neg q(x)\}]$

$\Leftrightarrow \neg\{\forall x \in Z, p(x)\} \vee \neg\{\exists x \in Q, \neg q(x)\}$

$\Leftrightarrow \{\exists x \in Z, \neg p(x)\} \vee \{\forall x \in Q, q(x)\}$

Internal Assessment Test 2 - Oct. 2019
Discrete Mathematical Structures - 18CS36

1. If n is an even integer then $n+3$ is an odd integer.

Let

p: n is an even integer.

q: $n+3$ is an odd integer.

Given statement is $p \to q$

Direct method:

Assume p is true.

$\Rightarrow$ n is an even integer.

$\Rightarrow$ $n = 2k$ for some $k \in Z$

$\Rightarrow$ $n+3 = 2k+3 = 2(k+1)+1$ which is odd

$\Rightarrow$ q is true.

$\therefore$ $p \to q$ is true.

Indirect method:

WKT $p \to q \Leftrightarrow \neg q \to \neg p$

Assume $\neg q$ is true.

$\Rightarrow$ $n+3$ is an even integer.

$\Rightarrow$ $n+3 = 2k$ for some $k \in Z$

$\Rightarrow$ $n = 2k-3 = 2(k-1)-1$ which is odd

$\Rightarrow$ $\neg p$ is true.

$\therefore$ $\neg q \to \neg p$ is true.

$\therefore$ $p \to q$ is true.

6	For the Lucas numbers $L_0, L_1, L_2 \ldots$ prove that $L_n = \left[\left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$ where $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n > 1$.	[07]	CO4	L3
7	Prove that $4n < (n^2 - 7)$ for all positive integers $n > 5$.	[07]	CO4	L3
8	How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000?	[07]	CO2	L3
9	A woman has 11 relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them if (i) there is no restriction on the choice? (ii) two particular persons will not attend separately? (iii) two particular persons will not attend together?	[07]	CO2	L3

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Internal Assessment Test 2 – Oct. 2019

Sub:	Discrete Mathematical Structures				Sub Code:	18CS36	Branch:	IS
Date:	14/10/2019	Duration:	90 mins.	Max Marks: 50	Sem / Sec:	III - A, B & C		OBE

Question 1 is compulsory and answer any six from Q.2 to Q.9

		MARKS	CO	RBT
1	Prove by direct method, indirect method and the method of contradiction that "If n is an even integer then n+3 is an odd integer."	[08]	CO4	L3
2	Find the negation of "All integers are rational numbers and some rational numbers are not integers."	[07]	CO1	L2
3	Let $p(x) : x^2 - 8x+15 = 0$, $q(x)$: x is odd, $r(x)$: $x > 0$ with the set of all integers as the universe. Find the truth values of the following statements. If a statement is false, give a counter example (i) $\forall x,[\{p(x) \vee q(x)\} \rightarrow r(x)]$, (ii) $\forall x,[q(x) \rightarrow p(x)]$, (iii) $\exists x,[p(x) \rightarrow \{q(x) \wedge r(x)\}]$, (iv) $\forall x,[\neg q(x) \rightarrow \neg r(x)]$	[07]	CO1	L3
4	(i) Prove by the method of exhaustion that "Every even integer x such that $3 < x < 29$ can be written as a sum of two prime numbers."	[3.5]		
	(ii) Disprove by the method of contradiction that "The square of an odd integer is an even integer."	[3.5]	CO4	L3
5	Find the coefficients of (i) $x^{11}y^4$ in $(2x^3 - 3xy^2 + z^2)^6$ (ii) x^{12} in the expansion of $x^3(1-2x)^{10}$.	[07]	CO2	L3

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