CBCS Scheme

USN 1CR16EC198

15EC43

Fourth Semester B.E. Degree Examination, June/July 2018

Control Systems

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Write the differential equations for the mechanical system shown in Fig.Q1(a) and obtain F-V analogy. **(06 Marks)**

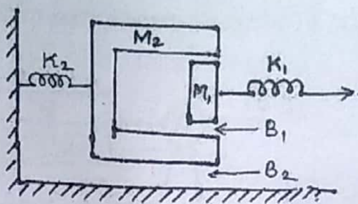


Fig.Q1(a)

b. Differentiate between open loop control system and closed-loop control system. **(06 Marks)**

c. For the rotational system shown in Fig.Q1(c). Draw torque-voltage analogous circuit. **(04 Marks)**

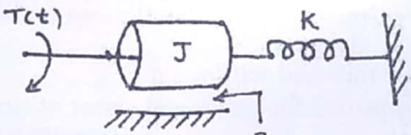


Fig.Q1(c)

OR

2 a. Reduce the following block diagram of the system shown on Fig.Q2(a) into a single equivalent block diagram by block diagram reduction rules. **(06 Marks)**

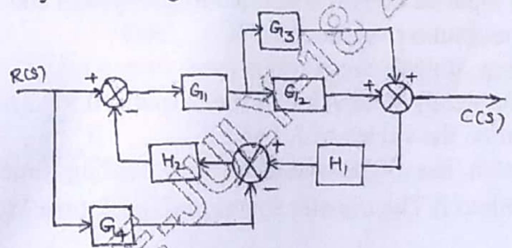


Fig.Q2(a)

b. Find $\frac{C(s)}{R(s)}$ for the following signal flow graph. [Refer Fig.Q2(b)] **(06 Marks)**

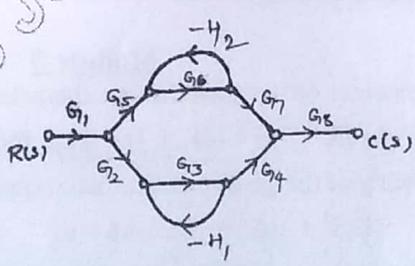


Fig.Q2(b)

c. For the following circuit write the signal flow graph. [Refer Fig.Q2(c)] **(04 Marks)**

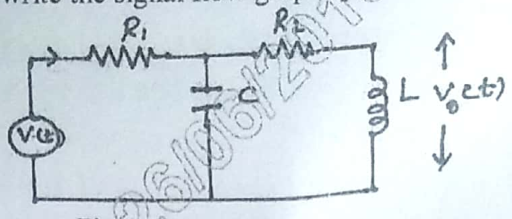


Fig.Q2(c)

Module-2

3 a. For the system shown in Fig.Q3(a). Find the : i) system type ii) static error constants k_p, k_v and k_a and iii) the steady state error for an input r(t) = 3 + 2t. **(06 Marks)**

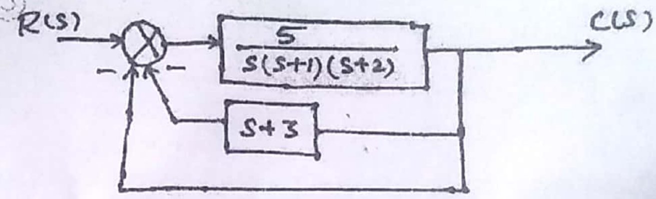


Fig.Q3(a)

b. Find the step-response, C(t) for the system described by $\frac{C(s)}{R(s)} = \frac{4}{S+4}$. Also find the time constant, rise time and settling time. **(05 Marks)**

c. Derive the equation for steady state error of simple closed loop system. **(05 Marks)**

OR

4 a. A second order system is represented by the transfer function.

$$\frac{Q(s)}{I(s)} = \frac{1}{JS^2 + fS + K}$$

A step input of 10 Nm is applied to the system and the test results are :

i) maximum overshoot = 6%

ii) time at peak overshoot = 1sec

iii) the steady state value of the output is 0.5 radian.

Determine the values of J, f and K. **(06 Marks)**

b. A system has 30% overshoot and settling time of 5 seconds for on unit step input. Determine: i) The transfer function ii) peak time 't_p' iii) output response (assume e_{ss} as 2%). **(06 Marks)**

c. Write the general block diagrams of the following :

i) PD type of controller

ii) PI type of controller. **(04 Marks)**

Module-3

5 a. Determine the ranges of 'K' such that the characteristic equation :

$S^3 + 3(K + 1)S^2 + (7K + 5)S + (4K + 7) = 0$ has roots more negative than S = –1. **(06 Marks)**

b. Check the stability of the given characteristic equation using Routh's method.

$S^6 + 2S^5 + 8S^4 + 12S^3 + 20S^2 + 16S + 16 = 0$. **(06 Marks)**

c. Mention few limitations of Routh's criterion. **(04 Marks)**

OR

6 a. Sketch the complete root locus of system having, $G(s)H(s) = \dfrac{K}{S(S+1)(S+2)(S+3)}$. **(12 Marks)**

b. Consider the system with $G(S)H(s) = \dfrac{K}{S(S+1)(S+4)}$. Find whether $S = -2$ point is on root locus or not using angle condition. **(04 Marks)**

Module-4

7 a. The open loop transfer function of a system is $G(s) = \dfrac{K}{s(1+s)(1+0.1s)}$. Determine the values of K such that i) gain margin = 10 dB ii) phase margin = 24°. Use Bode plot. **(10 Marks)**

b. Derive the expression for resonant peak 'M_r' and corresponding resonant frequency 'W_r' for a second–order underdamped system in frequency response analysis. **(06 Marks)**

OR

8 a. Sketch the Nyquist plot for a system with the open-loop transfer function :

$$G(s)H(s) = \frac{k(1+0.5s)(1+s)}{(1+10s)(s-1)}$$

Determine the range of values of 'k' for which the system is stable. **(08 Marks)**

b. Write the polar plot for the following open-loop transfer function :

$$G(S)H(s) = \frac{1}{1+0.1s}.$$

(04 Marks)

c. Explain Nyquist stability criteria. **(04 Marks)**

Module-5

9 a. Explain spectrum analysis of sampling process. **(06 Marks)**

b. Explain how zero–order hold is used for signal reconstruction. **(04 Marks)**

c. Find the state–transition matrix for $A = \begin{bmatrix} 0 & -1 \\ +2 & -3 \end{bmatrix}$. **(06 Marks)**

OR

10 a. Obtain an appropriate state model for a system represented by an electric circuit as shown in Fig.Q10(a).

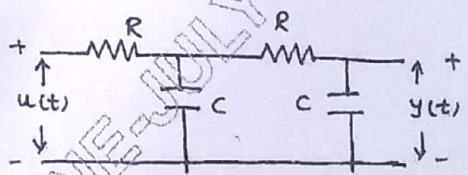


Fig.Q10(a) **(06 Marks)**

b. A linear time invariant system is characterized by the homogeneous state equation :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Compute the solution of homogeneous equation, assume the initial state vector.

$$X_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

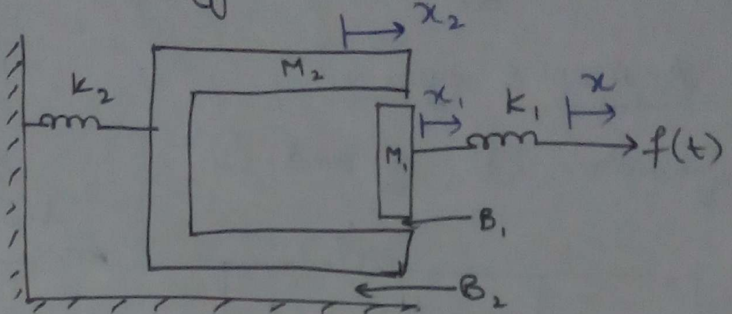
(06 Marks)

c. State the properties of state transition matrix. **(04 Marks)**

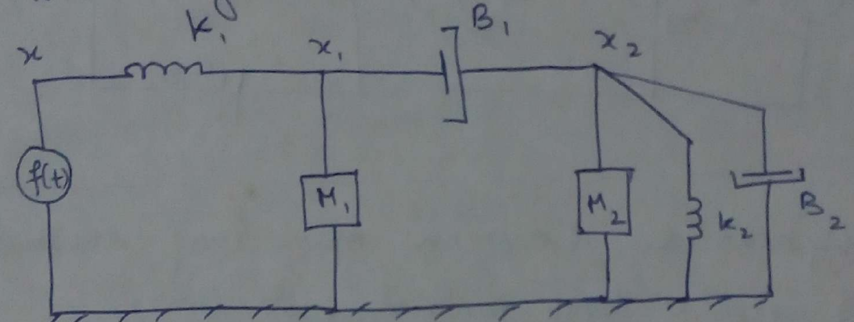
* * * * *

MODULE - 1

Q 1) a) Write the differential equations for the mechanical system shown in Fig Q1(a) and obtain F-V analogy.



