Modified

CBCS SCHEME

USN | | | | | | | | | |

15EE54

Fifth Semester B.E. Degree Examination, Dec.2018/Jan.2019

Signals and Systems

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Distinguish between
   i) Continuous and Discrete time signals
   ii) Even and Odd signals
   iii) Periodic and Non-periodic signals
   iv) Deterministic and Random signals
   v) Energy and Power signals. **(10 Marks)**

b. Determine and sketch the even and odd parts of the signal shown in Fig Q1(b)

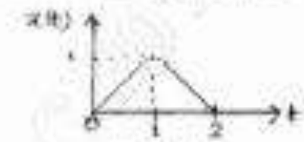


Fig Q1(b) **(06 Marks)**

OR

2 a. Determine whether the following signals are periodic, if periodic determine the fundamental period i) $x(t) = \cos 2t + \sin 3t$ ii) $x[n] = \cos\left(\frac{n\pi}{5}\right)\sin\left(\frac{n\pi}{3}\right)$ **(08 Marks)**

b. Using convolution integral, determine and sketch output of LTI system whose input and impulse response is $x(t) = e^{-2t}\left[u(t) - u(t-2)\right]$ and $h(t) = e^{-t}u(t)$ **(08 Marks)**

Module-2

3 a. Determine the convolution sum of two sequences
$x[n] = \left\{\underset{\uparrow}{3}, 2, 1, 2\right\}$ and $h[n] = \left\{1, \underset{\uparrow}{2}, 1, 2\right\}$. **(08 Marks)**

b. Find the step response of an LTI system, if impulse responses are
i) $h(t) = t^2 u(t)$ ii) $h[n] = \left(\frac{1}{2}\right)^n u[n]$ **(08 Marks)**

OR

4 a. Find the output response of the system described by a differential equation
$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 8y(t) = \frac{dx(t)}{dt} + 2x(t)$. The input signal $x(t) = e^{-t}u(t)$ and initial conditions are $y(0) = 2$, $\frac{dy(0)}{dt} = 3$. **(06 Marks)**

b. Draw the direct form I and direct form II implementation of the following differential equation. $\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt}$ **(06 Marks)**

c. Check whether the response of LTI system $y[n] = 2x[n+1] + 3x[n] + x[n-1]$ is causal and stable? **(04 Marks)**

Module-3

5 a. State and prove the following properties in continuous time Fourier transform i) Linearity ii) Time shift iii) Time differentiation. (10 Marks)

b. Find the Fourier Transformation of $x(t) = e^{-at} u(t), a > 0$. (06 Marks)

OR

6 a. Using partial fraction expansion and linearity to determine the inverse Fourier transform of $x(j\omega) = \dfrac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$ (08 Marks)

b. Find the frequency response and impulse response of the system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} + x(t)$$ (08 Marks)

Module-4

7 a. State and prove the following properties in Discrete time Fourier transform
i) Frequency shift ii) Parseval's theorem. (10 Marks)

b. Find DTFT of the following signal

i) $x[n] = \left(\frac{1}{2}\right)^{n+2} u[n]$ ii) $x[n] = 2(3)^n u[-n]$ (06 Marks)

OR

8 a. Using DTFT, find the total solution to the difference equation for discrete time $n \geq 0$.
$5y(n+2) - 6y(n+1) + y(n) = (0.8)^n u(n)$ (08 Marks)

b. Determine the difference equation description for the system with the following impulse response $h[n] = \delta[n] + 2\left(\frac{1}{2}\right)^n u(n) + \left(\frac{-1}{2}\right)^n u(n)$ (08 Marks)

Module-5

9 a. What is region of convergence? List any 5 properties of ROC. (07 Marks)

b. Find the z-transform and ROC of the signal $x[n] = -b^n u[-n-1]$ (05 Marks)

c. State and prove time shift property. (04 Marks)

OR

10 a. Determine the inverse z-transform of x(z)
$x(z) = \dfrac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$ for ROC $|z| > 1$, $\frac{1}{2} < |z| < 1$. (06 Marks)

b. Consider a causal discrete time sequence whose output y(n) and x(n) are related by
$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n)$
i) Find its system function ii) Find its impulse response h[n]. (10 Marks)

* * * * *

1(a)

A *Continuous-time signal* is a signal whose amplitude varies continuously with time, *i.e.* both the amplitude and time are continuous over their respective intervals. In other words, a signal $x(t)$ is said to be a continuous-time signal if it is defined for all time t.

