CBCS - SCHEME

FOURTH SEMESTER B.E. DEGREE EXAMINATION, JUNE | JULY 2018

SIGNALS & SYSTEMS

MODULE - 1

Ques:- 1 a) Sketch the even & odd part of signals shown in fig.Q1 a) - (i) and fig. Q1(a)-(ii)

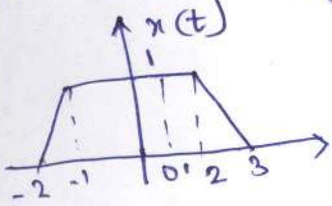


fig Q1(a)-(i)

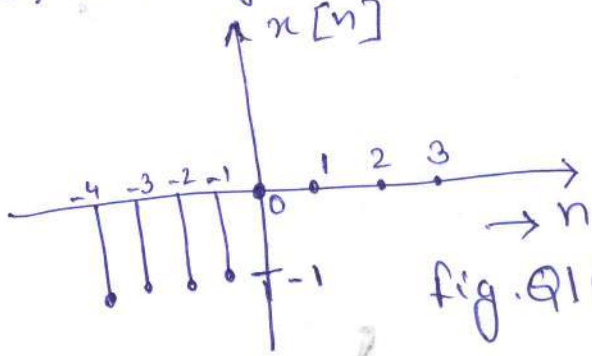
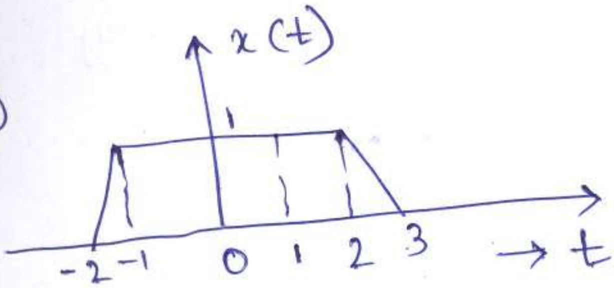


fig.Q1(a)-(ii)

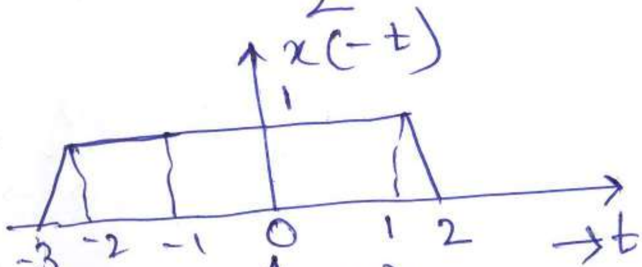
Sol^n:- (i)

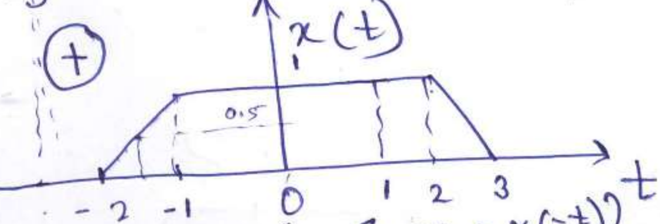


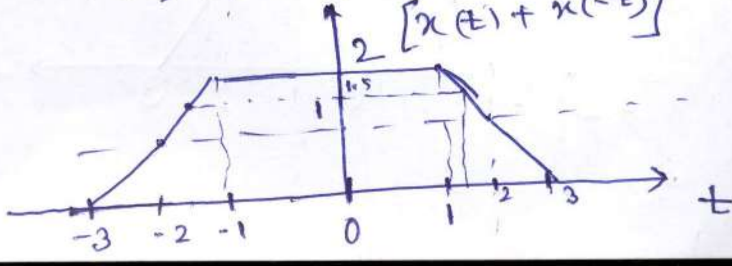
Even & odd parts are given by:-

$$x_e(t) = \frac{x(t) + x(-t)}{2}, \quad x_o(t) = \frac{x(t) - x(-t)}{2}$$

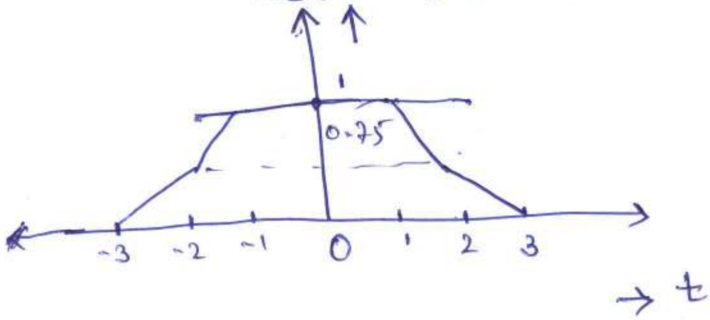
Even part :-





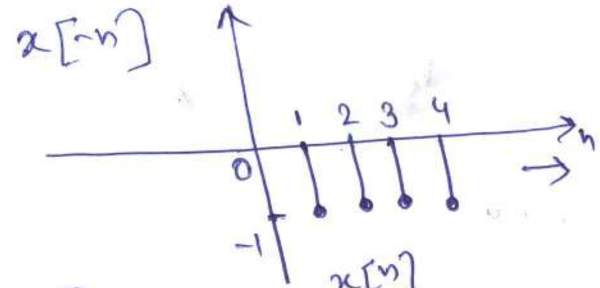


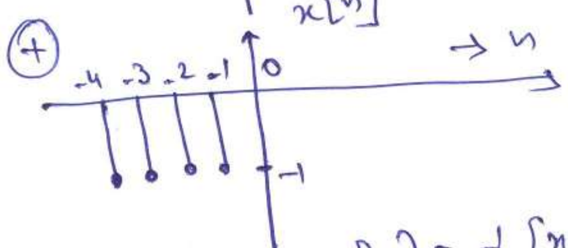
$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



