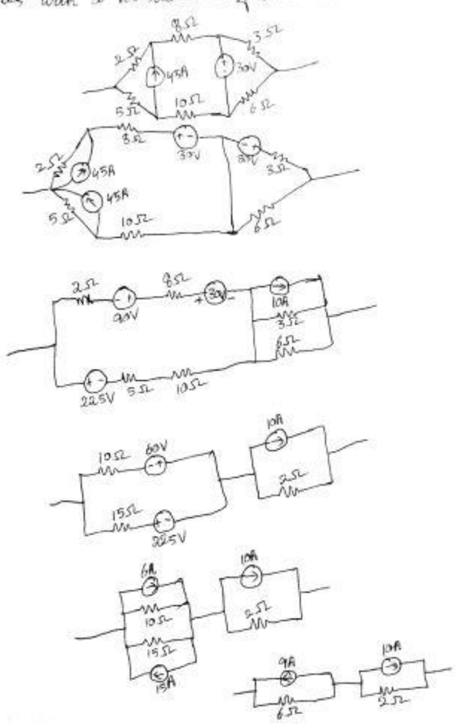
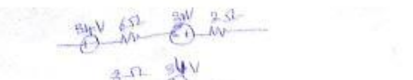
17EC35 Network Analysis Dec 2018/Jan 2019

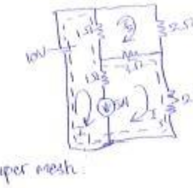
Module - 1

① ⓐ. Reduce the Network shown in Fig Q1(a) to a single voltage source in series with a resistance using source shift and source transformation.





① ⓑ. Use mesh analysis to determine the 3 mesh currents I_1, I_2 and I_3 in the circuit shown in Fig Q1(b)



Applying super mesh:

$10V - (I_1 - I_2)1 - 3(I_3 - I_2) - 2I_3 = 0$

$10 - I_1 + I_2 - 3I_3 + 3I_2 - 2I_3 = 0$

$10 - I_1 + 4I_2 - 5I_3 = 0$

$I_1 - 4I_2 + 5I_3 = +10$ — ①

KVL at 2nd loop:

$-(I_2 - I_1)1 - 2I_2 - (I_2 - I_3)3 = 0$

$-I_2 + I_1 - 2I_2 - I_2 3 + 3I_3 = 0$

$I_1 - 6I_2 + 3I_3 = 0$ — ②

~~$I_1 - I_3 = 5A$~~ $I_1 - I_3 = 5A$ — ③

~~$I_3 - I_1 = 5A$ — ③~~

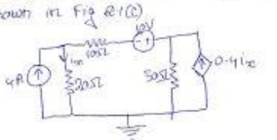
Solving equations simultaneously ①, ②, ③

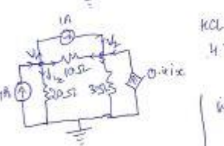
~~$I_1 = 1.5A$~~ $I_1 = 18.75A$

~~$I_2 = 1.5A$~~ $I_2 = 10A$

~~$I_3 = 3.5A$~~ $I_3 = 13.75A$

① ⓒ. Find current in 30Ω resistor using nodal analysis for the circuit shown in Fig Q1(C)





KCL at Node V_1:

$4 - 1 - i_x - \frac{(V_1 - V_2)}{10} = 0$

$i_x = \frac{V_1}{20}$ — ②

$3 - \frac{V_1}{20} - \frac{V_1}{10} + \frac{V_2}{10} = 0$

$-\frac{3V_1}{20} + \frac{V_2}{10} = -3$ — ①

KCL at Node 2:

$0.4i_x + 1 - \frac{(V_2 - V_1)}{10} - \frac{V_2}{30} = 0$

$\frac{0.4V_1}{20} + 1 - \frac{V_2}{10} + \frac{V_1}{10} - \frac{V_2}{30} = 0$

$\frac{0.4V_1}{20} + \frac{V_1}{10} - \frac{2V_2}{15} = -1$

$\frac{3V_1}{25} - \frac{2V_2}{15} = -1$ — ③

Solving eqns ③ & ①

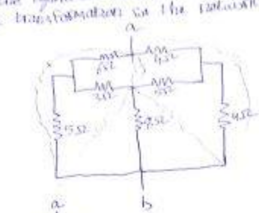
$V_1 = 62.5V$

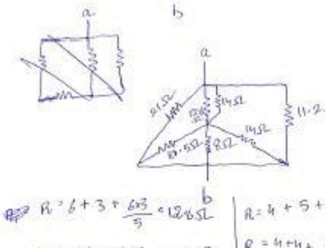
$V_2 = 63.75V$

Current flowing in 30Ω resistor $= \frac{V_2}{30} = \frac{63.75}{30} = 2.125A$

$i_x = \frac{V_1}{20} = \frac{62.5}{20} = 3.125A$

Q2. ⓐ. Find the equivalent resistance between a and b using star delta transformation in the network shown in Fig Q2(a)