Solution: nodal diagram:



Differential equations:

$$F(t) = K_1(x - x_1) \quad — (1)$$

$$0 = M_1 \frac{d^2(x_1)}{dt^2} + K_1(x_1 - x) + B_1 \frac{d(x_1 - x_2)}{dt} \quad — (2)$$

$$0 = M_2 \frac{d^2(x_2)}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_1 \frac{d(x_2 - x_1)}{dt} \quad — (3)$$

F-V: $F \rightarrow V$, $M \rightarrow L$, $D \rightarrow R$, $K \rightarrow 1/C$, $x \rightarrow q$

$$V(t) = \frac{1}{C_1}(q - q_1) \quad — (4)$$

$$0 = L_1 \frac{d^2(q_1)}{dt^2} + R_1 \frac{d(q_2 - q_1)}{dt} - \frac{1}{C_1}(q_1 - q) \quad — (5)$$

$$0 = L_2 \frac{d^2 q_2}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{1}{C_2} q_2 + R_1 \frac{d(q_2 - q_1)}{dt} \quad — (6)$$

substitute, $i = \frac{dq}{dt}$

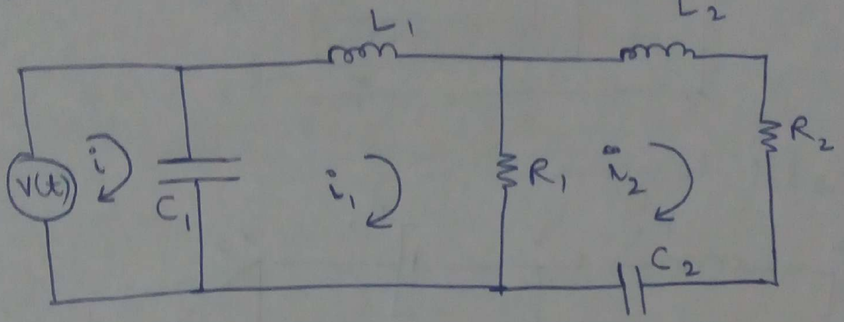
$$v(t) = \frac{1}{C_1}\int (i - i_2)\,dt \quad — (7)$$

$$0 = L_1 \frac{di_1}{dt} + R_1(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i)\,dt \quad — (8)$$

$$0 = L_2 \frac{di_2}{dt} + R_2(i_2) + \frac{1}{C_2}\int i_2\,dt + R_1(i_2 - i_1) \quad — (9)$$

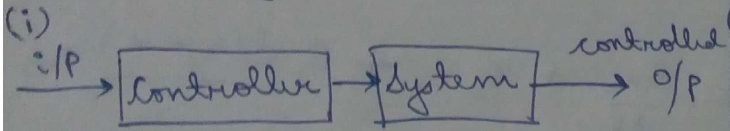
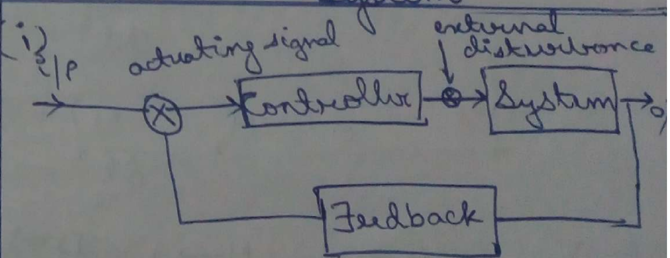
F-V Analogous system:

from equations, (7), (8) and (9),

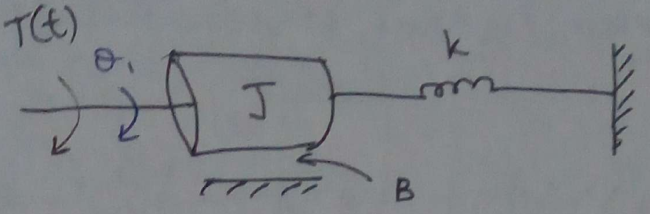


Q 2) b) Differentiate between open loop control system and closed loop control system.

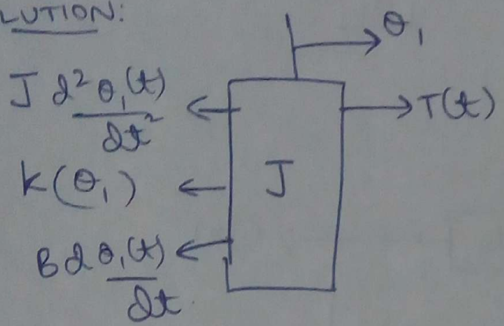
SOLUTION:

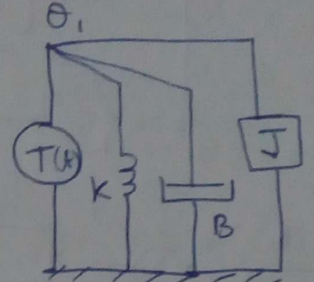
Open loop control system	Closed loop control system
(i) 	(i) 
(ii) Construction is simple	(ii) Construction is complex.
(iii) Cheaper when compared to closed loop systems.	(iii) Costlier when compared to open loop systems.
(iv) More stable.	(iv) Less stable.
(v) Optimisation is not possible.	(v) Optimisation is possible.
(vi) Accuracy depends on calibration.	(vi) More accurate due to feedback.

Q 1) c) For the rotational system shown in Fig. Q1(c). Draw torque-voltage analogous circuit.



SOLUTION:





$$T(t) = k\theta_1 + J\frac{d^2\theta_1}{dt} + B\frac{d\theta_1}{dt} \quad -(1)$$

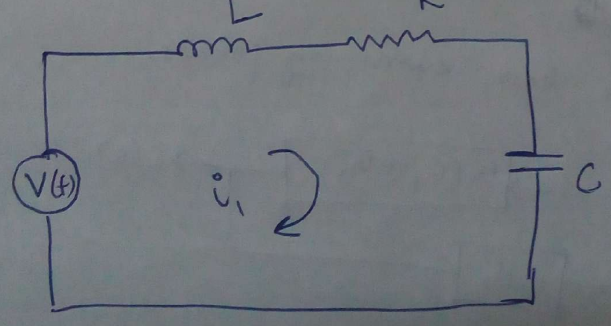
T-V $\quad T \to V, \quad J \to L, \quad D \to R, \quad k \to 1/C, \quad \theta(t) \to q(t)$

$$V(t) = \frac{1}{C}(q_1) + L\frac{d^2 q_1}{dt^2} + R\frac{dq_1}{dt} \quad -(2)$$