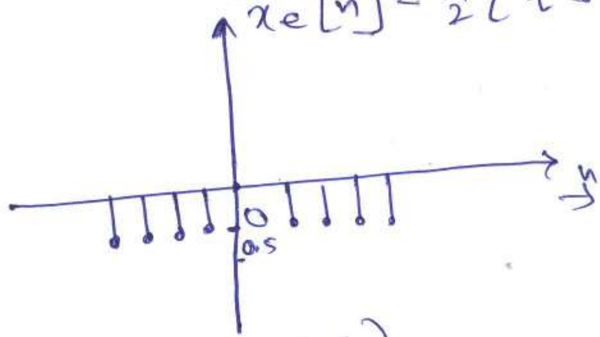
ii) Even part

~~Odd part~~

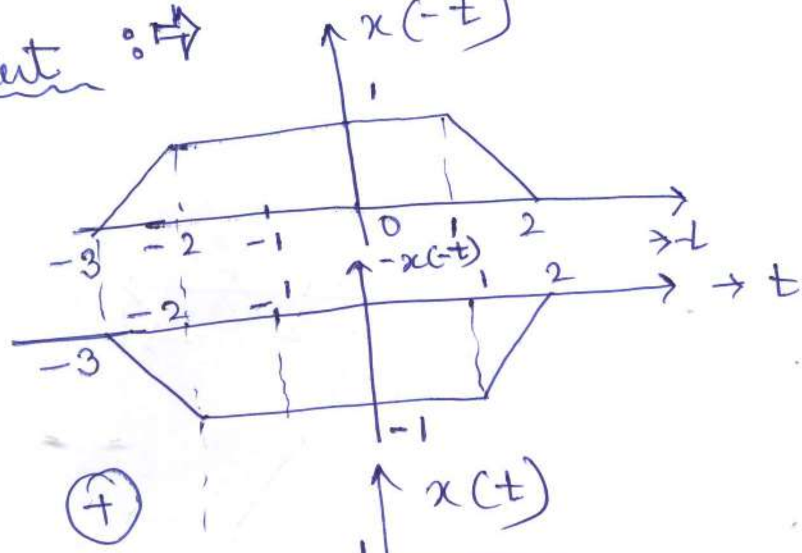


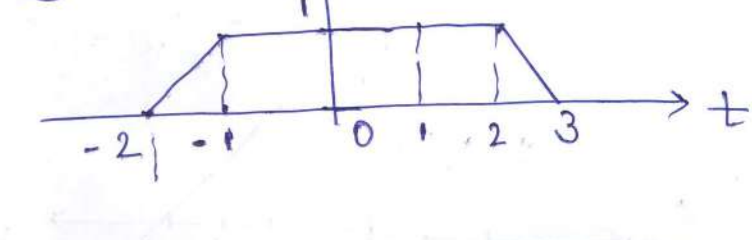


$x_e[n] = \frac{1}{2}[x[n] + x[-n]]$

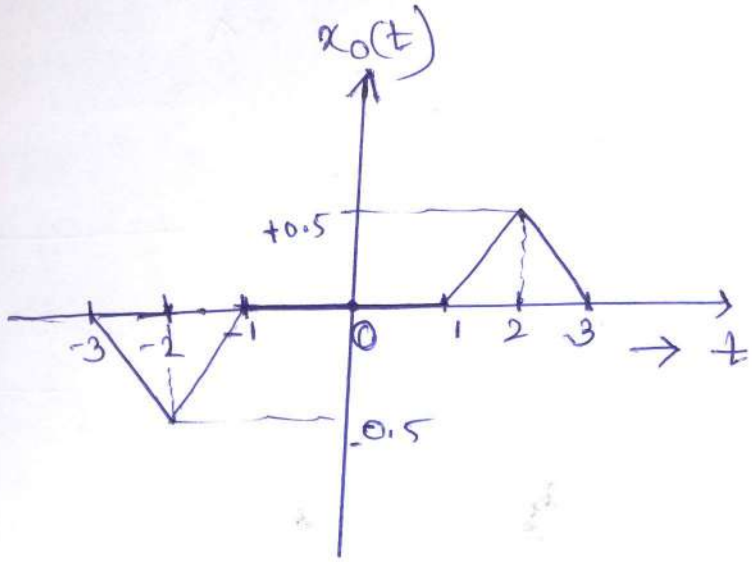


i) Odd Part :⇒

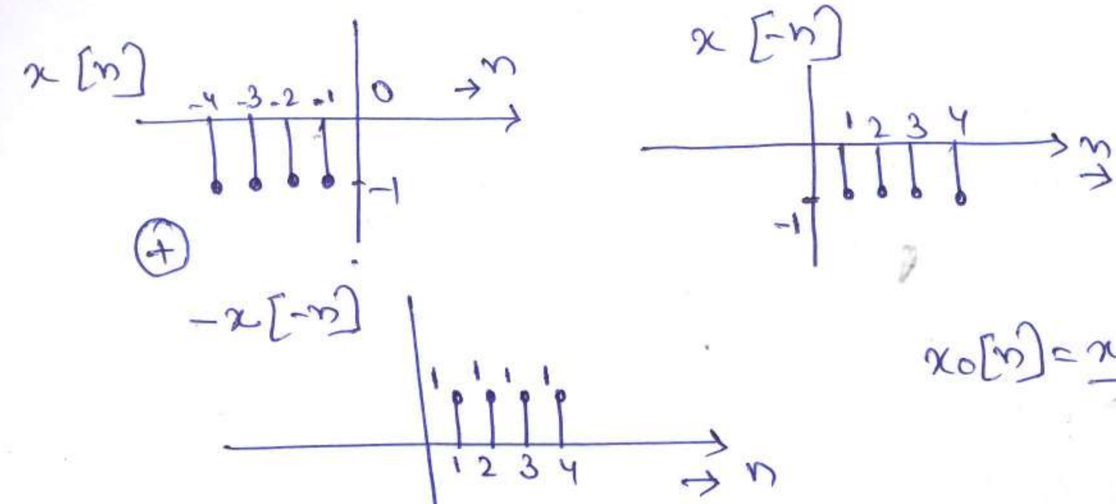




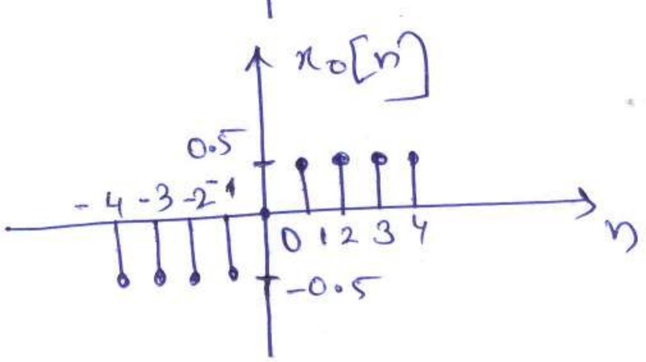
$$x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$



ii)



$$x_o[n] = \frac{x[n] - x[-n]}{2}$$



b) The trapezoidal pulse $x(t)$ shown in fig. Q1(b) is applied to a differentiator defined by

$$y(t) = \frac{d}{dt}\left[x(t)\right]$$

Determine the resulting output $y(t)$ & total Energy of $y(t)$

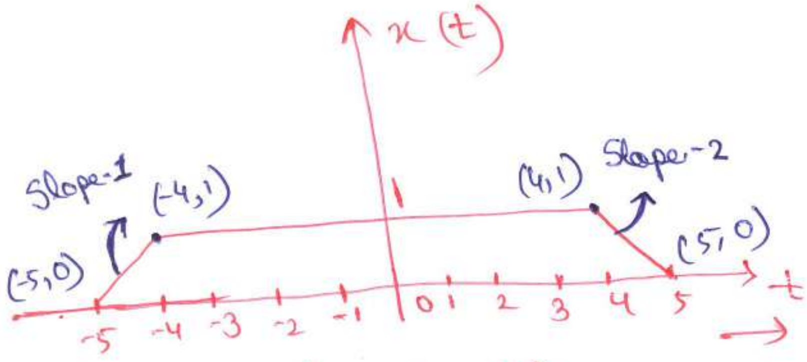


fig. Q 1(b)

Sol^n:-

$$x(t) \rightarrow \boxed{\text{Differentiator}} \rightarrow y(t) \rightarrow f = ?$$

for slope 1 :-

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$x(t) - 0 = \left(\frac{-4+5}{1-0}\right)(t+5)$$

$$\boxed{x(t) = t + 5, \quad -5 \le t \le -4}$$

for Slope 2 :-

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$x(t) - 1 = \left(\frac{0-1}{5-4}\right)(t-4)$$

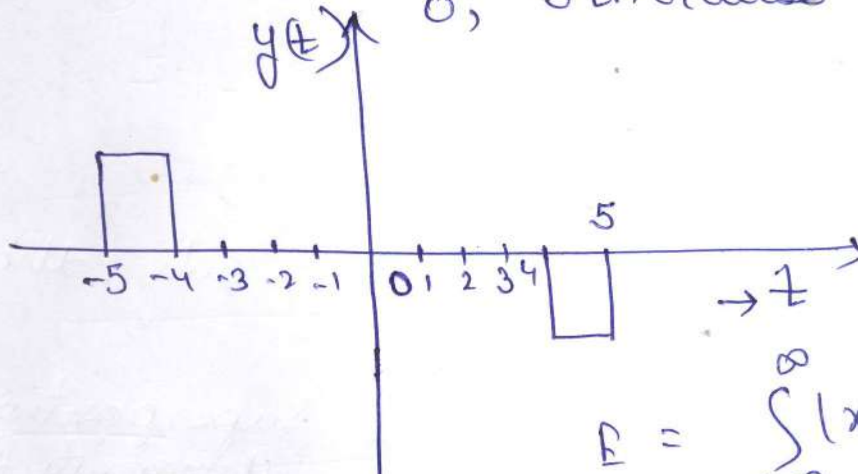
$$x(t) - 1 = -t + 4$$

$$\boxed{x(t) = 5 - t \quad , \quad 4 \le t \le 5}$$

$$x(t) = \begin{cases} 5-t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t+5, & -5 < t \le -4 \\ 0, & \text{Otherwise} \end{cases}$$

$$\therefore \; y(t) = \frac{d}{dt}[x(t)]$$

$$y(t) = \begin{cases} -1, & 4 \le t \le 5 \\ 0, & -4 \le t \le 4 \\ 1, & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$



$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$\therefore E_y(t) = \int_{-5}^{-4} (1)^2 dt + \int_{4}^{5} (-1)^2 dt$$

$$= [t]_{-5}^{-4} + [t]_{4}^{5} = -4-(-5)+(5-4)$$

$$= 2J$$

$$\boxed{E = 2J}$$ Ans

Ques:-2) a. Two systems are described by

i) $y(n) = (n+1)x(n)$, (ii) $y(t) = x(t)+10$. Test the systems for (i) Memory (ii) Causality (iii) linearity (iv) Time-invariance & (v) Stability

Sol^n:- (i) $y(n) = (n+1)x(n)$

(i) Memory :⇒

$y(0) = (0+1)x(0) = x(0)$

$y(1) = 2x(1)$

$y(-1) = 0$

$y(-2) = -x(-2)$

System doesn't depend on pasts values. Therefore, the system is memoryless.

(ii) Causality :⇒ $y(n) = (n+1)x(n)$

It depends on present values only, therefore the system is Causal.

(iii) linearity :⇒ According to Superposition & Homogenity Rule.

$y_1(n) = (n+1)x_1(n)$

$y_2(n) = (n+1)x_2(n)$

$y(n) = y_1(n)+y_2(n) = (n+1)x_1(n)+(n+1)x_2(n)$ — ①

$y(n) = y(n) = (n+1)[x_1(n)+x_2(n)]$

$y(n) = (n+1)x_1(n)+x_2(n)(n+1)$ — ②

Equation ① ≠ Equation ②.