$R = 6 + 3 + \frac{6\times3}{5} = 12.6Ω$

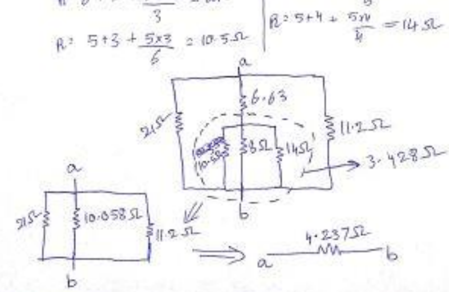
$R = 6 + 5 + \frac{6\times5}{3} = 21Ω$

$R = 5 + 3 + \frac{5\times3}{6} = 10.5Ω$

$R = 4 + 5 + \frac{4\times5}{4} = 14Ω$

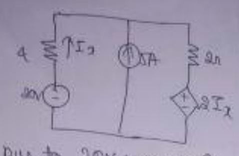
$R = 4 + 4 + \frac{4\times4}{5} = 11.2Ω$

$R = 5 + 4 + \frac{5\times4}{4} = 14Ω$

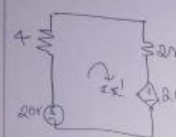


Module 2.

3a) For the circuit shown in fig Q3(a), Find the current I_x using superposition theorem



Case 1: Due to 20V source only

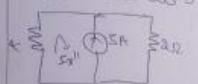


$20 + (2I_x') = 6I_x'$

$20 = 8I_x'$

$I_x' = \frac{20}{8} = 2.5A$.

Case 2: Due to 5A only



$I_x'' = \frac{5\times 2}{4+2} = \frac{10}{6} = 1.66A$.

Case 3 due to $2I_x'''$ only

$4I_x''' + 2I_x''' = -2I_x'''$

$8I_x''' = 0$

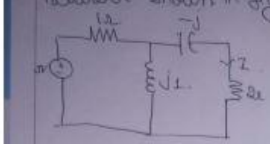
$I_x''' = 0$

$I_x = I_x' + I_x'' + I_x'''$

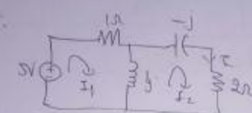
$= 2.5 + 1.66 + 0.0$

$I_x = 3.16A$.

3b) Verify Reciprocity theorem by calculating 'I' for the network shown in fig Q3(b)



Case 1:



Loop 1

$5 = I_1 + (I_1 - I_2)j$

$5 = (1+j)I_1 - jI_2$ —(1)

Loop 2

$0 = j(I_2 - I_1) + (-jI_2) + 2I_2$

$0 = jI_2 - jI_2 - jI_1 + 2I_2$

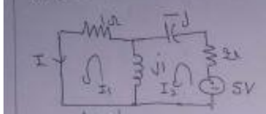
$0 = -jI_1 + 2I_2$

$$I_2 = I = \frac{\begin{vmatrix} 1+j & 5 \\ -j & 0 \end{vmatrix}}{\begin{vmatrix} 1+j & -j \\ -j & 2 \end{vmatrix}} = \frac{5j}{2+2j-j^2} = \frac{5j}{3+2j}$$

$I = 2+j1.4 = 1.386\angle 56.309°A$

$I = 2.23\angle -26.56A$.

Case 2



loop 1

$I_1 + (I_1 - I_2)j = 0$

$(1+j)I_1 - jI_2 = 0$

loop 2

$5 = 2I_2 - jI_2 + (I_2 - I_1)j$

$5 = 2I_2 - jI_2 + jI_2 - I_1 j$

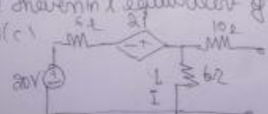
$5 = -I_1 j + 2I_2$

$$I_1 = \frac{\begin{vmatrix} 0 & -j \\ 5 & 2 \end{vmatrix}}{\begin{vmatrix} 1+j & -j \\ -j & 2 \end{vmatrix}} \qquad \frac{5j}{2+2j-j^2} \qquad \frac{5j}{3+2j}$$