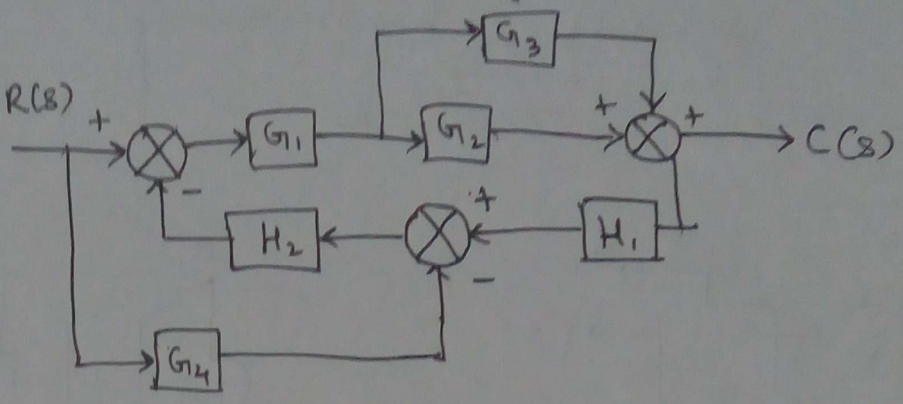
substitute, $i = \frac{dq}{dt}$

$$V(t) = \frac{1}{C}\int i_1\,dt + L\frac{di_1}{dt} + Ri_1 \quad -(3)$$

T-V Analogous circuit from (3),

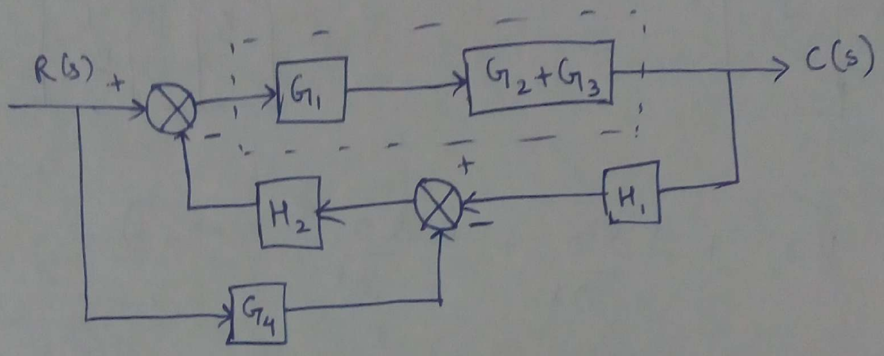


Q 2) a) Reduce the following block diagram of the system shown on Fig. Q2(a) into a single equivalent block diagram by block diagram reduction rules.

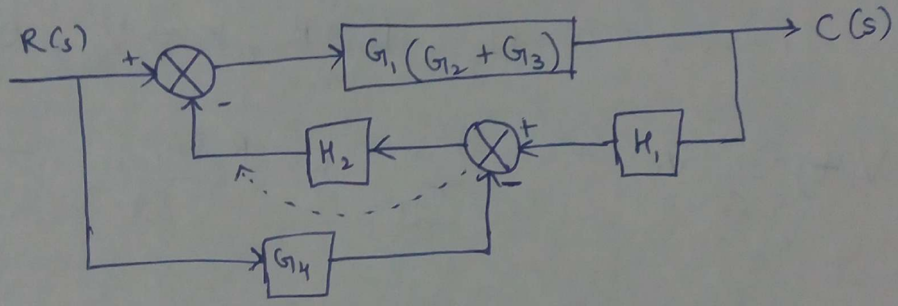


SOLUTION:

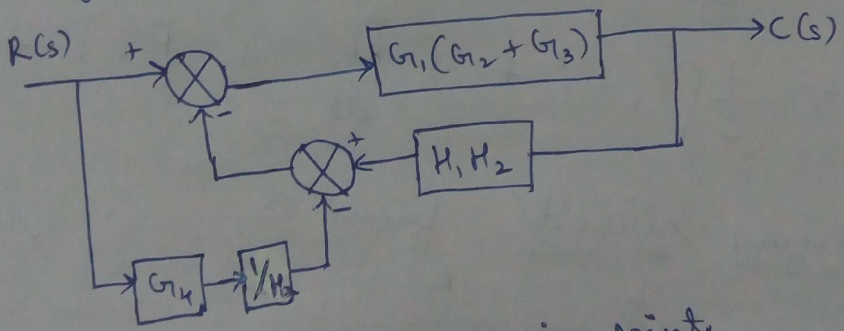
(i)



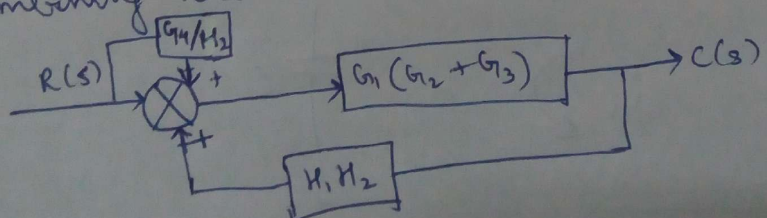
(ii)



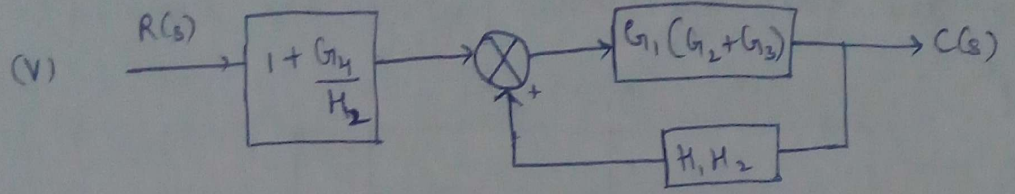
(iii) Moving the summer before H_2



(iv) Combining the two summing points.

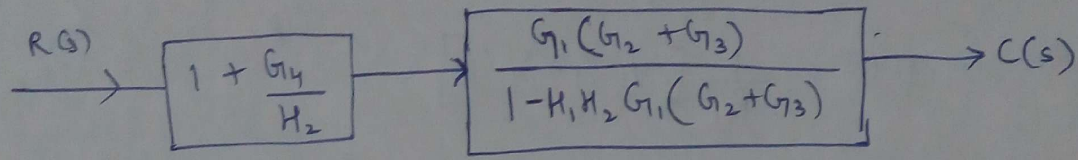


(v)

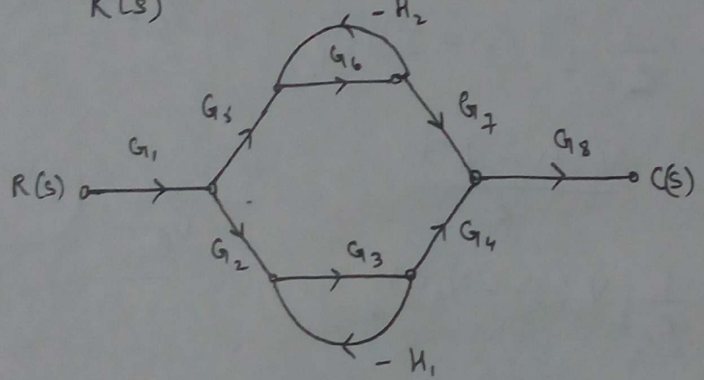


(vi) $$\frac{C(s)}{R(s)} = \frac{G(s)}{1-G(s)H(s)}$$ (positive feedback)

$$\frac{C(s)}{R(s)} = \left(1+\frac{G_4}{H_2}\right)\left(\frac{G_1G_2+G_1G_3}{1-H_1H_2G_1(G_2+G_3)}\right)$$



Q 2) b) Find $\frac{C(s)}{R(s)}$ for the following signal flow graph



SOLUTION:

→ Forward paths:

(i) $G_1G_5G_6G_7G_8$

(ii) $G_1G_2G_3G_4G_8$

→ Loops:

(i) $-G_6H_2$

(ii) $-G_3H_1$

→ Two non-touching loops.