Hence, the system is Non-linear.

(iv) Time- Invariance :⇒

$$y(n,k) = (n+1)\,x(n-k) \quad -①$$
$$y(n-k) = (n-k+1)\,x(n-k) - ②$$

Equation ① ≠ Equation ②

Thus, the System is time variant.

v) Stability :⇒

The system is stable because for bounded input, output is bounded.

(ii) $y(t) = x(t) + 10$

(i) Memory :⇒

The system is memoryless because ~~for bounded~~ output depends on only present input & not on past input.

(ii) Causal :⇒

The System is Causal because output depends only on present input.

(iii) Linearity :⇒ ~~$y_1(t)$~~ According to Homogeneity & Superposition,

$$y_1(t) = x_1(t) + 10$$
$$y_2(t) = x_2(t) + 10$$
$$y(t) = y_1(t) + y_2(t) = x_1(t) + x_2(t) + 20 \quad -①$$

&

$$y(t) = [x_1(t) + x_2(t)] + 10 - ②$$

Equation ① ≠ Equation ②

The System is Non - linear

iv) Time-Invariance :⇒

$$y(t,t_0) = x(t-t_0)+10 \quad \text{--- (1)}$$

$$y(t-t_0) = x(t-t_0)+10 \quad \text{--- (2)}$$

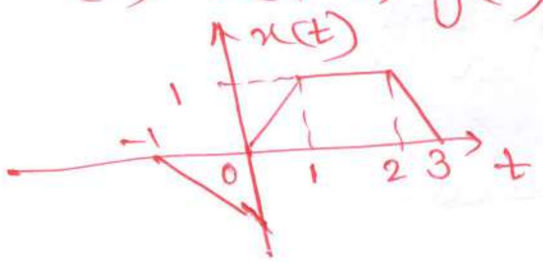
Equation (1) = Equation (2).

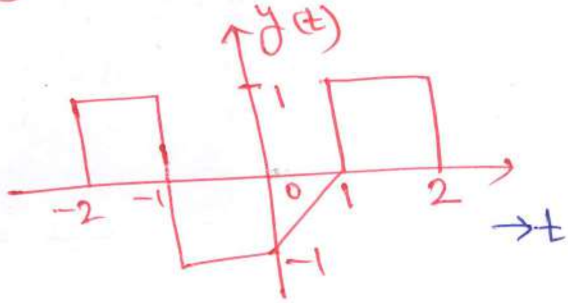
System is Time-Invariant.

v) Stability :⇒

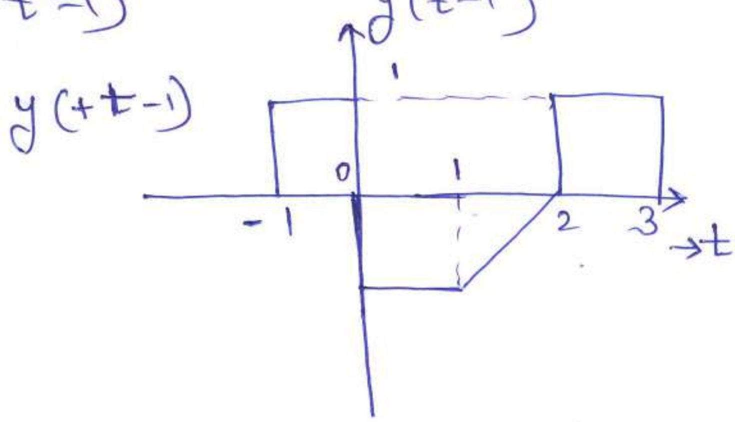
The bounded output depends on bounded input.

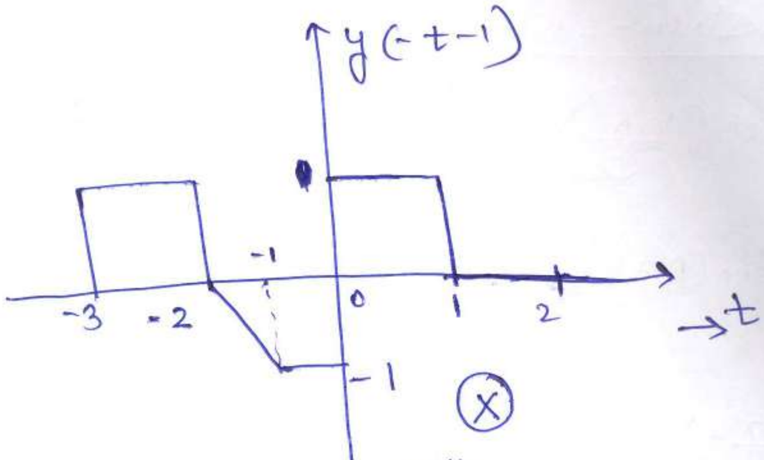
b) Let $x(t)$ & $y(t)$ be given in fig. Q-2(b) respectively. Sketch the following signals:- i) $x(t)y(-t-1)$ (ii) $x(4-t) \cdot y(t)$





Sol^n:- i) $x(t) \cdot y(-t-1)$





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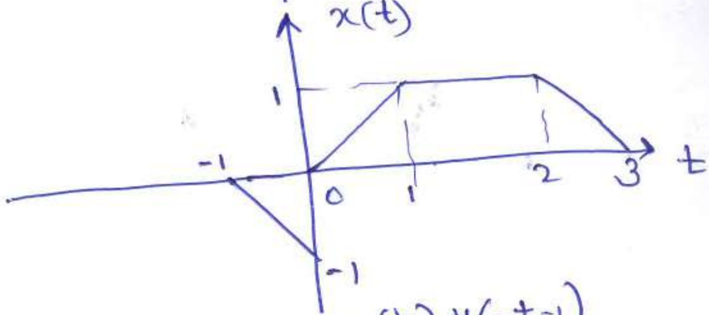
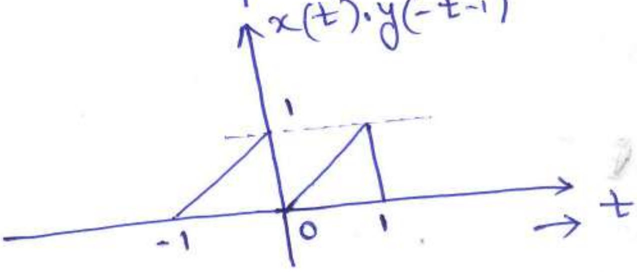
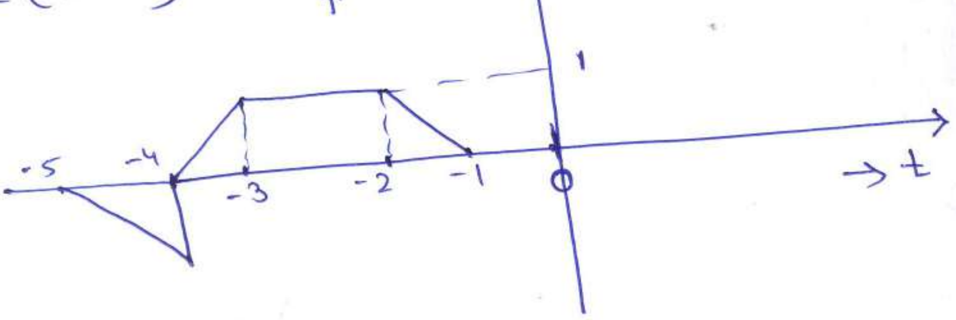


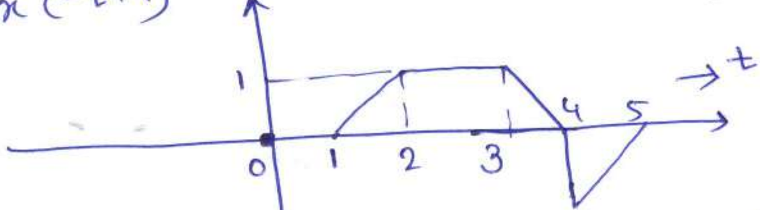


iii) $x(4-t)\cdot y(t)$

$x(t+4)$



$x(-t+4)$



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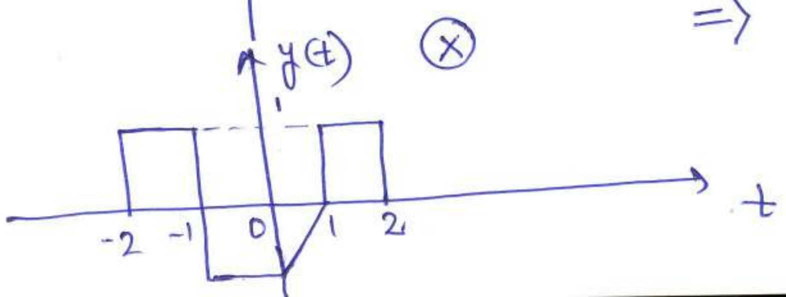
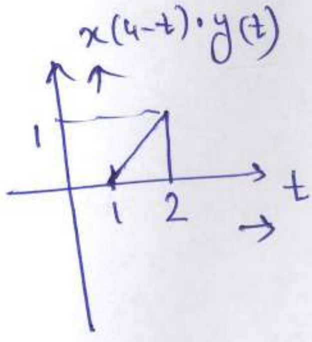