$$I = + I_1 = 1 - \frac{5j}{3+2j} = 1.38\angle 56.30^\circ A$$

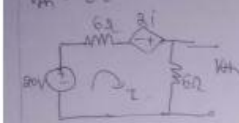
Hence Reciprocity theorem verified

2. Obtain the thevenin's equivalent of the circuit shown in Fig Q3d(c)



Step 1: To find Vth

$V_{th} = 6i$



$20 + 2i = 12I$

$20 + 2I = 12I$

$20 = 10I$

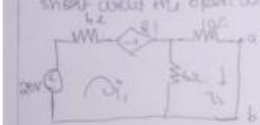
$I = 2A$

$i = 2A$

$V_{th} = 6 \times 2 = 12V$

Step 2: To find Rth

short circuit the open circuited circuit



$Z_{eq} = \frac{V_{th}}{I_{sc}}$

KVL @ loop 1 $\quad i = i_1 - i_2$

$20 + 2i = 6i_1 + 6(i_1 - i_2)$

$i = i_1 - i_2$

$20 = 6i_1 + 6i_1 - 6i_2 - 2i_1 + 2i_2$

$20 = 10i_1 - 4i_2 \rightarrow ①$

Loop 2.

$0 = 6(i_2 - i_1) + 10i_2$.

$0 = -6i_1 + 16i_2 = 0 \rightarrow (2)$

$i = i_1 - i_2 \Rightarrow (3)$

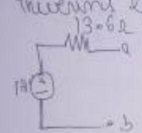
solving 1 & 2

$i_1 = 2.35A \quad i_2 = 0.882\Omega$

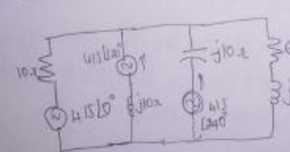
$I_{se} = i_2 = 0.883\Omega$.

$Z_{eq} = \frac{V_{th}}{I_{se}} = \frac{12}{0.8823} = 13.6\Omega$

Thevenin's equivalent circuit



4a For the circuit shown in fig 4(a) find the current in $(6+j8)\Omega$ impedance using Millman's theorem



Using Millman's theorem

$V_m = \frac{V_1Y_1 + V_2Y_2 + V_3Y_3}{Y_1 + Y_2 + Y_3}$

$Y_1 = \frac{1}{Z_1} = \frac{1}{10} = 0.1\Omega$.

$Y_2 = \frac{1}{Z_2} = \frac{1}{j10} \quad -0.1j\,\Omega$

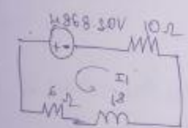
$Z_m = \frac{1}{Y_1 + Y_2 + Y_3}$

$Y_3 = \frac{1}{Z_3} = \frac{1}{-j10} = 0.1j\,\Omega$

$V_m = \frac{415\angle 0° \times 0.1 + 415\angle 120° \times -0.1j + 415 \times 0.1j\,\Omega}{0.1 + 0.1j - 0.1j}$

$V_m = 4868.80V$.

$Z_m = \frac{1}{Y_1 + Y_2 + Y_3} = \frac{1}{0.1 + 0.1j - 0.1j} = 10\Omega$



$I = \frac{4868.80}{10 + 6 + j8}$

$I = 272.1741\angle -26.565° A$

4b For the network shown in figure Q4(b) determine Norton's equivalent circuit A & B find current through impedance connected to the terminal A & B

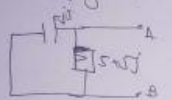
step 1: To find V_{AB}

$V_{th} = V_{AB} = V_{(5+5j)}$

$$V_{AB} = \frac{50\angle 0^\circ \times (5+5j)}{5+5j-5j} = \frac{\cancel{50}^{10}}{\cancel{5}}(5+5j)$$

$V_{AB} = 50 + 50j\,V = 70.71\angle 45^\circ V.$

step 2: To find $R_{th} = R_{AB}$



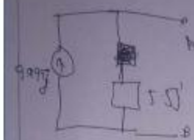
$$R_{AB} = \frac{-j5 \times (5+5j)}{-j5+5+5j}$$

$$= \frac{-25j - 25j^2}{5}$$

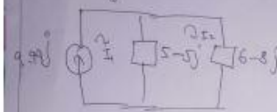
$$= \frac{5}{5}(-5j+5)$$

$R_{th} = 5 - 5j$

Norton's equivalent circuit



connecting $6-8j$ across AB

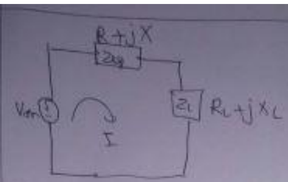


$$I_2 = \frac{9.99j \times (5-5j)}{5-5j+6-8j}$$

$$I_2 = \frac{9.999j \times (5-5j)}{11-13j} = 4.632\angle 103.39^\circ A$$

Q. State and prove Maximum Power transfer theorem for AC circuit when both R_L and X_L are varying.

It states that the maximum power delivered by a source represented by the thevenin's equivalent circuit is attain when the load (R_L) is equal to thevenin's equivalent