(i) $G_3G_6H_1H_2$

$$T.F = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta}$$

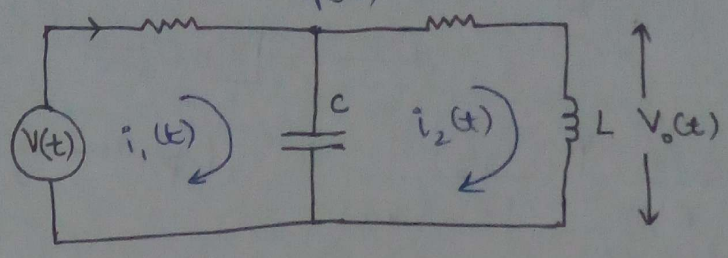
$$\Delta = 1+\{G_6H_2+G_3H_1\}+G_3G_6H_1H_2$$

$$\Delta_1 = 1+H_1G_3$$

$$\Delta_2 = 1+H_2G_6$$

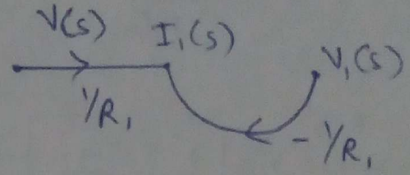
$$\boxed{T.F = \frac{G_1G_5G_6G_7G_8(1+H_1G_3) + G_1G_2G_3G_4G_8(1+H_2G_6)}{1+\{G_6H_2+G_3H_1\}+G_3G_6H_1H_2}}$$

Q 2 c) For the following circuit, write the signal flow graph.



SOLUTION:

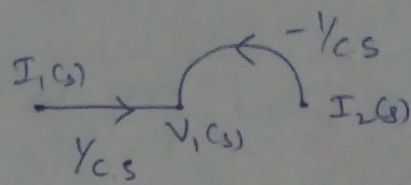
$$i_1(t) = \frac{V(t) - V_1(t)}{R_1}$$



Laplace transform:

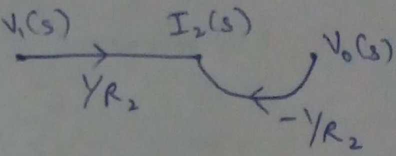
$$I_1(s) = \frac{1}{R_1}\left[V(s) - V_1(s)\right]$$

$$V_1(t) = \frac{1}{C}\int i_1(t) - i_2(t)\, dt$$



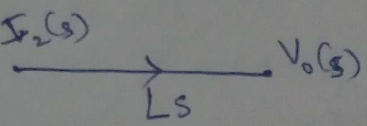
$$V_1(s) = \frac{1}{Cs}\left[I_1(s) - I_2(s)\right]$$

$$i_2(t) = \frac{V_1(t) - V_o(t)}{R_2}$$

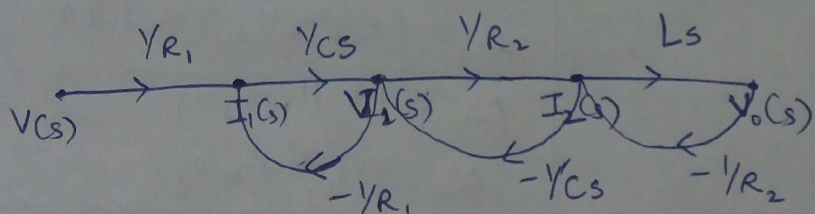


$$I_2(s) = \frac{1}{R_2}\left[V_1(s) - V_o(s)\right]$$

$$V_o(t) = L\frac{di_2(t)}{dt}$$



$$V_o(s) = Ls\, I_2(s)$$



MODULE - 2

Q 3) a) For the system shown in Fig Q 3(a), find the:
(i) system type (ii) static error constants K_p, K_v and K_a
and (iii) the steady state error for an input $x(t) = 3 + 2t$

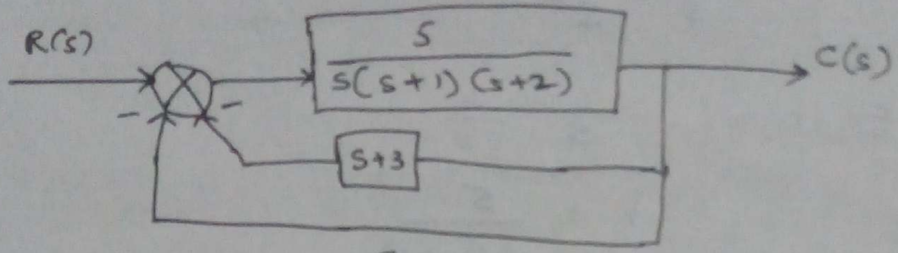
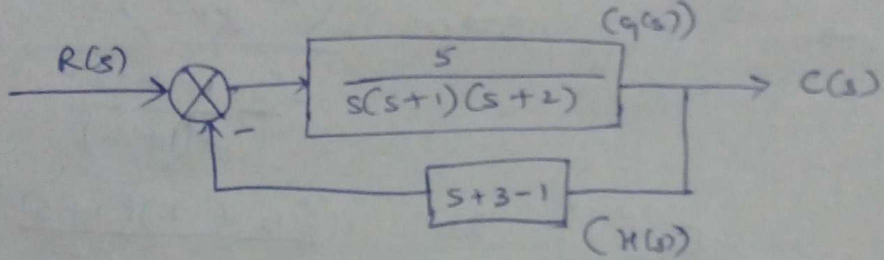


Fig. Q3(a)

SOLUTION:



(ii) $\rightarrow$ $K_p = \underset{s\to 0}{\text{lt}}\ G(s)H(s)$

$$K_p = \underset{s\to 0}{\text{lt}}\ \frac{5}{s(s+1)\cancel{(s+2)}}\ \cancel{(s+2)}$$

$$\boxed{K_p = \infty}$$

$\rightarrow$ $K_v = \underset{s\to 0}{\text{lt}}\ sG(s)H(s)$

$$K_v = \underset{s\to 0}{\text{lt}}\ \frac{\cancel{s}(5)}{\cancel{s}(s+1)\cancel{(s+2)}}\ \cancel{(s+2)}$$

$$\boxed{K_v = 5}$$

$\rightarrow$ $K_a = \underset{s\to 0}{\text{lt}}\ s^2G(s)H(s)$

$$K_a = \underset{s\to 0}{\text{lt}}\ \frac{s^{\cancel{2}}(5)}{\cancel{s}(s+1)\cancel{(s+2)}}\ \cancel{(s+2)}$$

$$\boxed{K_a = 0}$$

(iii) $r(t) = 3 + 2t$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

$$R(s) = \frac{3}{s} + \frac{2}{s^2}$$

$$E(s) = \frac{\frac{3}{s} + \frac{2}{s^2}}{1 + \frac{5}{s(s+1)}}$$

$$\lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{\cancel{s}\left(\frac{3s+2}{\cancel{s^2}}\right)}{\frac{s(s+1)+5}{\cancel{s}(1+s)}}$$

$$e_{ss} = \lim_{s \to 0} \frac{(3s+2)(1+s)}{s(s+1)+5}$$

$$e_{ss} = \frac{(2)(1)}{5} = 2/5$$

Q 3.) b.) Find the step response, c(t) for the system described by $\frac{C(s)}{R(s)} = \frac{4}{s+4}$. Also find the time constant, rise time and settling time.