$\Rightarrow$



Question-2 (c)

$$x(n) = \cos\left(\frac{n\pi}{5}\right)\sin\left(\frac{n\pi}{3}\right)$$

$$\cos A . \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$x(n) = \frac{1}{2}\left[\sin\left(\frac{n\pi}{3}+\frac{n\pi}{5}\right) - \sin\left(\frac{n\pi}{3}-\frac{n\pi}{5}\right)\right]$$

$$= \frac{1}{2}\left[\sin\left(\frac{8n\pi}{15}\right) - \sin\left(\frac{2n\pi}{15}\right)\right]$$

$$\frac{\Omega_{01}}{2\pi} = \frac{\frac{8\pi}{15}}{2\pi} = \frac{4}{15} = \frac{P}{q} \Rightarrow N_1 = 15$$

$$\frac{\Omega_{02}}{2\pi} = \frac{2\pi/15}{2\pi} = \frac{1}{15} = \frac{P}{q} \Rightarrow N_2 = 15$$

$\therefore$ ~~$N = \frac{N_1}{N_2}$~~ $x(n)$ is periodic with a fundamental period, $N = 15$.

MODULE-2

Ques:-3 a) (i) $x(t) * \delta(t-t_0) = x(t-t_0)$

$$x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-(\tau)-t_0)\,d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau-t_0)\,d\tau$$ Applying Shifting Property

$$= x(\tau)\Big|_{\tau = t-t_0} = x(t-t_0)$$

Hence, proved.

ii) $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

Applying shifting property of Impulse sequence,

$$\delta(n-k) = 1, \quad k = n$$

$$\therefore \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) = x(k)\Big|_{k=n} = x(n)$$

Hence, proved.

iii) $x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\cdot d\tau$

Since $u(t-\tau) = 1, \; t-\tau > 0$ or $\tau < t$

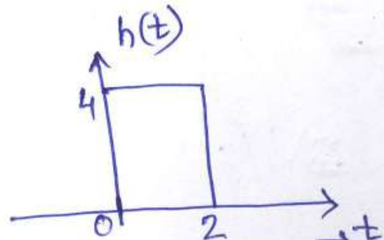
$\qquad\qquad\qquad\; 0, \; t-\tau < 0$ or $\tau > t$,

$$\therefore x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\,u(t-\tau)\,d\tau = \int_{-\infty}^{t} x(\tau)\,d\tau$$

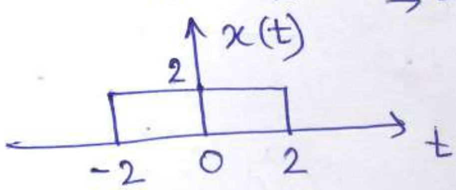
Hence, proved.

Question - 3 b)

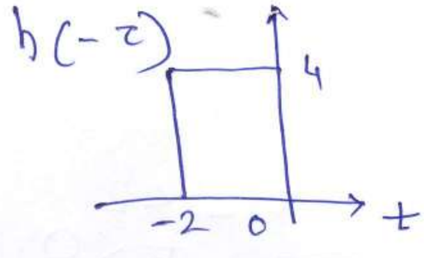
Sol

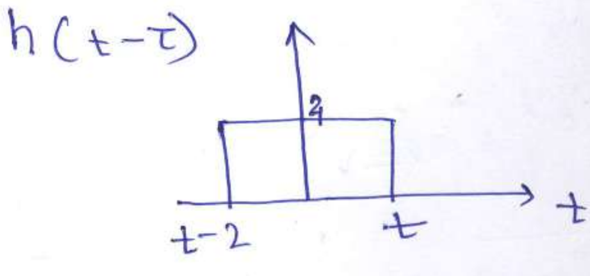


$$h(t) = \begin{cases} 4, & 0 \le t \le 2 \\ 0, & \text{Elsewhere} \end{cases}$$

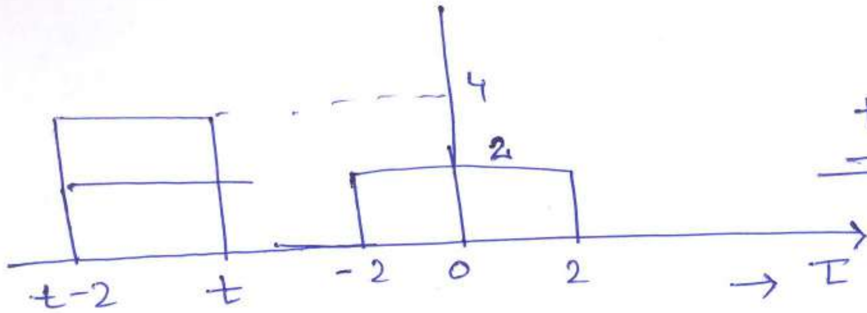


$$x(t) = \begin{cases} 2, & -2 \le t \le 2 \\ 0, & \text{Elsewhere} \end{cases}$$





i)



$t < -2$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

$$y(t) = 0.$$

ii)

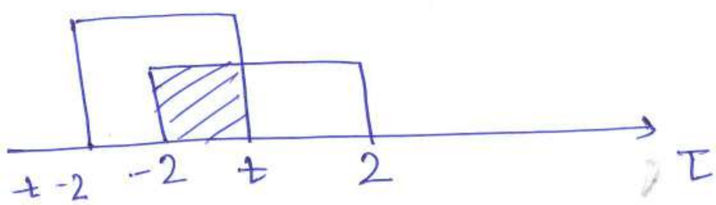
$t-2 < -2$

$t < 0$

&

$t > -2$

$-2 < t < 0$



$-2 < t < 0$

$$y(t) = \int_{-2}^{t} 2\cdot 4\cdot d\tau = 8\,[\tau]_{-2}^{t}$$

$$= 8\left[t-(-2)\right] = 8(t+2)$$

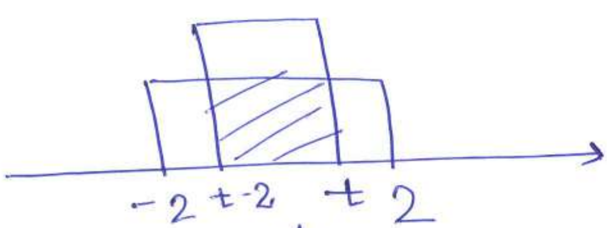
iii)

$t-2 > -2$

$t > 0$

& $t < 2$

∴ $0 < t < 2$



$$y(t) = \int_{t-2}^{t} 4\cdot 2\,d\tau = 8\,[\tau]_{t-2}^{t}$$

$$= 8\left[t-(t-2)\right] = 8\times 2 = 16.$$

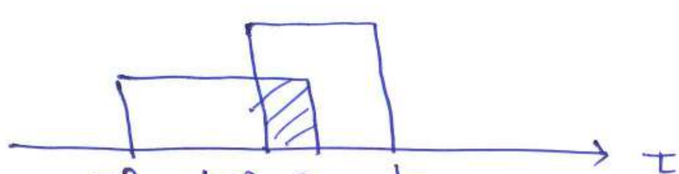
iv)

$t-2 < 2$

$t < 0$

$t > 2$

$0 < t < 2$

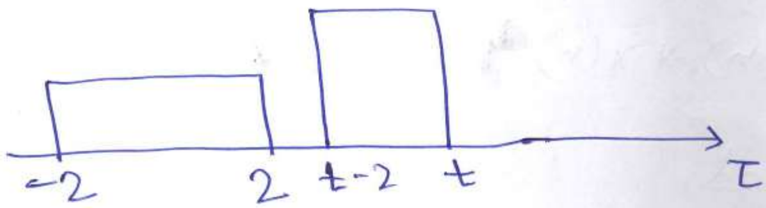


$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = \int_{t-2}^{2} 2\cdot 4\, d\tau = 8\left[\tau\right]_{t-2}^{2} = 8\left[t-2-2\right]$$

$$y(t) = 8(t-4)$$

v)



$t-2 > 2$

$t > 4$

$$\boxed{y(t) = 0}$$

$$\therefore \quad y(t) = \begin{cases} 0, & t < -2 \; \cancel{\text{or } t < 4} \\ 8(t+2), & -2 < t < 0 \\ 16, & 0 < t < 2 \\ 8(t-4), & 0 < t < 2 \\ 0, & t > 4 \end{cases}$$

Question-4 a)

i) $x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

Sol
$$x(n) * [h_1(n) + h_2(n)] = \sum_{k=-\infty}^{\infty} x(k)\left[h_1(n-k) + h_2(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h_1(n-k) + \sum_{k=-\infty}^{\infty} x(k)\, h_2(n-k)$$

$$= x(n) * h_1(n) + x(n) * h_2(n)$$

ii) $x(n) * h(n) = h(n) * x(n)$

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

let $n-k = m$

(4)

$$x(n) * h(n) = \sum_{m=\infty}^{-\infty} x(n-m)h(m)$$

$$= \sum_{m=-\infty}^{\infty} x(n-m)h(m)$$

$$= \sum_{m=-\infty}^{\infty} h(m)\,x(n-m)$$

$$= h(n) * x(n)$$

Hence, proved.