$$Z_{eq} = R + jX$$

$$Z_L = R_L + jX_L$$

$$I = \frac{V_{th}}{Z_{eq} + Z_L}$$

$$I = \frac{V_{th}}{R + jX + R_L + jX_L} \qquad \frac{V_{th}}{R + R_L + j(X + X_L)}$$

$$P_L = I^2 R_L$$

$$|I| = \frac{V_{th}}{\sqrt{(R+R_L)^2 + (X+X_L)^2}}$$

$$P_L = \frac{V_{th}^2 R_L}{(R+R_L)^2 + (X+X_L)^2}$$

maxima condition

$$\frac{\partial P_L}{\partial R_L} = 0 \qquad \frac{\partial P_L}{\partial X_L} = 0$$

$$\frac{\partial P_L}{\partial X_L} = \frac{\partial}{\partial X_L}\left(\frac{V_{th}^2 R_L}{(R+R_L)^2 + (X+X_L)^2}\right) = 0$$

$$\frac{\partial P_L}{\partial X_L} = \frac{-2V_{th}^2 R_L (X+X_L)}{\left((R+R_L)^2 + (X+X_L)^2\right)^2}$$

$$-2V_{th}^2 R_L (X+X_L) = 0$$

$$X + X_L = 0$$

$$\boxed{X = -X_L}$$

$$\frac{\partial P_L}{\partial R_L} = \frac{\partial}{\partial R_L}\left(\frac{V_{th}^2 R_L}{(R+R_L)^2 + (X+X_L)^2}\right) = \frac{(R+R_L)^2 + (X+X_L)^2 - V_{th}^2 R_L 2(R+R_L)}{\left((R+R_L)^2 + (X+X_L)^2\right)^2}$$

$$\frac{\partial P_L}{\partial R_L} = 0 = V_{th}^2\left((R+R_L)^2\right) - 2R_L(R+R_L)$$

$$= (R+R_L)^2 - 2R_L(R+R_L) = 0$$

$$= (R+R_L)^2 = 2R_L(R+R_L)$$

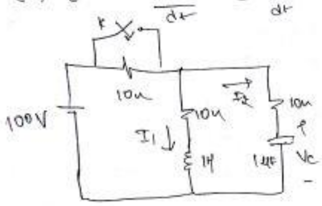
$$= R = R_L$$

$$P_{max} = I^2 R_L = \frac{V_{th}}{\sqrt{\left(\frac{R+R_L}{R_L}\right)^2 + \left(\frac{X+X_L}{-X_L}\right)^2}} \qquad I = \frac{V_{th}}{2R_L}$$

$$X = -X_L$$

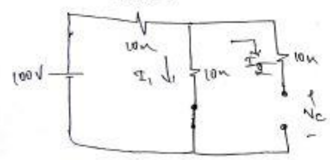
$$P_{max} = \frac{V_{th}^2 \times R_L}{4R_L^2} = \frac{V_{th}^2}{4R_L}$$

5a) In the Network shown in fig Q5(a), a steady state is reached with the switch k is open. At t = 0, the switch k is closed. Obtain the initial values of (i) i_1 (ii) i_2 (iii) V_c (iv) $\frac{di_1}{dt}$ (v) $\frac{di_2}{dt}$ & $\frac{di_1}{dt}$ at $t = \infty$



Ans

Case (i): At $t = 0^-$, k is open, steady state is reached.



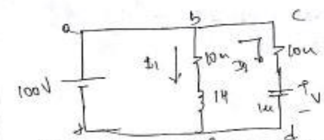
(i)
$$i_1(0^-) = \frac{V}{10+10} = \frac{100}{20} = \underline{\underline{5A}} = i_1(0^+)$$

(ii) $i_2(0^-) = \underline{0A}$

(iii) $$V_c(0^-) = \frac{100[10]}{10+10} = \underline{50V} = V_c(0^+)$$

(iv)

Case (ii): At $t = 0^+$, k is closed.