Continuous-time signals arise naturally when a physical waveform like light wave or acoustic wave is converted into an electrical signal.

A *discrete-time signal* is defined only at discrete instants of time. Since a discrete-time signal is defined at discrete times, it is often identified as a sequence of numbers denoted by $\{x_n\}$ or $x[n]$, where n is an integer. A discrete-time signal $x[n]$ can be derived from a continuous-time signal by sampling it at a uniform rate.

Examples of discrete-time signals include the rainfall in all the months of a year, the number of goods produced by a company every year, etc.

A signal $x(t)$ or $x[n]$ is said to be an even signal if it satisfies the condition:

$$x(-t) = x(t) \quad \text{for all } t,$$

$$x[-n] = x[n] \quad \text{for all } n.$$

A signal $x(t)$ or $x[n]$ is said to be an odd signal if it satisfies the conditions:

$$x(-t) = -x(t) \quad \text{for all } t,$$

$$x[-n] = -x[n] \quad \text{for all } n$$

A signal which repeats after every time interval T is called a *periodic signal*. Mathematically, a signal $x(t)$ is called periodic if and only if

$$x(t + T) = x(t) \quad \text{for all } t$$

where t denotes time and T is a constant. The smallest value of T that satisfies this condition is called the *fundamental period* or simply *period* of $x(t)$.

The reciprocal of fundamental period T is called the *fundamental frequency* f of $x(t)$, *i.e.*

$$f = \frac{1}{T}$$

The angular frequency is given by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\therefore \qquad T = \frac{2\pi}{\omega}$$

A signal $x(t)$ for which there is no value of T satisfying the condition $x(t + T) = x(t)$ is called *non-periodic* or *aperiodic signal*.

A *deterministic signal* is the one where no uncertainity occurs w.r.t. its value at any time. They are modelled by explicit mathematical expressions. For example,

$$x(t) = 100 \sin 50t \quad \text{(Continuous case)}$$

$$x[n] = 100 \sin 50n \quad \text{(Discrete case)}$$

A *random signal* is the one about which there is some degree of uncertainty before it actually occurs. They must be modelled probabilistically. For example, the output of a TV/radio receiver when tuned to a frequency where there is no broadcast.

1. The signals for which the total energy is finite ($0 < E < \infty$) are called *energy signals*. They have zero average power. Examples include deterministic and non-periodic signals.
2. The signals for which the average power is finite ($0 < P < \infty$) are called *power signals*. They have infinite energy. Examples include, random and periodic signals.
3. Both the energy and power signals are mutually exclusive.
4. Signals that satisfy neither property are referred to as neither energy signals nor power signals.

(b)

Solution : We have $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

$$x_e(t) = \frac{1}{2}x(t) + \frac{1}{2}x(-t)$$

The signal $x_e(t)$ consists 2 terms. i.e. ½ $x(t)$ and ½ $x(-t)$. The signal ½ $x(t)$ is obta from $x(t)$ by multiplying its strength (amplitude) by ½ at all 't' as shown below in fig. P

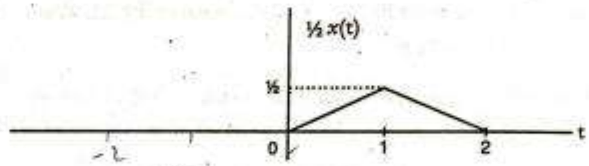


Fig P1.5.1

Similarly, the signal ½ $x(-t)$ is obtained by taking the mirror image of $x(t)$ to ob (-t), then multiplying its strength by ½ at all 't' as shown below in fig P1.5.2