Question :- 4(b)

$$x(n) = \alpha^n [u(n) - u(n-8)], \quad |\alpha| < 1$$

$$h(n) = u(n) - u(n-5)$$

Sol

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\,h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k [u(k) - u(k-8)][u(n-k) - u(n-k-5)].$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k \begin{Bmatrix} u(k)\cdot u(n-k) - u(k)\cdot u(n-k-5) \\ - u(k-8)u(n-k) + u(k-8)u(n-k-5) \end{Bmatrix}$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k u(k)\cdot u(n-k) - \sum_{k=-\infty}^{\infty} \alpha^k u(k)\cdot u(n-k-5)$$

$$- \sum_{k=-\infty}^{\infty} \alpha^k u(k-8)\,u(n-k) + \sum_{k=-\infty}^{\infty} \alpha^k u(k-8)\,u(n-k-5)$$

$$\because u(k)\cdot u(n-k) = \begin{cases} 1, & 0 \le k \le n, \; n \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$u(k-8)\cdot u(n-k-5) = u(k-8)\cdot u(-k+n-5), \quad 8 < k < n-5$$

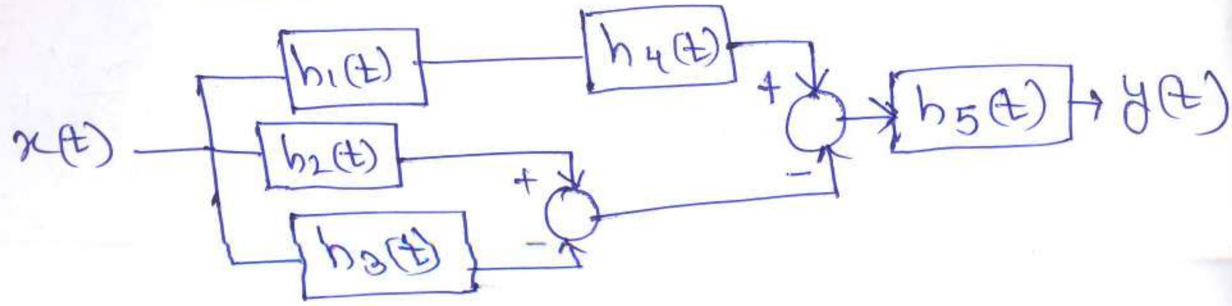
$$= \sum_{k=0}^{n} \alpha^k - \sum_{k=0}^{n-5} \alpha^k - \sum_{k=8}^{n} \alpha^k + \sum_{k=8}^{n-5} \alpha^k$$

$$= \cancel{\sum_{k=0}^{n} \alpha^k} - \cancel{\sum_{k=0}^{n-5} \alpha^k} - \cancel{\sum_{k=0}^{n} \alpha^k} + \cancel{\sum_{k=0}^{7} \alpha^k} + \cancel{\sum_{k=0}^{n-5} \alpha^k} + \cancel{\sum_{k=0}^{7} \alpha^k}$$

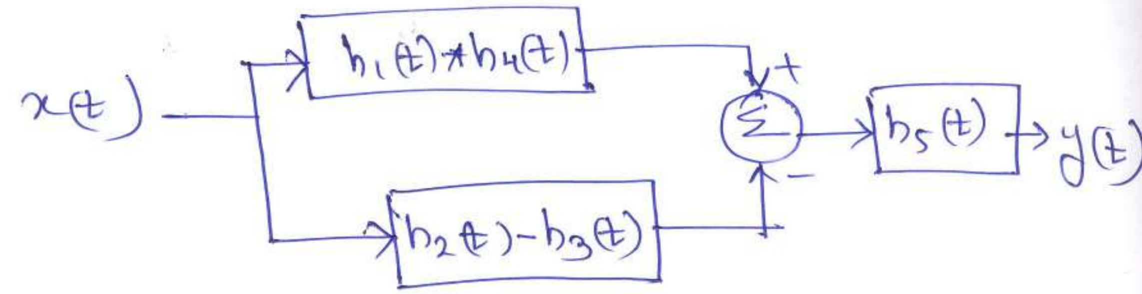
$$= \cancel{\frac{1-\alpha^8}{1-\alpha}} \quad 0$$

Module-3

Question-5 a)



$\equiv$



$$\equiv \; x(t) \rightarrow \boxed{[h_1(t) * h_4(t)] - [h_2(t) - h_3(t)]} \rightarrow \boxed{h_5(t)} \rightarrow y(t)$$

$$h_{eq}(t) = \Big([h_1(t) * h_4(t)] - [h_2(t) - h_3(t)] \Big) * h_5(t)$$

$$y(t) = h_{eq}(t) * x(t)$$

$$y(t) = \Big\{ [h_1(t) * h_4(t)] - [h_2(t) - h_3(t)] \Big\} * h_5(t) * x(t)$$

Question - 5 b)

i) $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$h(n)$ is zero for $n<0$ because of $u(n)$ function.
Therefore, the system is Causal

$$\sum_{n=-\infty}^{\infty} |h(n)| = 1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots \infty = \frac{1}{1-\frac{1}{2}}$$

$$= 2$$

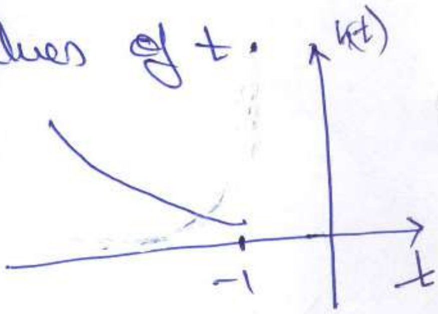
Since $h(n)$ is absolutely summable, the system is stable.

The system is memoryless as it doesn't store the past values for present input.

ii) $h(t) = e^{t} u(-1-t)$

h(t) is not zero for negative values of t.

Therefore system is non-Causal.



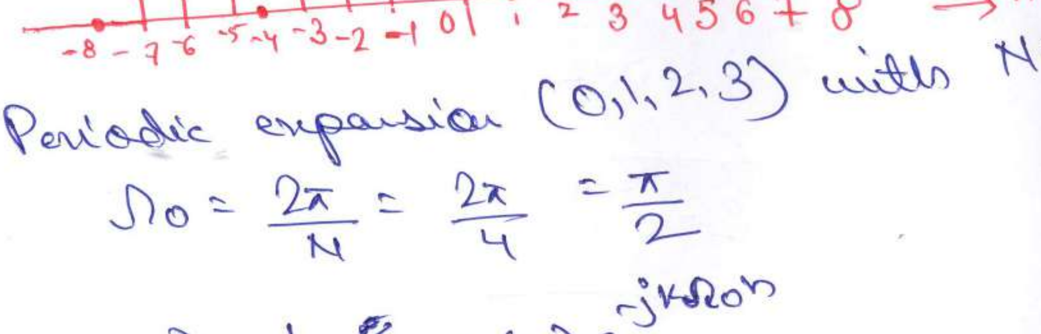
It is memory system as it stores previous values ~~too~~.

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

$$\Rightarrow \int_{-\infty}^{-1} |e^{t}| dt = \left\{ \frac{e^{t}}{1} \right\}_{-\infty}^{-1} = e^{-1} - e^{-\infty} = e^{-1} < \infty.$$

Stable.

Question - 5(c)



find DTFS Coefficients.