Applying KVL to the loop abefa

$$100 = 10 I_1(t) + L\frac{dI_1(t)}{dt} \quad \text{— ①}$$

$$100 = 10 I_1(t) + 1\frac{dI_1(t)}{dt}$$

At $t = 0^+$

$$100 = 10 I_1(0^+) + \frac{dI_1(0^+)}{dt} \qquad [I_1(0^-) = I_1(0^+)]$$

$$\frac{dI_1(0^+)}{dt} = 100 - 10(5)$$

$$\frac{dI_1(0^+)}{dt} = \underline{\underline{50\ A/s}}$$

Applying KVL to the loop abcdefa

$$100 = 10 I_2(t) + \frac{1}{C}\int I_2(t)\,dt \quad \text{— ②}$$

$$100 = 10 I_2(t) + 10^6 \int_0^t I_2(t)\,dt \qquad [V_c(0^-) = V_c(0^+)]$$

At $t = 0^+$

$$I_2(0^+) = \frac{100 - 50}{10} = \underline{5A}$$

Differentiating eqn ②

$$0 = 10\frac{dI_2(t)}{dt} + \frac{1}{C} I_2(t)$$

At $t = 0^+$

$$0 = 10\frac{dI_2(0^+)}{dt} + 10^6 I_2(0^+)$$

$$\frac{dI_2(0^+)}{dt} = -\frac{10^6 \times \cancel{5}}{\cancel{10}\ 2} = \underline{\underline{-10^6/2\ A/s}}$$

(v) At $t = \infty$,

$$I_1(\infty) = \frac{100}{10} = \underline{10A}$$

$$\frac{dI_1(\infty)}{dt} = \underline{0\ A/s}$$

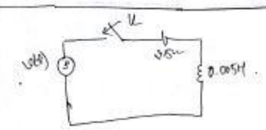
[illegible]

$$I_2(\infty) = 0$$

$$\underline{\underline{\frac{dI_2(\infty)}{dt} = 0\ A/s}}$$

6a) In the circuit of Fig Q6(a). The source voltage is $v(t) = 50 \sin 250t$. Using Laplace transform, determine the current when switch k is closed at $t = 0$.

Ans



At $t = 0^-$, k is open,

$$\underline{\underline{I(0^-) = 0A}}$$

At $t = 0^+$, k is closed,

Applying KVL to the loop

$$v(t) = 2.5 I(t) + \left[L\frac{dI(t)}{dt} - I(0^-)\right]$$

$$50 \sin 250t = 2.5 I(t) + 0.005\frac{dI(t)}{dt}$$

Applying Laplace transform,

$$\frac{50 \cdot 250}{s^2 + 250^2} = 2.5 I(s) + 0.005 s I(s)$$

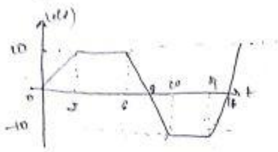
$$I(s) = \frac{50 \cdot 250}{s^2 + 250^2} \cdot \frac{1}{(2.5 + 0.005s)}$$

$$I(s) = \frac{50 \cdot 250}{s^2 + 250^2} \cdot \frac{1}{0.005\left(s + \frac{2.5}{0.005}\right)}$$

$$I(s) = \frac{10{,}000}{(s+500)} \cdot \frac{250}{s^2 + 250^2}$$

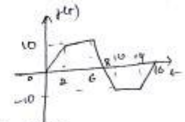
$$I(t) = L^{-1}[I(s)].$$

(b) Synthesize the periodic waveform shown in fig Q6(b) & find its Laplace transform & prove any formula used.



Ans

$v(t) = f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t) + f_6(t)$



$$f(t) = \frac{10}{2} t\,u(t) + \left(\frac{-10}{2}(t-2)u(t-2)\right) + \left[-\frac{20}{2}(t-6)u(t-6)\right] + \left(\frac{20}{2}(t-10)u(t-10)\right] + \left[+\frac{10}{2}(t-14)u(t-14)\right] + \left[-\frac{10}{2}(t-16)u(t-16)\right]$$

$$f(t) = 5t\,u(t) - 5(t-2)u(t-2) - 10(t-6)u(t-6) + 10(t-10)u(t-10) + 5(t-14)u(t-14) - 5(t-16)u(t-16)$$

Taking Laplace transform

$$F(s) = \frac{5}{s^2} - \frac{5}{s^2}e^{-2s} - \frac{10}{s^2}e^{-6s} + \frac{10}{s^2}e^{-10s} + \frac{5}{s^2}e^{-14s} - \frac{5}{s^2}e^{-16s}$$

$$V_1(s) = F(s) = \frac{5}{s^2}\left[1 - e^{-2s} - 2e^{-6s} + 2e^{-10s} + 5e^{-14s} - 5e^{-16s}\right]$$

The periodic function is given by

$$V(s) = \frac{V_1(s)}{1-e^{-Ts}}$$

where $T = 16$

$$V(s) = \frac{5/s^2\left[1 - e^{-2s} - 2e^{-6s} + 2e^{-10s} + 5e^{-14s} - 5e^{-16s}\right]}{1 - e^{-16s}}$$

we have used Ramp function & periodic function

Proof for Ramp function & periodic function.