SOLUTION:

$$\frac{C(s)}{R(s)} = \frac{4}{s+4}$$

$$C(s) = R(s)\left(\frac{4}{s+4}\right)$$

$$C(s) = \frac{4}{(s+4)s}$$

$$C(s) = \frac{A}{s} + \frac{B}{s+4}$$

$$C(s) = \frac{4}{(s+4)s} = \frac{A(s+4) + B(s)}{s(s+4)}$$

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$$4 = A(s+4) + Bs$$

$$4 = As + 4A + Bs$$

$$A + B = 0$$

$$\boxed{B = -1}$$

$$4A = 4$$

$$\boxed{A = 1}$$

$$C(s) = \frac{1}{s} - \frac{1}{s+4}$$

Taking inverse laplace transform,

$$c(t) = 1 - e^{-4t}, \quad t \geq 0$$

Time constant, $T = \frac{1}{a}$

$$C(s) = R(s)G(s) = R(s)\frac{4}{s+4}$$

in general, $G(s) = \frac{a}{s+a}$

$$T = 1/4 = 0.25s$$

rise time, $t_r = \frac{2.2}{a}$

$$t_r = \frac{2.2}{4} = 0.55\,s$$

settling time; $t_s = \frac{4}{a}$

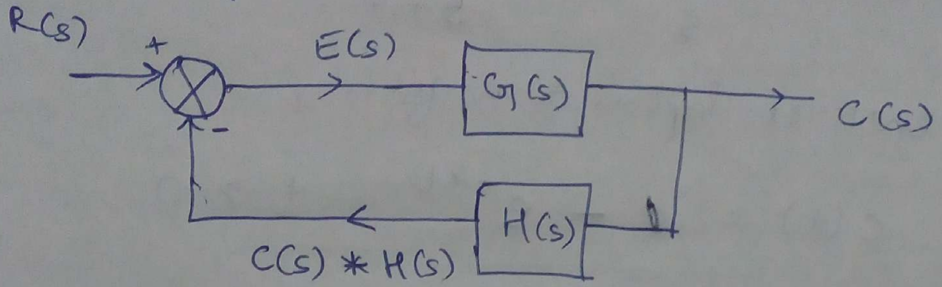
$$t_s = \frac{4}{4} = 1s$$

Q 3> c> Derive the equation for steady state error of simple closed loop system.

SOLUTION:

$$\text{Transfer function} = \frac{P(s)}{Q(s)} = \frac{k(s+z_1)(s+z_2)\dots}{s^N(s+P_1)(s+P_2)\dots}$$

Consider a simple closed loop system,



where $C(s) * H(s)$ is the feedback signal

$E(s)$ is the actuating signal or error signal.

$$E(s) = R(s) - C(s)H(s)$$

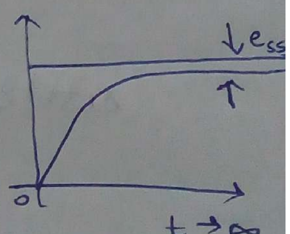
$$C(s) = E(s) * G(s)$$

$$E(s) = R(s) - E(s)G(s)H(s)$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

e_{ss} - steady state error which occurs when $t \to \infty$.

$\therefore$ final value theorem can be applied.



$$F(s) = L\{f(t)\}$$

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$e(t) = L^{-1}[E(s)] = L^{-1}\left[\frac{R(s)}{1+G(s)H(s)}\right]$$

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

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$$\boxed{e_{ss} = \lim_{s \to 0} s\left(\frac{R(s)}{1+G(s)H(s)}\right)}$$

Q 4) a) A second order system is represented by the transfer function : $\frac{\theta(s)}{I(s)} = \frac{1}{Js^2+fs+K}$

A step input of 10Nm is applied to the system and the test results are :

(i) maximum overshoot = 6%

(ii) time at peak overshoot = 1sec

(iii) the steady state value of output is 0.5 radian

Determine the values of J, f and K.

SOLUTION: Given : $\% M_p = 6\%$ $\quad M_p = 0.06$

$t_p = 1s.$ $\quad I(s) = 10Nm$

$e_{ss} = 0.5.$

$$\frac{\theta(s)}{I(s)} = \frac{1/J}{s^2 + \frac{f}{J}s + \frac{K}{J}}$$

$$\omega_n^2 = \frac{k}{J} \qquad \omega_n = \sqrt{\frac{k}{J}}$$

$$2\xi\omega_n = \frac{f}{J}$$

$$2\xi\sqrt{\frac{k}{J}} = \frac{f}{J}$$

$$\xi = \frac{1}{2}\frac{f}{J}\sqrt{\frac{J}{k}} = \frac{1}{2}\frac{f}{\sqrt{JK}}$$

$$e_{ss} = \lim_{s \to 0} s\,\theta(s) = 0.5$$

$$G(s) = \frac{10/J}{s\left(s^2 + \frac{f}{J}s + \frac{K}{J}\right)}$$

$$e_{ss} = \lim_{s \to 0} s\,G(s) = \lim_{s \to 0} \frac{10/J}{s^2 + \frac{f}{J}s + \frac{K}{J}}$$

$$0.5 = \frac{10/\cancel{J}}{K/\cancel{J}}$$

$$K = \frac{10}{0.5}$$

$$\boxed{K = 20}$$

$$M_p = 0.06 = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = -2.8134$$

$$\xi^2\pi^2 = (2.8134)^2\left[1-\xi^2\right]$$

$$\xi^2\pi^2 = 7.9153\left[1-\xi^2\right]$$

$$\xi^2\left[\pi^2 + 7.9153\right] = 7.9153$$

$$\xi^2 = 0.445$$

$$\boxed{\xi = 0.667.}$$

$$t_p = 1s$$

$$1 = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\omega_n = \frac{\pi}{\sqrt{1 - 0.445}} = 4.217 \text{ rad/s.}$$

$$\boxed{\omega_n = 4.217 \text{ rad/s}}$$

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$$\omega_n = \sqrt{\frac{k}{J}}$$

$$J = \frac{k}{\omega_n^2} = \frac{20}{17.783} = 1.125$$

$$\boxed{J = 1.125}$$

$$\xi = \frac{1}{2}\frac{f}{\sqrt{Jk}}$$

$$f = 2\xi\sqrt{kJ}$$

$$\boxed{f = 6.3277}$$

Q 4) b.) A system has 30% overshoot and settling time of 5 seconds for an unit step input. Determine (i) the transfer function (ii) peak time 't_p' (iii) output response (assume e_{ss} as 2%).

<u>SOLUTION:</u>

$$M_p = 0.30$$

$$t_s = 5s$$

$$e_{ss} = 2\% = 0.02$$

$$t_s = \frac{4}{\xi\omega_n} \quad \text{(for } \pm 2\% \text{ tolerance band)}$$

$$M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$0.30 = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

$$\frac{-\xi\pi}{\sqrt{1-\xi^2}} = -1.20397$$

$$\xi^2 = \frac{1.4496}{\pi^2 + 1.4496} = 0.128$$

$$\boxed{\xi = 0.3578}$$

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$$5 = \frac{4}{\xi \omega_n}$$

$$\omega_n = \frac{4}{5 \times 0.3578} = 2.235$$

(i) $$T.F = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$T.F. = \frac{4.997}{s^2 + 1.5993s + 4.997}$$

(ii) $$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$t_p = \frac{\pi}{2.235\sqrt{1-0.128}} = 1.505s$$

(iii) $$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left[\sin(\omega_d t + \theta)\right]$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 1.2048^c = 69.03^\circ$$

$$c(t) = 1 - \frac{e^{-0.7996t}}{0.9338}\left[\sin(2.087t + 1.2048\right.$$

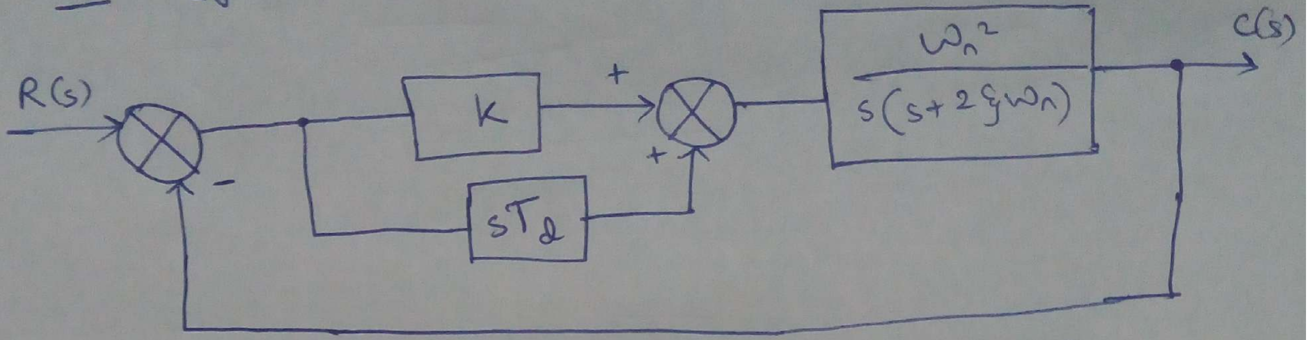
Q 4.) c.) Write the general block diagrams of the following:

(i) PD type of controller

(ii) PI type of controller

SOLUTION:

(i) PD type of controller

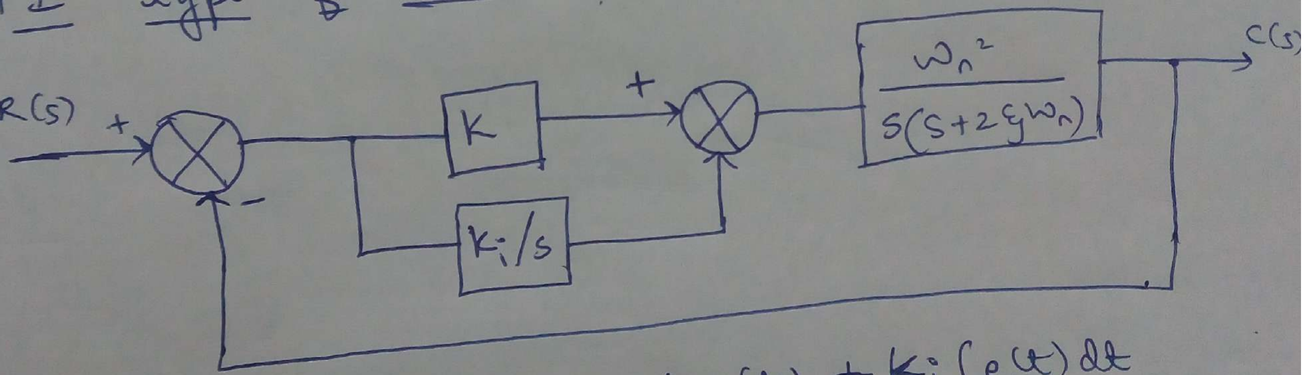


Output of controller $= Ke(t) + T_d \frac{de(t)}{dt}$

Taking Laplace, $= K\,E(s) + T_d\, s\, E(s)$

$$= E(s)\left[K + sT_d\right]$$

(ii) PI type of controller



Output of controller $= Ke(t) + K_i \int e(t)\,dt$

Taking Laplace $= K\,E(s) + \frac{K_i}{s} E(s) = E(s)\left[K + \frac{K_i}{s}\right]$

Module - 3

5 a) Given,
Char. eq : $s^3 + 3(k+1)s^2 + (7k+5)s + (4k+7) = 0$

$$\sigma = -1$$

$$s \rightarrow s_1 - 1$$

$$(s_1 - 1)^3 + 3(k+1)(s_1 - 1)^2 + (7k+5)(s_1 - 1) + 4k + 7 = 0$$

$$[\ (a-b)^3 = a^2 - 3a^2 b + 3ab^2 - b^3\ ]$$

$$(s_1^3 - 3s_1^2 + 3s_1 - 1) + 3(k+1)(s_1^2 - 2s_1 + 1)$$
$$+ (7k+5)s' - (7k+5) + (4k+7) = 0$$

Char. eq.:

$$s_1^3 + s_1^2(3k) + s_1(k+2) + 14 = 0.$$

+	s_1^3	1	$k+2$
+	s_1^2	$3k$	14
+	s_1	$\frac{3k(k+2)-14}{3k}$	0
+	s_1^0	14	

$$\frac{3k(k+2)-14}{3k} > 0$$

$$3k^2 + 6k - 14 > 0$$

$$k > 1.38 \quad \& \quad k > -3.38$$

Ranges of 'k' : $\boxed{k > 1.38}$ & $\boxed{k > -3.38}$

b) Char. eq:

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 16s + 16 = 0$$

s^6	1	8	20	16
s^5	2	12	16	
s^4	2	12	16	
s^3	0	0	0	

$$A(s) = 2s^4 + 12s^2 + 16$$

$$\frac{dA(s)}{ds} = 8s^3 + 24s.$$

$A(s) = 0$

$2s^4 + 12s^2 + 16 = 0$

$S_{1,2} = \pm 2i \quad , \quad S_{3,4} = \pm\sqrt{2}i$

$$\boxed{S = +2i, \sqrt{2}i, -\sqrt{2}i, -2i}$$

2 Single pair of roots of poles lie on imaginary axis hence, the given system is <u>marginally Stable</u> / <u>Conditionally stable</u>.

c) <u>Limitations of Routh's criterion</u>:

- It is valid only if the characteristic equation is algebraic
- If any co-efficient of the characteristic equation is complex or contains power of 'e' this criterion can not be applied.
- It gives information about how many roots are lying in the RHS of s-plane ; values of the roots are not available. Also it cannot distinguish between real and complex roots.

(OR)

6 a) Given, $G(s).H(s) = \dfrac{K}{s(s+1)(s+2)(s+3)}$

<u>Step 1</u>: Poles : $0, -1, -2, -3$

zeros : NIL

$P = 4 \qquad z = 0$

No. of root locus branches,

For $P > z$ $\qquad N = P = 4$

Starting point : 0 , -1 , -2 , -3

Terminal Point : ∞ , ∞ , ∞ , ∞

Step 2: Asymptotic lines

Centroid , $\sigma = \dfrac{\Sigma RPP - \Sigma RPZ}{P - Z} = \dfrac{(0-1-2-3)-0}{4-0}$

$= -\dfrac{3}{2} = -1.5$

$q = 0, 1, \ldots (P - Z - 1)$

$q = 0, 1, 2, 3$

$\theta = \dfrac{(2q+1)\,180^\circ}{P-Z}$

For $q = 0$, $\theta_1 = 45^\circ$

$q = 1$, $\theta_2 = 135^\circ$

$q = 2$, $\theta_3 = 225^\circ$

$q = 3$, $\theta_4 = 315^\circ$

Step 3: Break away points

Char. eq : $1 + G(S).H(S) = 0$.

$1 + \dfrac{K}{S(S+1)(S+2)(S+3)} = 0$.

$S(S+1)(S+2)(S+3) + K = 0$.

Char. eq : $S^4 + 6S^3 + 11S^2 + 6S + K = 0$.

$K = -S^4 - 6S^3 - 11S^2 - 6S$

$\dfrac{dK}{dS} = -4S^3 - 18S^2 - 22S - 6 = 0$.

$S_1 = -2.61$	$S_2 = -0.381$	$S_3 = -1.5$
$K = 0.999$	$K = 0.99$	$K = -0.5625$

Since K is positive for $\boxed{s = -2.61}$ and $\boxed{s = -0.381}$ are valid BAP.

$\frac{180^\circ}{n}$ ($n \rightarrow$ no. of branches approaching BAP)