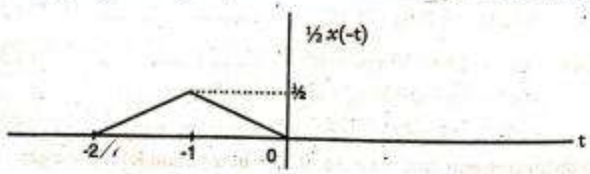


Fig P1.5.2

Adding ½ $x(t)$ and ½ $x(-t)$, we get $x_e(t)$ as shown in fig P1.5.3

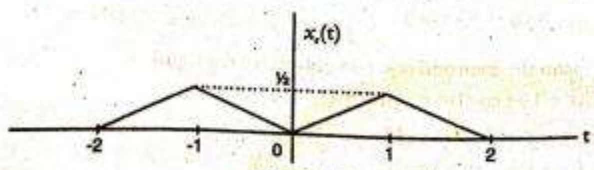
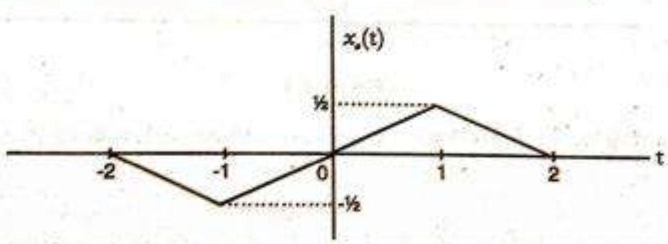


Fig P1.5.3

Similarly, we have $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

$$x_o(t) = \frac{1}{2}x(t) - \frac{1}{2}x(-t)$$

Substracting ½ $x(-t)$ from ½ $x(t)$, we get $x_o(t)$ as shown below in fig P1.5.4



2(a)

i) $x(t) = \cos 2t + \sin 3t$ 2)

$2\pi f_1 = 2$

$f_1 = \frac{2}{2\pi} = \frac{1}{\pi}$ $T_1 = \pi$

$2\pi f_2 = 3$ $f_2 = \frac{3}{2\pi}$ $T_2 = \frac{2\pi}{3}$

$\frac{T_1}{T_2} = \frac{\cancel{\pi}}{2\cancel{\pi}/3} = 3/2$ $\frac{T_1}{T_2} = \frac{3}{2}$

$2T_1 = 3T_2$

$2 \times \pi$ $\cancel{3} \times 2\pi/\cancel{3}$

$\boxed{T = 2\pi}$

Given $x[n] = \cos(\frac{1}{5}\pi n)\sin(\frac{1}{3}\pi n)$

$[\sin A \cos B = \frac{1}{2}\{\sin(A+B) + \sin(A-B)\}]$

$\therefore x[n] = \frac{1}{2}\{\sin\frac{8\pi}{15}n + \sin\frac{2\pi}{15}n\}$

Angular frequency of $\sin\frac{8\pi}{15}n$: $\Omega_1 = \frac{8\pi}{15}$

$\Omega_1 = \frac{2\pi m}{N} = \frac{2\pi.4}{15}$

$m = 4$ & $N = 15$

Fundamental period $N_1 = 15$.

Similarly fundamental period of $\sin\frac{2\pi}{15}n$: $N_2 = 15$

$\frac{N_1}{N_2} = \frac{15}{15} = \frac{1}{1}$

l.c.m. of the denominator $l = 1$

$\therefore$ Period of $x[n]$ $N = N_1(1)$

$= 15(1)$

$= 15$

(b)