Module - 4.

7) (a) Bandwidth of series RLC ckt. is defined as the band of frequencies over which the power in ckt is half of its max value.

At resonance frequency, max. current I_0 is given by.

$$I_0 = \frac{V}{R}.$$

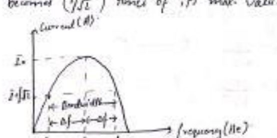
At resonance frequency, Power in ckt is max. & given by.

$$P_0 = P_{max} = (I_0)^2 R.$$

half of max power is given by.

$$P' = \frac{P_0}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R.$$

∴ Frequencies where power in ckt. is half of max. value, current becomes $(1/\sqrt{2})$ times of its max. values.



At resonant frequency, power in ckt. is given by $P_0 = P_{max} = (I_0)^2 R$

At frequency f_1, power in ckt. is half & it is given by $P' = \frac{1}{2}(I_0)^2 R$.

Similarly, at frequency f_2, power in ckt is half & it is given by $P' = \frac{1}{2}(I_0)^2 R$.

Thus f_1 is lower half-power frequency &

f_2 is upper half-power frequency.

Bandwidth $= (f_2 - f_1)$ Hz.

Now, current in series RLC ckt is given by.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \rightarrow ①$$

At half power point,

$$I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{V}{R} \rightarrow ② \left[\because I_0 = \frac{V}{R} \text{ at resonance}\right].$$

∴ equating eq ① & eq ②.

$$\frac{1}{\sqrt{2}}\frac{V}{R} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\Rightarrow \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R.$$

Squaring bothsides, we get.

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2.$$

$$\Rightarrow \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

Above eq. is quadratic in ω, which gives 2 values of ω as ω_1 & ω_2 whose half-power frequency (f_1 & f_2) are.

$$\therefore \quad \omega_2 L - \frac{1}{\omega_2 C} = R \longrightarrow ③.$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \longrightarrow ④.$$

Adding eq ③ & ④.

$$\omega_2 L - \frac{1}{\omega_2 C} + \omega_1 L - \frac{1}{\omega_1 C} = 0.$$

$$(\omega_2 + \omega_1)L - \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right)\frac{1}{C} = 0.$$

$$(\omega_2 + \omega_1)L = \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right)\frac{1}{C} = \left(\frac{\omega_1 + \omega_2}{\omega_1\omega_2}\right)\frac{1}{C}$$

$$\Rightarrow \omega_1\omega_2 = \frac{1}{LC}$$

but from resonance condition, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \omega_1\omega_2 = (\omega_0)^2.$$

$$\Rightarrow \omega_0 = \sqrt{\omega_1\omega_2}.$$

i.e, $f_1 f_2 = f_0^2$.

$$\Rightarrow \boxed{f_0 = \sqrt{f_1 f_2}}$$

7) (b) $V(t) = 10\cos 1000t$; $C = 10\mu F \Rightarrow I = I_m$

$\Rightarrow \omega = 1000$. $C = 12.5\mu F \Rightarrow I = I_m/\sqrt{2}$

find L, R & Q.

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C}$$

$$= \frac{1}{(1000)^2(10\mu)} = \frac{1}{10} = \underline{0.1H = L}$$

$$\Rightarrow \boxed{L = 0.1H}$$

When $C = 12.5\mu F$.

$$I = \frac{I_m}{\sqrt{2}} = \frac{V}{\sqrt{2}R} \rightarrow (1) \quad \left[\because I_m = \frac{V}{R} \text{ at resonance}\right].$$

$$\& \quad I = \frac{V}{Z} = \frac{V}{\sqrt{R^2+X^2}} \rightarrow (2).$$

equating (1) & (2).

$$\sqrt{R^2+X^2} = \sqrt{2}R.$$

Comparing LHS & RHS

$$X^2 = R^2 \Rightarrow X = R.$$

but $X = X_L - X_C$.

$$\Rightarrow X_L - X_C = R.$$

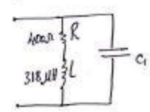
$$\omega L - \frac{1}{\omega C} = R.$$

$$1000(0.1) - \frac{1}{1000(12.5\mu)} = R.$$

$$\Rightarrow \boxed{R = 20\Omega}.$$

Q factor $= \frac{\omega L}{R} \Rightarrow Q = \frac{(1000)(0.1)}{20} \Rightarrow \boxed{Q = 5}$

7) (c). $R = 400\Omega$; $L = 318\mu H$; $f_r = 1MHz$.