step 4: Intersection of Root locus with imaginary axis

Char. eq: $s^4 + 6s^3 + 11s^2 + 6s + K = 0$

+	s^4	1	11	K
+	s^3	6	6	0
+	s^2	10	K	0
+	s	$\frac{60-6K}{10}$	0	0
+	s^0	K		

$K > 0$ & $\frac{60-6K}{10} > 0$

$60 - 6K > 0$

$K < 10$

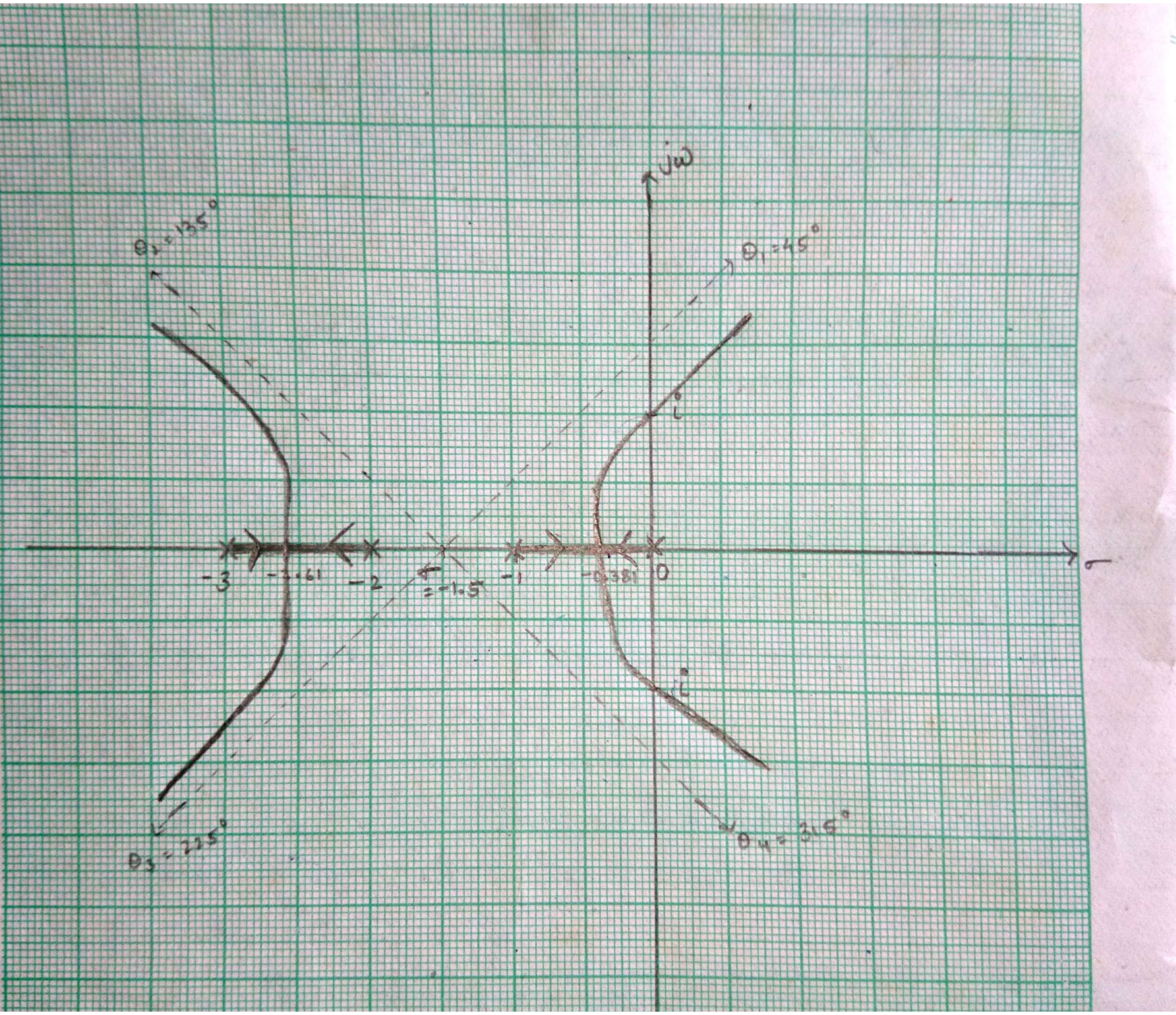
$\boxed{0 < K < 10}$

$K_{max} = 10$

$A(s) = 10s^2 + K_{max}$

$10s^2 + 10 = 0$

$\boxed{s_{1,2} = i, -i}$



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Module -4

7 a) Given, $G(s) = \dfrac{K}{s(1+s)(1+0.1s)}$

Step 1:

take $K = 1$

$$G(j\omega)\cdot H(j\omega) = \frac{1}{j\omega(j\omega+1)(0.1j\omega+1)}$$

$\dfrac{1}{j\omega} \longrightarrow \omega_c = \text{NIL}$

$\dfrac{1}{j\omega+1} \Longrightarrow \omega_{c1} = \dfrac{1}{1} = 1$ rad/s.

$\dfrac{1}{0.1j\omega+1} \longrightarrow \omega_{c2} = \dfrac{1}{0.1} = 10$ rad/s.

Step 2: Magnitude table

Term	ω_c (rad/s)	Slope (dB/dec)	Chang in slope (dB/dec)
$1/j\omega$	–	-20	-20
$\dfrac{1}{j\omega+1}$	1	-20	-40
$\dfrac{1}{0.1j\omega+1}$	10	-20	-60

Step 3: Starting point,

$$|Y(j\omega)| = 20\log\left|\frac{1}{j\omega}\right|_{@\,\omega=0.1}$$

$$= 20\log(1) - 20\log 0.1$$

$$\boxed{|Y(j\omega)| = 20\text{ dB}}$$

Step 4: Phase plot

$$\angle G(j\omega) = \phi$$

$$\phi = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(0.1\omega).$$

ω	ϕ (deg)
0.1	-96.28
1	-140.71
2	-164.74
3	-178.28
4	-187.76
5	-195.25
7	-206.861
8	-21[illegible].28
10	-219.28

From plot,

$$\omega_{gc} = 1 \text{ rad/s}$$

$$\omega_{pc} = 3.2 \text{ rad/s}$$

$$\phi_{gc} = -140^\circ$$

$$PM = 180 + \phi_{gc}$$

$$PM = 40^\circ$$

$$GM = (-1 \times -21) = 21 \text{ dB}.$$

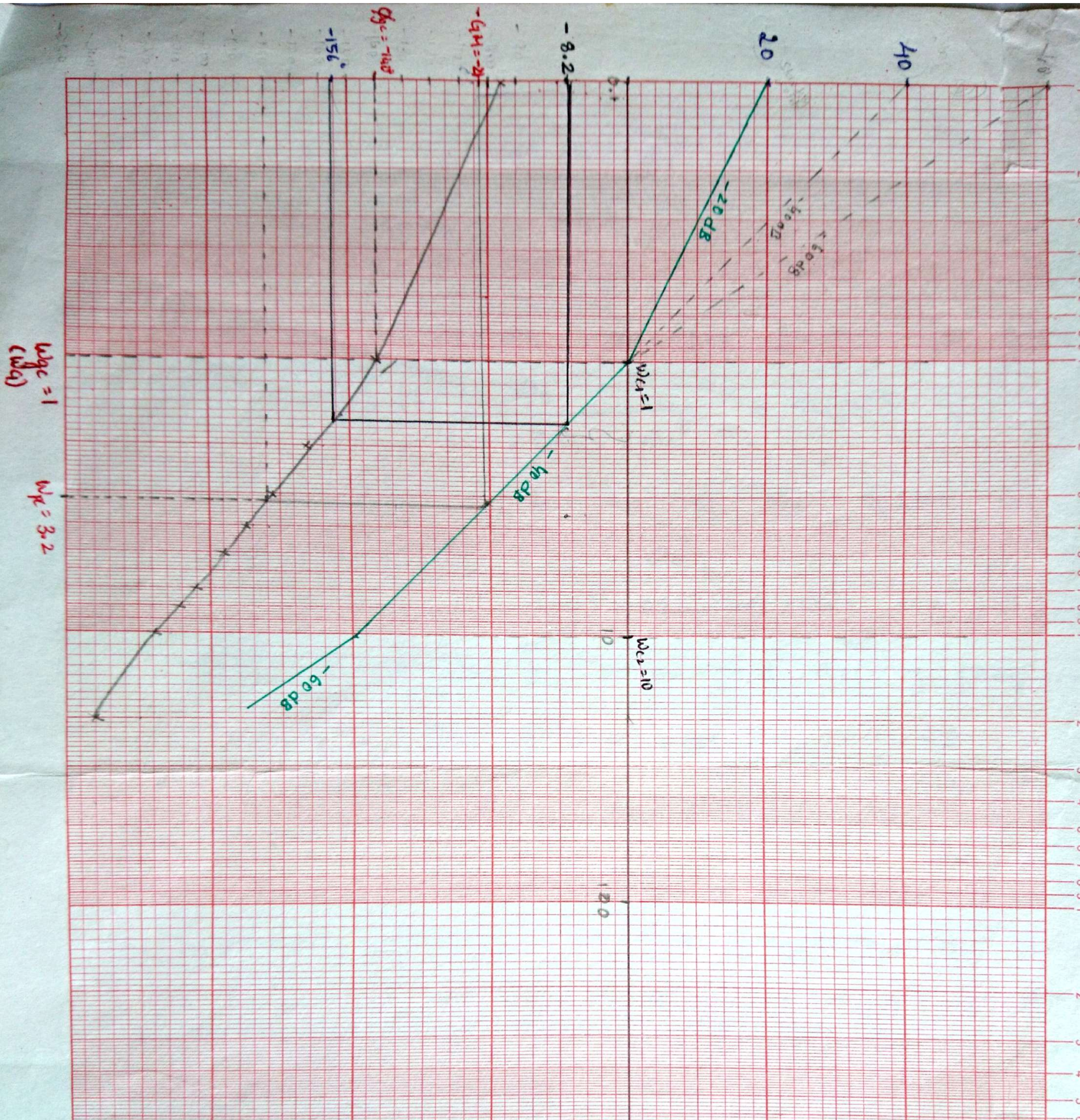
i) GM = 10 dB.