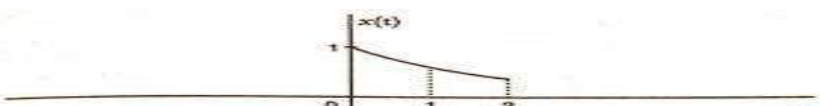


Fig. P2.14.1

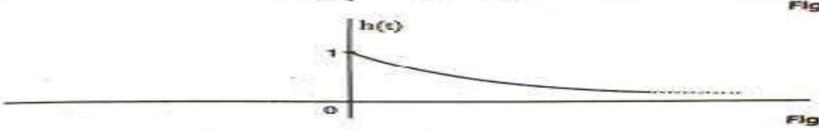


Fig. P2.14.2

We know that $y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

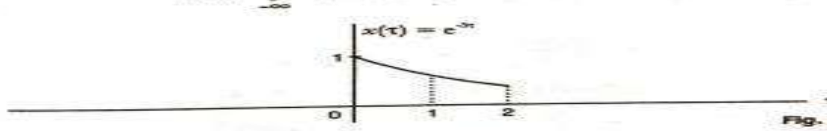


Fig. P2.14.3

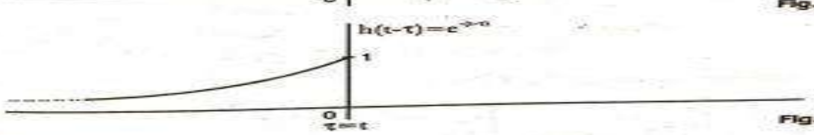


Fig. P2.14.4

When $t<0$

$x(\tau)\, h(t-\tau) = 0$

$\therefore y(t) = 0$; $t<0$

When $t \geq 0$ and $t<2$, (i.e. $0 \leq t < 2$), we get,

$$= \int_0^t e^{-2\tau} e^{-(t-\tau)}\, d\tau$$

$$= \int_0^t e^{-t-2\tau}\, d\tau$$

$$= e^{-t} \int_0^t e^{-2\tau}\, d\tau$$

$$= e^{-t} \left[\frac{e^{-2\tau}}{-2}\right]_0^t$$

$$y(t) = \tfrac{1}{2}\,[1-e^{-2t}]\, e^{-t} \qquad ; 0 \le t < 2$$

When $t > 2$

$$y(t) = \int_0^2 e^{-2\tau} e^{-(t-\tau)}\, d\tau$$

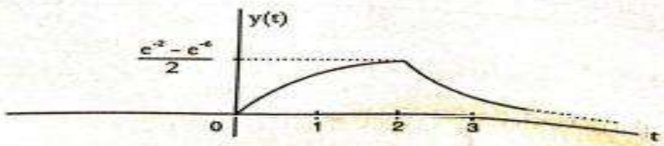
$$= e^{-t} \int_0^2 e^{-2\tau}\, d\tau$$

$$= e^{-t} \left[\frac{e^{-2\tau}}{-2}\right]_0^2$$

$$y(t) = \tfrac{1}{2}\,[1-e^{-4}]\, e^{-t} \qquad ; t > 2$$

$$\therefore\ y(t) = \begin{cases} 0 & ; t < 0 \\ \tfrac{1}{2}(1-e^{-2t})e^{-t} & ; 0 \le t < 2 \\ \tfrac{1}{2}(1-e^{-4})e^{-t} & ; t > 2 \end{cases}$$

The signal $y(t)$ is plotted in Fig. P2.14.5

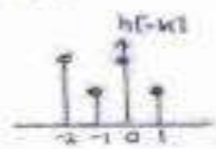


3(a)

3 a)

2

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$



1+1

$$y[n] = \{3, \underset{\uparrow}{8}, 8, 12, 9, 4, 4\} \longrightarrow$$ 4

(b)

3 (b) $h(t) = t^2 u(t)$

$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau = \int_{-\infty}^{t} \tau^2 u(\tau)\, d\tau$$

$$= \int_0^t \tau^2\, d\tau = \left.\frac{\tau^3}{3}\right|_0^t = \underline{\underline{\frac{t^3}{3}}} \quad t \ge 0$$

$$h(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$$s(n) = h(n) * u(n) = \left(\tfrac{1}{2}\right)^n u(n) * u(n)$$

$$= \sum_{K=0}^{n} \left(\tfrac{1}{2}\right)^K = \frac{1-\left(\tfrac{1}{2}\right)^{n+1}}{1-\tfrac{1}{2}}$$

$$= \frac{1-\left(\tfrac{1}{2}\right)^{n+1}}{\tfrac{1}{2}} = \underline{\underline{2\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)}}$$

4(a)

4 a) $\lambda^2 + 6\lambda + 8 = 0 \quad \lambda_1 = -2$ and $\lambda_2 = -4$ (1)

$y_n(t) = C_1 e^{-2t} + C_2 e^{-4t}$ (1)

$y_p(t) = C e^{-t}$. $\frac{d^2 y_p(t)}{dt^2} + 6\frac{dy_p(t)}{dt} + 8y_p(t) = \frac{dx(t)}{dt} + 2x(t)$

$C\left[e^{-t} - 6e^{-t} + 8e^{-t}\right] = -e^{-t} + 2e^{-t}$

$C = \frac{e^{-t}}{3e^{-t}} \quad \boxed{C = \frac{1}{3}}$ (1)