Solⁿ: Periodic expansion (0,1,2,3) with N = 4.

$$\Omega_0 = \frac{2\pi}{N} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$X(k) = \frac{1}{N} \sum_{n=\langle N \rangle} x(n) e^{-jk\Omega_0 n}$$

$$= \frac{1}{4} \sum_{n=0}^{3} x(n) e^{-jk\left(\frac{2\pi}{4}\right)n}$$

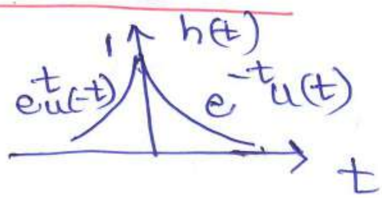
$$= \frac{1}{4}\left[x(0) + x(1)e^{-j\frac{2\pi k}{4}} + x(2)\, e^{-j\frac{2\pi 2k}{4}} \cdot x(3) e^{-j\frac{6\pi k}{4}}\right]$$

$$X(0) = \frac{3}{2}, \quad X(1) = \frac{-1}{2} + j\frac{1}{2}, \quad X(2) = \frac{-1}{2},$$

$$X(3) = -\frac{1}{2} - \frac{j}{2}$$

Question-6(a)

(ii) $h(t) = e^{-|t|}$



Step response is given by:-

$$S(t) = \int_{-\infty}^{t} h(\tau)\cdot d\tau$$

for $t<0$
$$S(t) = \int_{-\infty}^{t} e^{\tau} d\tau = e^{t}$$

for $t \geq 0$
$$S(t) = \int_{-\infty}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau$$

$$= \left[e^{\tau}\right]_{-\infty}^{0} + \left[\frac{e^{-\tau}}{-1}\right]_{0}^{t}$$

$$= e^{0} - e^{-\infty} + \left[\frac{e^{-t} - e^{0}}{-1}\right]$$

$$= 1 - 0 - e^{-t} + 1 = 2 - e^{-t}$$

$$\boxed{S(t) = 2 - e^{-t}}$$

ii) $h(n) = \left(\frac{1}{2}\right)^n u(n-2)$

step response is given by :-

$$S(n) = \sum_{k=-\infty}^{n} h(k)$$

for $n < 2$, $S(n) = 0$, since $h(n)$ is present only for positive values of n.

for $n \geq 2$,

$$S(n) = \sum_{k=2}^{n} \left(\frac{1}{2}\right)^k = \frac{\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{2}\right)^{2+1}}{\frac{1}{2} - 1}$$

$$= 2\left[\left(\frac{1}{2}\right)^{n+1} - \left(\frac{1}{2}\right)^3\right]$$

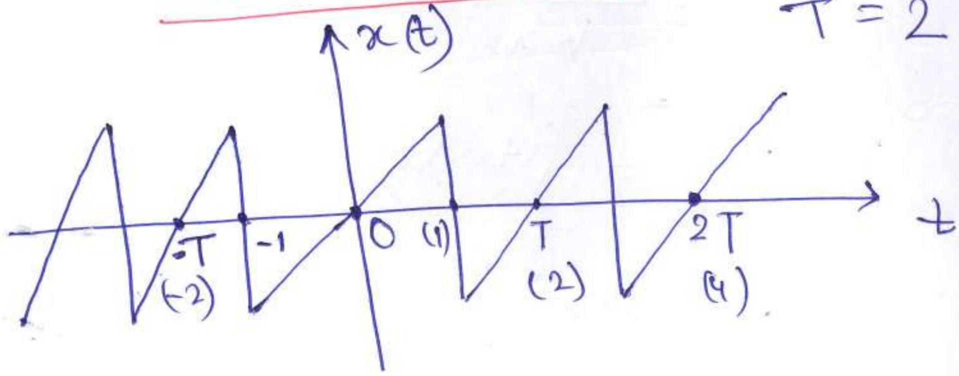
$$= 2\left[\left(\frac{1}{2}\right)^{n+1} - \frac{1}{8}\right]$$

$$\therefore \quad S(n) = \begin{cases} 0, & n < 2 \\ 2\left[\left(\frac{1}{2}\right)^{n+1} - \frac{1}{8}\right], & n > 2 \end{cases}$$

Question- 6(b)

$T = 2$

f.S. = ?



$T = 2, \quad \omega = \frac{2\pi}{2} = \pi.$

$$X(k) = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} t\left[\cos(k\pi t) - j\sin(k\pi t)\right] dt$$

$\because \; x(t) = t \;, \; -1 < t < 1$

$t \cdot \cos(k\pi t)$ is odd,
$t \cdot \sin(k\pi t)$ is even & the limits of integration are symmetric.

Waveform $x(t)$ is odd & hence $X(k)$ is purely imaginary i.e. $X(k) = jB(k)$ & $A(k) = 0$.

$$X(k) = \frac{2}{T}\int_{0}^{1} -jt \cdot \sin(k\pi t)\, dt$$

$$= -j\left[-\frac{t \cdot \cos(k\pi t)}{k\pi} + \frac{\sin(k\pi t)}{k^2\pi^2}\right]_0^1$$

$$= j\,\frac{\cos(k\pi)}{k\pi} = \frac{j\,(-1)^k}{k\pi} \quad (k \neq 0)$$

for $k=0$, $X(k) = \frac{1}{2}\int_{-1}^{1} t \cdot dt = 0$.

$$X(k) = \begin{cases} \frac{j(-1)^k}{k\pi}, & k \neq 0 \\ 0, & k = 0 \end{cases}$$

Module-4

Question :- 7 (a)

i) frequency shifting property of DTFT.

Sol

$$x(n) \xleftrightarrow{DTFT} X(e^{j\Omega})$$

let $y(n) = e^{j\beta n} x(n) \xleftrightarrow{DTFT} X\left(e^{j(\Omega-\beta)}\right)$

$$Y\left(e^{j\Omega}\right) = \sum_{n=-\infty}^{\infty} y(n) e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} e^{j\beta n} \cdot e^{-j\Omega n} \cdot x(n)$$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-j(\Omega-\beta)n}$$

$$= X\left(e^{j(\Omega-\beta)}\right)$$

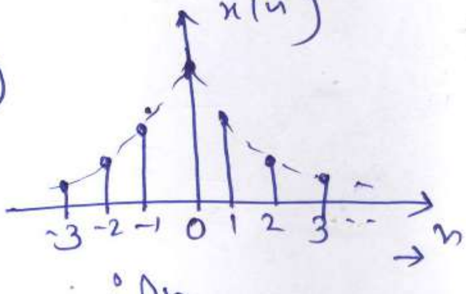
Hence, proved.

Question:- 7 b)

(i) $x(n) = a^{|n|}, \ |a| < 1$

Sol

$$x(n) = a^{-n} u(-n-1) + a^{n} u(n)$$



$$DTFT\left[x(n)\right] = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{-1} a^{-n} e^{-j\Omega n} + \sum_{n=0}^{\infty} a^{n} e^{-j\Omega n}$$

Put $n = -m$ in first summation.

$$X(e^{j\Omega}) = \sum_{m=1}^{\infty} \left(a e^{j\Omega}\right)^{m} + \sum_{n=0}^{\infty} \left(a e^{-j\Omega}\right)^{n}$$

<u>formulae</u> :-

$$\sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \; ; \; |b|<1$$

$$\sum_{n=n_0}^{\infty} b^n = \frac{b^{n_0}}{1-b} \; ; \; |b|<1$$

$$X(e^{j\Omega}) = \frac{ae^{j\Omega}}{1-ae^{j\Omega}} + \frac{1}{1-ae^{-j\Omega}}$$

$$\boxed{X(e^{j\Omega}) = \frac{1-a^2}{1+a^2-2a\cos\Omega}} \; ; \; |a|<1$$

(ii) $x(n) = 2^n u(-n)$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$$

$u(-n) = 1 , n \leq 0$
$ 0 , n > 0$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{0} 2^n \cdot e^{-j\Omega n}$$

Put $n = -m$,

$$X(e^{j\Omega}) = \sum_{m=\infty}^{0} 2^{-m} e^{j\Omega m}$$

$$= \sum_{m=0}^{\infty} \left(\frac{e^{j\Omega}}{2}\right)^m$$

$$= \frac{1}{1-e^{j\Omega}/2} = \frac{2}{2-e^{j\Omega}} \; \underline{\text{Ans}}$$

Question :- 8 (a)

i) $x(t) = 1 + \cos 2000\pi t + \sin 4000\pi t$.

$f = \max [f_1, f_2]$

$f_1 = 1000\,Hz$, $f_2 = 2000\,Hz$

$\therefore f = \max.[f_1, f_2] = 2000\,Hz$

Nyquist rate, $f_s = 2f = 2 \times 2000 = 4000\,Hz$

Nyquist Interval, $T_s = \frac{1}{2f} = \frac{1}{4000}$ Secs.

ii) $x(t) = 25e^{j500\pi t} = 25[\cos 500\pi t + j \sin 500\pi t]$

$f = 250\,Hz$.

f_s = Nyquist Rate $= 2 \times 250 = 500\,Hz$.

Nyquist Interval, $T_s = \frac{1}{2f} = \frac{1}{500}$ Secs.