For $K = 1$, GM = 21 dB.

For GM = 10 dB,

$$20 \log_{10} K = 11$$

$$\boxed{K = 3.548}$$



20
40
-8.2
-156°
-20 dB
-40 dB
-60 dB
ωc1=1
ωc2=10
ωgc =1
ωpc = 3.2
10

ii) $PM = 24^\circ$.

$$\phi_{gc} = PM - 180^\circ = -156^\circ$$

$$20 \log_{10} K = +8.2$$

$$\boxed{K = 2.57}$$

b) Second order system transfer function is given as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \& \quad T(j\omega).$$

Sinusoidal transfer function of the system is

$$\frac{C}{R}(j\omega) = T(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2} = \frac{1}{(1-u^2) + j2\xi u}$$

where $u = \omega/\omega_n$ is normalized driving frequency.

$$|T(j\omega)| = M = 1/\sqrt{(1-u^2)^2 + (2\xi u)^2} \rightarrow (1)$$

$$\angle T(j\omega) = \phi = -\tan^{-1}(2\xi u / 1-u^2)$$

The frequency where M has a peak value is known as resonant frequency. At this frequency, the slope of the magnitude curve is zero. Let ω_r be the resonant frequency and $u_r = \omega_r/\omega_n$ be the normalized resonant frequency. Then

$$\left.\frac{dM}{du}\right|_{u=u_r} = -\frac{1}{2}\frac{[-4(1-u_r^2)u_r + 8\xi^2 u_r]}{[(1-u_r^2)^2 + (2\xi u_r)^2]^{3/2}} = 0$$

gives,

$$4u_r^3 - 4u_r + 8\xi^2 u_r = 0 \quad \text{or} \quad u_r = \sqrt{1-2\xi^2}$$

ie, $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$

Maximum value of magnitude known as resonant peak is given by (from eq (1)).

$$\boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

8 b) Given, $G(s).H(s) = \frac{1}{1+0.1s}$.

Magnitude : $|GH(j\omega)| = \frac{1}{\sqrt{1+(0.1\omega)^2}}$

Angle : $\phi \angle GH(j\omega) = -\tan^{-1}(0.1\omega)$.

ω	Magnitude $\lvert GH(j\omega)\rvert$	Angle $\phi \angle GH(j\omega)$ (deg)
0	1	0
2	0.9805	-11°
5	0.8944	-26.56°
10	0.7071	-45°
40	0.2425	-75.95°
∞	0	-90°

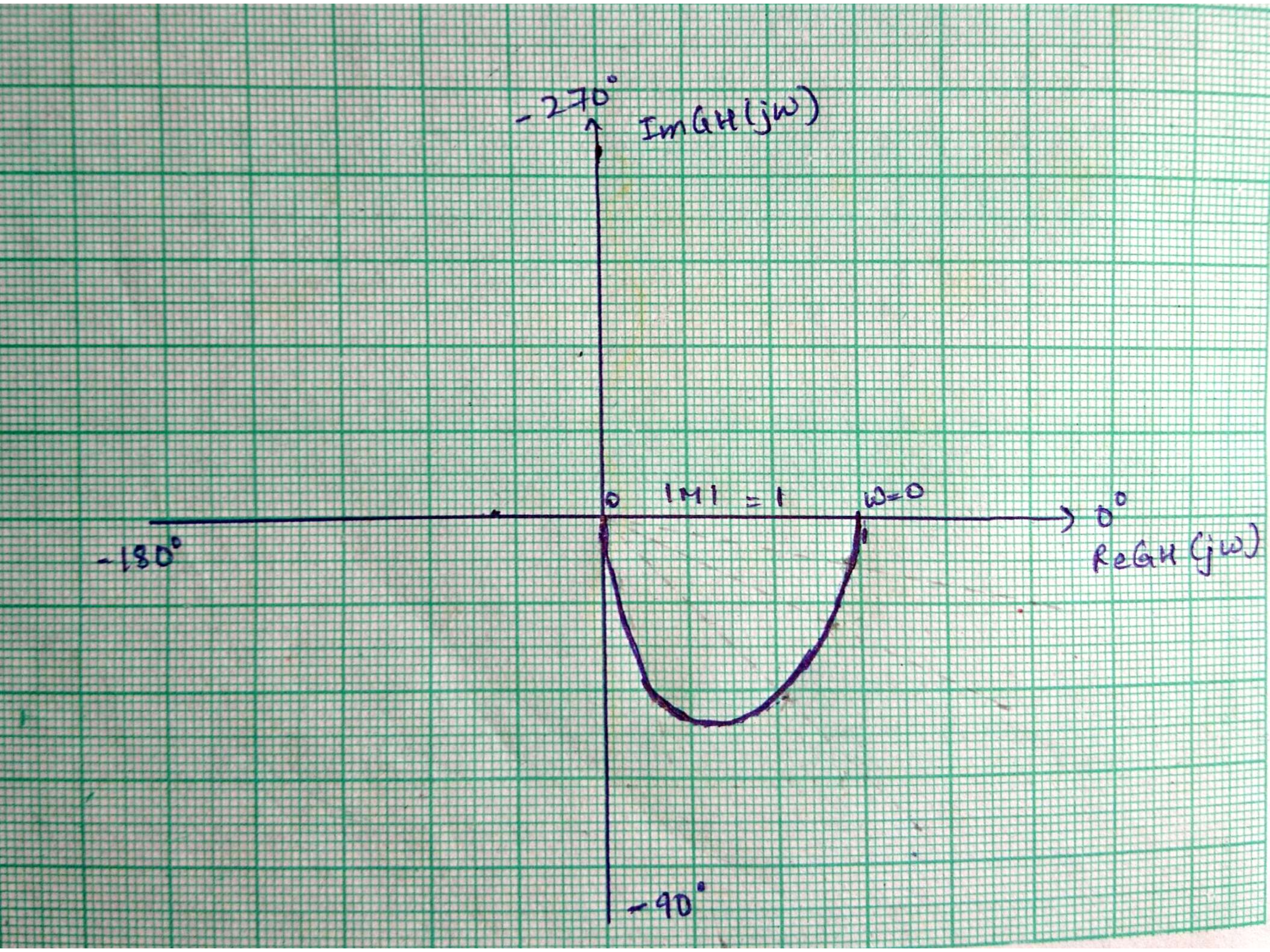
c) Nyquist stability criteria :

- The Nyquist plot allows us also to predict the stability and performance of a closed loop system by observing its open-loop behavior.

  $F(s) = 1 + G(s).H(s)$ wher $G(s).H(s)$ is OLTF

- Poles of CLTF $(1+G(s)H(s))$
  = Poles of $G(s).H(s)$ (OLTF)

- Zeros of $1+G(s).H(s)$ = close loop poles of the system.



-270°
ImGH(jw)
0
|M| = 1
w=0
0°
ReGH(jw)
-180°
-90°