$y(t) = y_n(t) + y_p(t)$

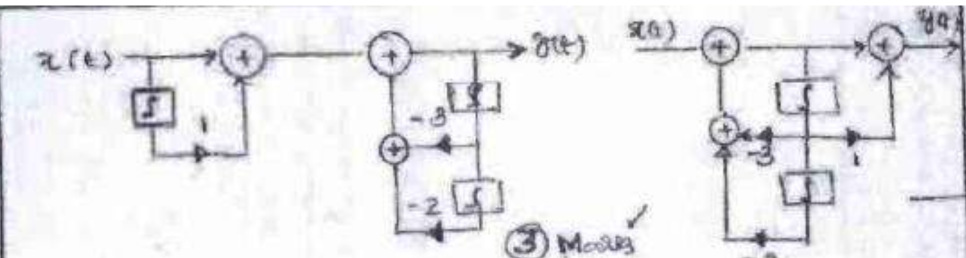
$y(t) = C_1 e^{-2t} + C_2 e^{-4t} + \frac{1}{3} e^{-t}$ (1)

$C_1 = 5$ and $C_2 = -\frac{10}{3}$ (1)

$y(t) = 5e^{-2t} - \frac{10}{3} e^{-4t} + \frac{1}{3} e^{-t}$ (1)

(b)

4 b)



(2)

$y(t) + 3\int y(t)\,dt + 2\iint y(t)\,dt^2 = x(t) + \int x(t)\,dt$ (1)

(c)

A c) $y[n] = \{2, \underset{\uparrow}{3}, 1\}$ → ① marks

(i) Non causal — ①, (ii) $6 < \infty$ Stable → ②

5(a)

5 a) i) $a x_1(t) + b x_2(t) \xleftrightarrow{FT} a X_1(j\omega) + b X_2(j\omega)$ → ③

ii) $x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega)$ → ③

iii) $\frac{d}{dt} x(t) \xleftrightarrow{FT} j\omega X(j\omega)$ → ④

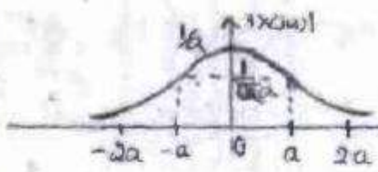
(b)

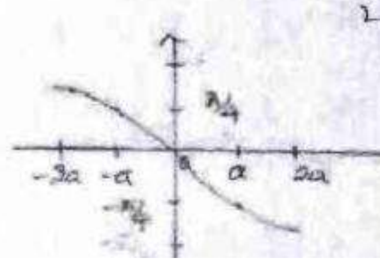
b) $X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \Rightarrow \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt$ ②

$X(j\omega) = \int_0^{\infty} e^{-(a+j\omega)t} dt = \frac{1}{a+j\omega}$ ②

$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$ phase angle $\arg|X(j\omega)| = -\tan^{-1}\left(\frac{\omega}{a}\right)$ ① $\cancel{②}$

1 → ②





Plot is not asked in QP.

6(a)

6 a) $\frac{-j\omega}{(j\omega)^2 + 3j\omega + 2} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2}$ ③

③

$A = 1$ and $B = -2$

$X(j\omega) = \frac{1}{j\omega+1} - \frac{2}{j\omega+2}$ ②

$$x(t) = e^{-t}u(t) - 2e^{-2t}u(t) \qquad (2)$$

(b)

6(b)

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{\sum_{k=0}^{M} b_k (j\omega)^k}{\sum_{k=0}^{N} a_k (j\omega)^k} \qquad (1)$$

$N = 2$ and $M = 1$ $\quad \therefore \; H(j\omega) = \dfrac{1+2j\omega}{2+3j\omega+(j\omega)^2} \qquad (2)$

$$\frac{1+2j\omega}{(j\omega)^2+3j\omega+\underset{1\times 2}{2}} = \frac{A}{j\omega+1} + \frac{B}{j\omega+2} \qquad (2)$$

$A = -1$ and $B = 3$ $\quad (1) \qquad h(t) = -e^{-t}u(t) + 3e^{-2t}u(t) \qquad (2)$

7(a)

Frequency Shift : If $\quad x(n) \xleftrightarrow{\text{DTFT}} X(e^{j\Omega})$

then $\quad y(n) = e^{j\beta n}\, x(n) \xleftrightarrow{\text{DTFT}} Y(e^{j\Omega}) = X(e^{j(\Omega-\beta)})$

Proof: $\quad$ We have, $\quad X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$

$$\therefore\; Y(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} y(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} e^{j\beta n} x(n) \cdot e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\Omega-\beta)n}$$

| Parseval's ...

$$\text{If} \quad x(n) \xleftrightarrow{\text{DTFT}} X(e^{j\Omega})$$

$$\text{then} \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\Omega})|^2 \, d\Omega \quad \text{.........................} \quad (3.19)$$

In eqn. 3.19, $|X(e^{j\Omega})|^2$ is known as 'energy density spectrum' of the signal $x(n)$. We know that the LHS of eqn. 3.19 is the energy of the signal $x(n)$.