Question :- 8(b)

i) $x(t) = e^{-3t} u(t-1)$

$FT[e^{-at} u(t)] = \frac{1}{j\omega + a}$

$\therefore e^{-3t} u(t) \xleftrightarrow{FT} \frac{1}{j\omega + 3}$

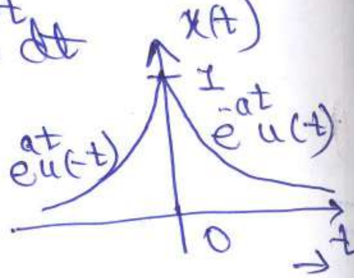
Using time shifting property,

$x(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} X(j\omega)$

$$\therefore\ e^{-3t}u(t-1) \xleftrightarrow{FT} e^{-j\omega\cdot 1}\,\frac{1}{j\omega+3} = \frac{e^{-j\omega}}{j\omega+3}$$

(ii) $x(t) = e^{-a|t|}$, $a>0$.

$$X(j\omega) = \int_{-\infty}^{0} e^{at}\cdot e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{-at}\cdot e^{-j\omega t}\,dt$$



$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{2}{a^2+\omega^2}$$

$$\boxed{X(j\omega) = \frac{2}{a^2+\omega^2}}$$

Question:- 8 (c)

$$X(j\omega) = \frac{j\omega+1}{(j\omega+2)^2}$$ {Using Partia fraction Method}

$$X(j\omega) = \frac{A}{j\omega+2} + \frac{B}{(j\omega+2)^2}$$

$$\frac{A(j\omega+2)+B}{(j\omega+2)^2} = \frac{j\omega+1}{(j\omega+2)^2}$$

$$\Rightarrow (2A+B) + Aj\omega = j\omega+1$$ {Equating Coefficients}

$A = 1$ &

$2A+B = 1$

$B = 1-2 = -1$

$$\therefore X(j\omega) = \frac{1}{j\omega+2} - \frac{1}{(j\omega+2)^2}$$

W/K/T,

$$e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{j\omega + a}$$

$$t \cdot e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{(j\omega + a)^2}$$ Taking IFT,

$$\therefore x(t) = e^{-2t}u(t) - te^{-2t}u(t)$$

$$\boxed{x(t) = (1-t)e^{-2t}u(t)}$$

MODULE-5

Question-9 (a)

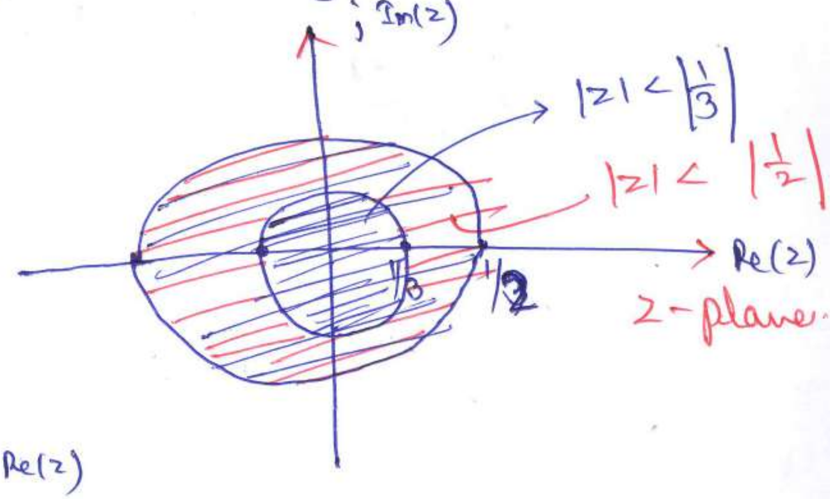
i) $x(n) = -\left(\frac{1}{2}\right)^n u(-n-1) - \left(-\frac{1}{3}\right)^n u(-n-1)$

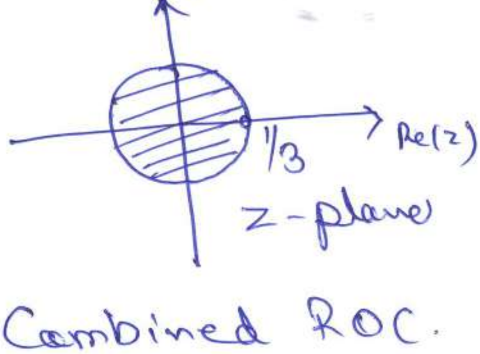
We know that,

$$-a^n u(-n-1) \xleftrightarrow{Z} \frac{z}{z-a} \quad ; \text{ROC} : |z| < |a|$$

$$\therefore -\left(\frac{1}{2}\right)^n u(-n-1) \xleftrightarrow{Z} \frac{z}{z-\frac{1}{2}} \quad ; \text{ROC} : |z| < \left|\frac{1}{2}\right|$$

$$-\left(\frac{1}{3}\right)^n u(-n-1) \xleftrightarrow{Z} \frac{z}{z-\frac{1}{3}} \quad ; \text{ROC} : |z| < \left|\frac{1}{3}\right|$$





Combined ROC.

(26)

$$\therefore X(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{1}{3}}$$

$$= \frac{z\left(z-\frac{1}{3}\right) + z\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

$$= \frac{z^2 - \frac{z}{3} + z^2 - \frac{z}{2}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} = \frac{2z^2 - \frac{5z}{6}}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

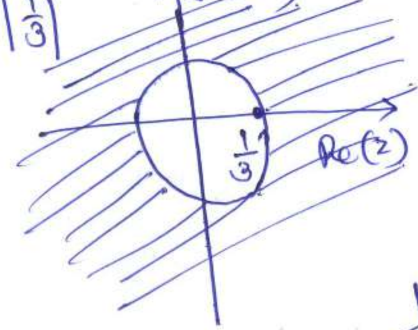
$$X(z) = \frac{z\left(2z-\frac{5}{6}\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$$

(ii) $x(n) = -\left(\frac{3}{4}\right)^n u(-n-1) + \left(-\frac{1}{3}\right)^n u(n)$

$$-\left(\frac{3}{4}\right)^n u(-n-1) \xleftrightarrow{z} \frac{z}{z-\frac{3}{4}} \;;\text{ROC: } |z| < \left|\frac{3}{4}\right|$$

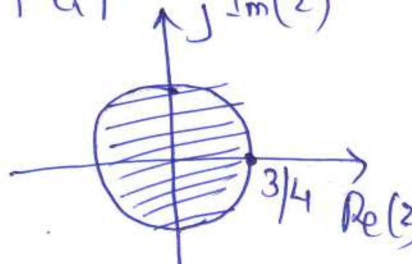
$$\left(-\frac{1}{3}\right)^n u(n) \xleftrightarrow{z} \frac{z}{z-\left(-\frac{1}{3}\right)} = \frac{z}{z+\frac{1}{3}} \;;$$

$|z| > \left|\frac{1}{3}\right|$

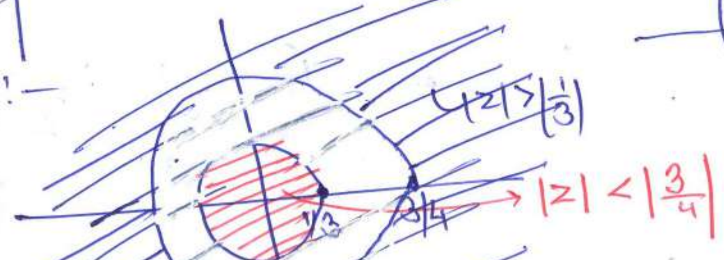


ROC: $|z| > \left|\frac{1}{3}\right|$

$|z| < \left|\frac{3}{4}\right|$



Combined :-



No overlapping of ROC's.

$$X(z) = \frac{z}{z-\frac{3}{4}} + \frac{z}{z+\frac{1}{3}} = \frac{2z^2 - \frac{5z}{12}}{\left(z-\frac{3}{4}\right)\left(z+\frac{1}{3}\right)}$$

Question:- 9 (b)

i) **Time-shift:-** If $x(n) \xleftrightarrow{z} X(z)$; ROC: R

then $x(n-n_0) \xleftrightarrow{z} z^{-n_0}X(z)$; ROC: R.

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} \quad , \text{let } y(n) = x(n-n_0)$$

$$Y(z) = \sum_{n=-\infty}^{\infty} x(n-n_0) z^{-n}, \quad \text{let } n-n_0 = m$$

$$Y(z) = \sum_{m=-\infty}^{\infty} x(m) z^{-(m+n_0)} = \sum_{m=-\infty}^{\infty} x(m) z^{-m-n_0}.$$

$$Y(z) = z^{-n_0} \sum_{m=-\infty}^{\infty} x(m) z^{-m}.$$

$$Y(z) = z^{-n_0} X(z) \quad ; \text{ROC; R}$$

Hence, proved.

ii) **Convolution:-** If $x(n) \xleftrightarrow{z} X(z)$; ROC: R_1

$h(n) \xleftrightarrow{z} H(z)$; ROC: R_2

then $y(n) = x(n) * h(n) \xleftrightarrow{z} X(z) \cdot H(z)$; ROC: $R : R_1 \cap R_2$.