Here $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC_1} - \frac{R^2}{L^2}}$

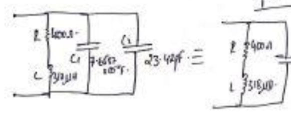
$$\Rightarrow f_r^2 = \frac{1}{4\pi^2}\left(\frac{1}{LC_1} - \frac{R^2}{L^2}\right).$$

$$4\pi^2 f_r^2 + \frac{R^2}{L^2} = \frac{1}{LC_1}$$

$$\Rightarrow C_1 = \frac{1}{L\left(4\pi^2 f_r^2 + \frac{R^2}{L^2}\right)}$$

$$= \frac{1}{(318\mu H)\left(4\pi^2(1M)^2 + \left(\frac{400}{318\mu}\right)^2\right)}$$

$$\Rightarrow \boxed{C_1 = 7.6587 \times 10^{-11} F}$$



$3.9478 \times 10^{?} + 1.5822 \times 10^{12}$
$(1.7567\times10^{12}) - (1.5822\times10^{12})$
1.8057×10^{10} $\left(\frac{1.7956\times10^{-20}}{1.0000\times10^{-?}}\right)$
$1319423\,3.59$
$C_1 = 7.6587\times10^{-11}$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} \Rightarrow C_{eq} = 1.79\times10^{-11} F.$$

$$\Rightarrow f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC_{eq}} - \frac{R^2}{L^2}} = \frac{1}{2\pi}\sqrt{\frac{1}{(318\mu)(1.79\times10^{-11})} - \frac{(400)^2}{(318\mu)^2}}$$

$$\Rightarrow \boxed{f_r = 2.099\ MHz.}$$

9 a. Z parameters of a network are obtained from an experiment. Explain how y parameters and ABCD parameters can be computed from experimental data.

Z parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2 \rightarrow (1)$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2 \rightarrow (2)$$

Y parameters

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \rightarrow (3)$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2 \rightarrow (4)$$

To obtain Y parameters

From (1) & (2)

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\Delta Z} \qquad \Delta Z = Z_{11}Z_{22} - Z_{21}Z_{12}$$

$$I_1 = \frac{V_1 Z_{22} - V_2 Z_{12}}{\Delta Z}$$

$$I_1 = \frac{Z_{22}}{\Delta Z}V_1 + \left(-\frac{Z_{12}}{\Delta Z}\right)V_2 \rightarrow (5)$$

$$I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\Delta Z}$$

$$I_2 = \frac{Z_{11}V_2}{\Delta Z} - \frac{Z_{21}V_1}{\Delta Z}$$

$$I_2 = \left(-\frac{Z_{21}}{\Delta Z}\right)V_1 + \frac{Z_{11}}{\Delta Z}V_2 \rightarrow (6)$$

Compare (3) & (4) with (5) & (6)

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = -\frac{Z_{12}}{\Delta Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{Z_{11}}{\Delta Z}$$

$$\therefore Y = \begin{bmatrix} \frac{Z_{22}}{\Delta Z} & \frac{-Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{bmatrix}$$

ABCD parameters

$$V_1 = AV_2 - BI_2 \rightarrow (7)$$
$$I_1 = CV_2 - DI_2 \rightarrow (8)$$

From (2)

$$V_2 - Z_{22}I_2 = Z_{21}I_1$$

$$I_1 = \frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}}I_2 \rightarrow (9)$$

Sub (9) in (1)

$$V_1 = Z_{11}\left(\frac{V_2}{Z_{21}} - \frac{Z_{22}}{Z_{21}}I_2\right) + Z_{12}I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{11}Z_{22}}{Z_{21}}I_2 + Z_{12}I_2$$

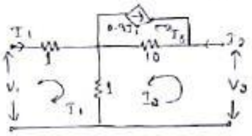
$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 + \left(\frac{Z_{21}Z_{12} - Z_{11}Z_{22}}{Z_{21}}\right)I_2$$

$$V_1 = \frac{Z_{11}}{Z_{21}}V_2 - \frac{(Z_{11}Z_{22} - Z_{21}Z_{12})}{Z_{21}}I_2 \rightarrow (10)$$

Compare (9) & (10) with (7) & (8)

$$T = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22} - Z_{21}Z_{12}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

9(b) Find Z and Y parameters of the network show in fig



→ apply KVL to loop1

$$V_1 = 1I_1 + 1(I_1 + I_2)$$
$$V_1 = 2I_1 + I_2 \quad \text{—} ①$$

$$\boxed{I_3 = 0.9I_1}$$

apply KVL to loop2

$$V_2 = 10(I_2 + I_3) + 1(I_1 + I_2)$$
$$V_2 = 11I_2 + 10I_3 + I_1$$
$$V_2 = 11I_2 + 10(0.9I_1) + I_1$$
$$V_2 = 9I_1 + 11I_2 \quad \text{—} ②$$