Proof: We have $E = \sum_{n=-\infty}^{\infty} |x(n)|^2$

$$= \sum_{n=-\infty}^{\infty} x(n)\, x^*(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n) \left[\frac{1}{2\pi}\int_{2\pi} X^*(e^{j\Omega})\, e^{-j\Omega n}\, d\Omega\right]$$

Changing the order of summation and integration, we get,

$$E = \frac{1}{2\pi}\int_{2\pi} X^*(e^{j\Omega}) \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}\, d\Omega$$

$$= \frac{1}{2\pi.}\int_{2\pi} X^*(e^{j\Omega}) \,.\, X(e^{j\Omega})\, d\Omega$$

$$E = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\Omega})|^2\, d\Omega$$

$$\therefore \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\Omega})|^2\, d\Omega$$

(b)

b) i) $X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+2} e^{-j\Omega n} = \dfrac{\frac{1}{4}}{1-\frac{1}{2}e^{-j\Omega}}$ (3)

ii) $X(\Omega) = \sum_{n=-\infty}^{0} 2\,(3)^n e^{-j\Omega n} = 2\sum_{n=0}^{\infty} \left(\frac{e^{j\Omega}}{3}\right)^n = \dfrac{2}{1-\frac{e^{j\Omega}}{3}}$ (3)

8(a)

8 a) $5e^{j2\Omega}Y(e^{j\Omega}) - 6e^{j\Omega}Y(e^{j\Omega}) + Y(e^{j\Omega}) = \dfrac{e^{j\Omega}}{e^{j\Omega}-0.8}$ (2)

$Y(e^{j\Omega})\left[5e^{j2\Omega} - 6e^{j\Omega} + 1\right] = \dfrac{e^{j\Omega}}{e^{j\Omega}-0.8}$

$Y(e^{j\Omega})\; 5\left[e^{j2\Omega} - \frac{6}{5}e^{j\Omega} + \frac{1}{5}\right] = \dfrac{e^{j\Omega}}{e^{j\Omega}-0.8}$

$\dfrac{Y(e^{j\Omega})}{e^{j\Omega}} = \dfrac{1}{5\,(e^{j\Omega}-0.8)(e^{j2\Omega}-1.2e^{j\Omega}+0.2)}$ (2)

$\dfrac{Y(e^{j\Omega})}{e^{j\Omega}} = \dfrac{1}{5\,(e^{j\Omega}-0.8)(e^{j\Omega}-0.2)(e^{j\Omega}-1)}$ (2)

$y[n] = -1.67\,(0.8)^n + 0.417\,(0.2)^n + 1.25\,(1)^n$ (2)

(b)

8 b) $H(e^{j\Omega}) = 1 + \dfrac{2}{1-\frac{1}{2}e^{-j\Omega}} + \dfrac{1}{1+\frac{1}{2}e^{-j\Omega}}$ (2)

$H(e^{j\Omega}) = \dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \dfrac{-\frac{1}{4}e^{-j2\Omega} + \frac{1}{2}e^{-j\Omega} + 4}{1-\frac{1}{4}e^{-2j\Omega}}$ (4)

$\therefore\; y[n] - \frac{1}{4}y[n-2] = -\frac{1}{4}x[n-2] + \frac{1}{2}x[n-1] + 4x[n]$ — (2)

9(a)

The range of 'r' for which this condition is satisfied is known as "region of convergence" (ROC).

The set of $|z|$ for which the summation in eqn. 5.9 converges is known as ROC.

Property 1 : The ROC of X(z) consists of a ring in the Z-plane centered about origin.

Property 2 : The ROC does not contain any poles.

Property 3 : If $x[n]$ is of finite duration, then the ROC is the entire Z-plane, except possibly $z=0$ and/or $|z|=\infty$.