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [x(n) * h(n)] z^{-n} = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k) h(n-k) z^{-n}$$

Substitute $n-k=m$ & interchange summation order

$$Y(z) = \sum_{k=-\infty}^{\infty} x(k) z^{-k} \sum_{m=-\infty}^{\infty} h(m) z^{-m}$$

$$= X(z) \cdot H(z)$$

Hence, proved.

ROC: $R : R_1 \cap R_2$

Question-10(a)

$$X(z) = \frac{z(z^2-4z+5)}{(z-3)(z-2)(z-1)}$$

Using Partial fraction,

$$\frac{X(z)}{z} = \frac{z^2-4z+5}{(z-3)(z-2)(z-1)}$$

$$\frac{X(z)}{z} = \frac{A}{z-3} + \frac{B}{z-2} + \frac{C}{z-1}$$

$$\frac{z^2-4z+5}{\cancel{(z-3)(z-2)(z-1)}} = \frac{A(z-2)(z-1)+B(z-3)(z-1)+C(z-3)(z-2)}{\cancel{(z-3)(z-2)(z-1)}}$$

$$z^2-4z+5 = A\left[z^2-2z-z+2\right]+B\left[z^2-3z-z+3\right]+ C\left[z^2-3z-2z+6\right]$$

$$z^2-4z+5 = (A+B+C)z^2+(-3A-4B-5C)z + (2A+3B+6C)$$

Comparing Coefficients,

$$A+B+C = 1 \quad -(1)$$

$$-3A-4B-5C = -4 \quad -(2)$$

$$2A+3B+6C = 5 \quad -(3)$$

Solving above, we get,

$$A = 1 \quad B = -1 \quad \& \quad C = 1$$

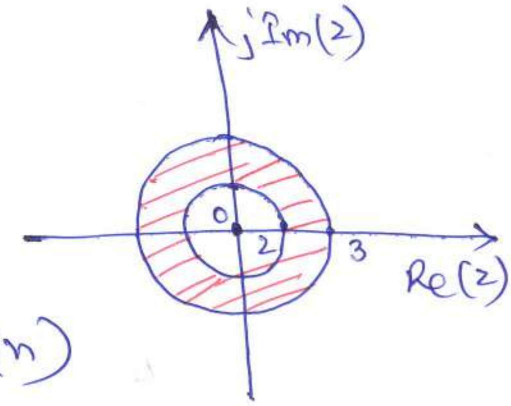
$$\frac{X(z)}{z} = \frac{1}{z-3} - \frac{1}{z-2} + \frac{1}{z-1}$$

$$X(z) = \frac{z}{z-3} - \frac{z}{z-2} + \frac{z}{z-1}$$

(i) $2 < |z| < 3$.

means,

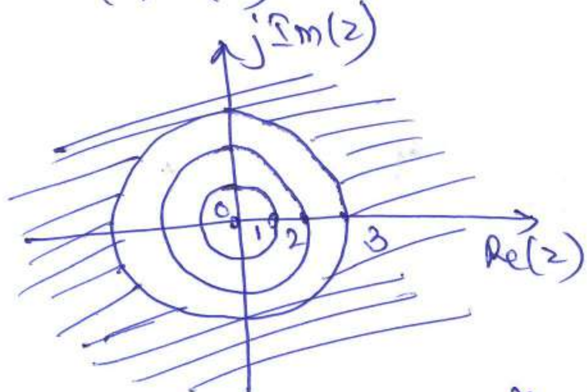
$|z| > 2$ & $|z| < 3$



IFT,

$$x(n) = -(3)^n u(-n-1) - (2)^n u(n) + (1)^n u(n)$$

ii) $|z| > 3$



Taking IFT,

$$x(n) = (3)^n u(n) - (2)^n u(n) - (1)^n u(n)$$

$$\boxed{x(n) = (3)^n u(n) - (2)^n u(n) - u(n)}$$

Question:- 10(b)

$$x(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2)$$

$$y(n) = \delta(n) - \frac{3}{4}\delta(n-1)$$

$$y(n) = h(n) * x(n) \xleftrightarrow{z} H(z) \cdot X(z)$$

$$x(n) \xleftrightarrow{z} 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}$$

$$y(n) \xleftrightarrow{z} 1 - \frac{3}{4}z^{-1}$$

$$\therefore Y(z) = H(z) \cdot X(z)$$

$$H(z) = \frac{1 - \frac{3}{4}z^{-1}}{1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}} = \frac{z\left(z - \frac{3}{4}\right)}{\left(z^2 + \frac{1}{4}z - \frac{1}{8}\right)}$$

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$$\frac{H(z)}{z} = \frac{\left(z-\frac{3}{4}\right)}{z^2+\frac{z}{4}-\frac{1}{8}} = \frac{\left(z-\frac{3}{4}\right)}{z^2+\frac{z}{2}-\frac{z}{4}-\frac{1}{8}}$$

$$= \frac{\left(z-\frac{3}{4}\right)}{z\left(z+\frac{1}{2}\right)-\frac{1}{4}\left(z+\frac{1}{2}\right)} = \frac{\left(z-\frac{3}{4}\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{2}\right)} \quad \text{—(1)}$$

Using Partial fraction,

$$\frac{H(z)}{z} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z+\frac{1}{2}} = \frac{\left(z-\frac{3}{4}\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{2}\right)}$$

$$A\left(z+\frac{1}{2}\right)+B\left(z-\frac{1}{4}\right) = \left(z-\frac{3}{4}\right)$$

$$(A+B)z + \frac{A}{2} - \frac{B}{4} = z - \frac{3}{4}$$

$$\left.\begin{aligned} A+B &= 1 \\ \frac{A}{2}-\frac{B}{4} &= -\frac{3}{4} \end{aligned}\right\}$$ Solving above.

$A = -0.667$.

$B = 1.667$

$$\therefore H(z) = -0.667\left(\frac{z}{z-\frac{1}{4}}\right) + 1.667\left(\frac{z}{z+\frac{1}{2}}\right) \quad \text{—(2)}$$

Taking Inverse Z- Transform,

$$h(n) = -0.667\left(\frac{1}{4}\right)^n u(n) + 1.667\left(-\frac{1}{2}\right)^n u(n) \quad \text{—(3)}$$

Again, we need to find out,

$y(n)$ if $x(n) = \left(\frac{1}{2}\right)^n u(n) \xleftrightarrow{z} \left(\frac{z}{z-\frac{1}{2}}\right)$

$$\therefore y(n) = h(n) * x(n) \xleftrightarrow{z} Y(z) = H(z)\cdot X(z)$$

$$Y(z) = \frac{z\left(z-\frac{3}{4}\right)}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{2}\right)} \times \left(\frac{z}{z-\frac{1}{2}}\right)$$

$$Y(z) = \frac{z^2\left(z-\frac{3}{4}\right)}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

$$\therefore \frac{Y(z)}{z} = \frac{z\left(z-3/4\right)}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z+\frac{1}{2}} + \frac{C}{z-\frac{1}{4}}$$

$$\therefore A = \left.\frac{z\left(z-3/4\right)}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right|_{z=\frac{1}{2}} = \frac{\frac{1}{2}\left(\frac{1}{2}-\frac{3}{4}\right)}{\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{1}{2}-\frac{1}{4}\right)} = \frac{-1}{2}$$

$$B = \left.\frac{z\left(z-3/4\right)}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right|_{z=-\frac{1}{2}} = \frac{\left(\frac{-1}{2}\right)\left(-\frac{1}{2}-\frac{3}{4}\right)}{(-1)\left(-\frac{1}{2}-\frac{1}{4}\right)} = \frac{5}{6}$$

$$C = \left.\frac{z\left(z-3/4\right)}{\left(z-\frac{1}{2}\right)\left(z+\frac{1}{2}\right)}\right|_{z=\frac{1}{4}} = \frac{2}{3}$$

$$\therefore \frac{Y(z)}{z} = \frac{-1/2}{z-\frac{1}{2}} + \frac{5/6}{z+\frac{1}{2}} + \frac{2/3}{z-\frac{1}{4}}$$

$$Y(z) = -\frac{1}{2}\left(\frac{z}{z-\frac{1}{2}}\right) + \frac{5}{6}\left(\frac{z}{z+\frac{1}{2}}\right) + \frac{2}{3}\left(\frac{z}{z-1/4}\right)$$

Taking Inverse Z-Transform, for ROC $|z| > \left|\frac{1}{2}\right|$

$$y(n) = -\frac{1}{2}\left(\frac{1}{2}\right)^n u(n) + \frac{5}{6}\left(-\frac{1}{2}\right)^n u(n) + \frac{2}{3}\left(\frac{1}{4}\right)^n u(n)$$

Ans