Comparing ① & ②

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$Z_{11} = 2 \quad Z_{12} = 1 \quad Z_{21} = 9 \quad Z_{22} = 11$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 9 & 11 \end{bmatrix}$$

Y parameters

$$Y_{11} = \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{11}{2 \times 11 - 9 \times 1} = 0.846 \ \mho$$

$$Y_{12} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{21}Z_{12}} = \frac{1}{2 \times 11 - 9} = \frac{1}{13} = -0.076 \ \mho$$

$$Y_{21} = \frac{-Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = \frac{-9}{13} = -0.692 \ \mho$$

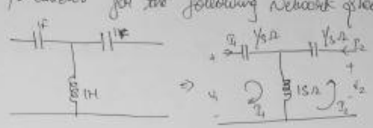
$$Y_{22} = \frac{Z_{11}}{Z_{11}Z_{22} - Z_{21}Z_{12}} = \frac{2}{13} = 0.153 \ \mho$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.846 & -0.076 \\ -0.692 & 0.153 \end{bmatrix} \mho$$

Network Analysis

Module - 5 (Question -10)

10)
a) Find Z and h- Parameters for the following Network given



Applying KVL to mesh 1:

$$V_1 - \frac{1}{s} I_1 - 1(I_1 + I_2) = 0$$

$$V_1 - I_1\left(\frac{1}{s}+1\right) - I_2 = 0$$

$$V_1 = \left(\frac{1}{s}+1\right) I_1 + I_2 \rightarrow (1)$$

Applying KVL to mesh 2:

$$V_2 - \frac{1}{s} I_2 - 1(I_2 + I_1) = 0$$

$$V_2 = +I_1 + I_2\left(\frac{1}{s}+1\right) \rightarrow (2)$$

we know the equations for Z-parameters.

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad , \quad V_2 = Z_{21} I_1 + Z_{22} I_2$$

comparing these equations with eqn (1) and (2)

$$\boxed{Z_{11} = \left(\frac{1}{s}+1\right)\Omega , \quad Z_{12} = 1\Omega , \; Z_{22} = \left(1+\frac{1}{s}\right)\Omega , \; Z_{21} = 1\,\Omega}$$

H-parameters in terms of Z-parameters

we get $H_{11} = \frac{\Delta Z}{Z_{22}}$, $H_{12} = \frac{Z_{12}}{Z_{22}}$.

$$H_{22} = 1/Z_{22} \, , \quad H_{21} = -Z_{21}/Z_{22}.$$

So we know,

$$Z_{11} = \frac{1}{s} + 1\,\Omega , \quad Z_{12} = 1\,\Omega , \quad Z_{22} = 1/s + 1\,\Omega$$

$$Z_{21} = 1\,\Omega$$

$$H_{12} = \frac{1}{\frac{1}{s}+1} = \frac{1}{\frac{1+s}{s}} \Rightarrow \boxed{h_{12} = \frac{s}{s+1}}$$

$$H_{22} = \frac{1}{Z_{22}} = \frac{1}{\frac{1}{s}+1} \Rightarrow \boxed{H_{21} = \frac{s}{s+1}}$$

$$H_{21} = -Z_{21}/Z_{22} = \frac{-1}{\frac{1}{s}+1} \Rightarrow \boxed{H_{21} = \frac{-s}{s+1}}$$

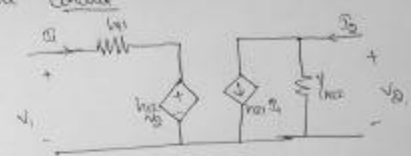
$$\boxed{H_{11} = \frac{\left(\frac{1+s}{s}\right)^2 - 1}{\frac{1+s}{s}}}$$

10)
b) Write a note on hybrid parameters with its equivalent circuit

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

Equivalent circuit



10) c) Explain symmetry and reciprocal property of 2-port network

Ans Condition for Reciprocity:

If the ratio of voltage at one port to the current at the other port is same to the ratio if the positions of voltage and current are interchanged then the network is said to be reciprocal.

Z-Parameters: $Z_{12} = -Z_{21}$

H-parameters: $h_{12} = -h_{21}$

T-parameters: $AD - BC = 1$

Y-Parameters: $Y_{12} = Y_{21}$

Condition for symmetry:

Z-parameters: $Z_{11} = Z_{22}$

Y-Parameters: $Y_{11} = Y_{22}$

h-Parameters: $\Delta h = 1$ $(h_{11}h_{22} - h_{21}h_{12} = 1)$