Property 4 : If $x[n]$ is a right-sided sequence, and if the circle $|z|=r_0$ is in the ROC, then all finite values of 'z' for which $|z|>r_0$ will also be in the ROC. i.e. the ROC of a right-handed signal is outside the circle.

Property 5 : If $x[n]$ is a left-sided sequence and if the circle $|z|=r_0$ is in the ROC, then all values of 'z' for which $0 < |z| < r_0$ will also be in the ROC. That is the ROC of a left handed signal is inside the circle.

Property 6 : If $x[n]$ is two sided and if the circle $|z|=r_0$ is in the ROC, then the ROC will consist of a ring in the Z-plane that includes the circle $|z|=r_0$ is in the ROC, then the ROC will consist of a ring in the Z-plane that includes the circle $|z|=r_0$. That is the ROC of a two sided signal is a concentric ring.

Property 7 : If the Z-transform X(z) of $x[n]$ is rational, then its ROC is bounded by poles or extends to infinity.

(b)

b)

$$X(z) = \sum_{n=-\infty}^{\infty} -b^n u(-n-1)\, z^{-n} = -\sum_{n=-\infty}^{1} (b z^{-1})^n \qquad (1)$$

$$X(z) = 1 - 1 - \sum_{n=\infty}^{1} (b^{-1} z)^n = 1 - \sum_{n=0}^{\infty} \left(\frac{z}{b}\right)^n \qquad (2)$$

$$X(z) = \frac{-z}{b-z} \quad \text{OR} \quad \frac{z}{z-b} \qquad |z| < |b| \qquad (2)$$

(c)

If $\quad x(n) \xleftrightarrow{z} X(z) \quad$ with RC

then $\quad x(n-n_0) \xleftrightarrow{z} z^{-n_0} X(z) \quad$ with RC addition

Proof: $Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

$$Z[x(n-n_0)] = \sum_{n=-\infty}^{\infty} x(n-n_0)\, z^{-n}$$

Put $n-n_0 = l$

$$= \sum_{l=-\infty}^{\infty} x(l).\, z^{-(l+n_0)}$$

$$= \sum_{l=-\infty}^{\infty} x(l).\, z^{-l}.\, z^{-n_0}$$

$$= z^{-n_0} \sum_{l=-\infty}^{\infty} x(l)\, z^{-l}$$

$$Z[x(n-n_0)] = z^{-n_0} X(z)$$

10(a)

10 a) $\dfrac{z^2}{z^2 - \frac{3}{2}z + \frac{1}{2}}$ $\qquad X(z) = \dfrac{z^2}{(z-1)(z-\frac{1}{2})}$ (1)

-1 & $-\frac{1}{2}$

$$\frac{X(z)}{z} = \frac{z^2}{z(z-1)(z-\frac{1}{2})} = \frac{z}{(z-1)(z-\frac{1}{2})} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}} \quad -(2)$$

$A = 2$ and $B = -1$ $\longrightarrow$ (1)

Case(i) $|z| > 1$ $\qquad X(z) = 2\dfrac{z}{z-1} - \dfrac{z}{z-\frac{1}{2}}$

$$x(n) = 2u(n) - \left(\tfrac{1}{2}\right)^n u(n) \quad (2)$$

Case(ii) $\frac{1}{2} < |z| < 1$

$$x(n) = 2(-1)^n u(-n-1) - \left(\tfrac{1}{2}\right)^n u(n) \quad (2)$$

8

(b)

b) $Y(z) - \frac{5}{6} z^{-1} Y(z) + \frac{1}{6} z^{-2} Y(z) = X(z)$ ——— (2)

$$\frac{Y(z)}{X(z)} = \frac{z^2}{z^2 - \frac{5}{6} z + \frac{1}{6}} = \frac{z^2}{(z-\frac{1}{2})(z-\frac{1}{3})} \quad \text{— (2)}$$

$-\frac{1}{2} \times + \frac{1}{3}$

$$\frac{H(z)}{z} = \frac{z}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z-\frac{1}{3}} \qquad A = 3, \; B = -2 \quad (2)$$

$$H(z) = 3\,\frac{z}{z-\frac{1}{2}} - 2\,\frac{z}{z-\frac{1}{3}} \quad (2)$$

$$h(n) = \left[3\left(\tfrac{1}{2}\right)^n - 2\left(\tfrac{1}{3}\right)^n\right] u(n) \quad (2)$$

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