Modified

USN | | | | | | | | | | 10ME74

Seventh Semester B.E. Degree Examination, Dec.2017/Jan.2018
Operations Research

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Use of normal distribution tables is permitted.

PART – A

1 a. Define operations research. Give the historical development of operations research. **(06 Marks)**

b. A farmer has 100 acre land. He can sell all the tomatoes, lettuce or radishes that he can raise. The price he can obtain is ₹ 10/- per kg of tomatoes, ₹ 7/- a head of lettuce and ₹ 10/- per kg of radishes. The average yield per acre is 2000 kg of tomatoes, 3000 heads of lettuce and 1000 kg of radishes. Labour required for Sowins, Cultivating and harwesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour is available at ₹ 100/- per man day. Formulae this problem as LPP to maximize the farmer's profit. **(08 Marks)**

c. Define the following terms with reference to LPP:
(i) Feasible solution. (ii) Infeasible solution. (iii) Unbounded solution. **(06 Marks)**

2 a. Explain the concepts of degeneracy in simplex method. **(05 Marks)**

b. Solve the following LPP using simplex method,

$z_{min} = x_1 - 3x_2 + 2x_3$

Subject to: $3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$ **(15 Marks)**

3 a. Larsen and Toubro company needs 3, 3, 4 and 5 million cubic feet of fill at four earthen dam sites I, II, III and IV in Karnataka. It can transfer the fill from three mounds A, B, C where 2, 6, 7 million cubic feet of fill is available respectively. Costs of transportation of one million cubic feet of fill from mounds to the four sites in lakhs of rupees are given in the following table.

From \ To	I	II	III	IV
A	15	10	17	18
B	16	13	12	13
C	12	17	20	11

Determine the optimal transportation plan which minimizes the total transportation cost to the company. **(12 Marks)**

b. A batch of 4 jobs can be assigned to 5 different machines. The following table shows the installation time in hours for each job on various machines. Find the optimal assignment of jobs to machines which will minimize the total installation time. **(08 Marks)**

Job \ Machine	M_1	M_2	M_3	M_4	M_5
J_1	10	11	4	2	8
J_2	7	11	10	14	12
J_3	5	6	9	12	14
J_4	13	15	11	10	7

4 a. What is an integer programming problem? Explain the importance of integer programming. **(05 Marks)**

b. Use branch and bound method to solve the following integer programming problem:

$Z_{max} = 7x_1 + 9x_2$

Subject to,

$-x_1 + 3x_2 \le 6$

$7x_1 + x_2 \le 35$

$x_2 \le 7$

$x_1, x_2 \ge 0$ and are integers. **(15 Marks)**

PART – B

5 a. Explain the basic steps involved in PERT/CPM. **(04 Marks)**

b. Write short notes on crashing of a project network. **(04 Marks)**

c. An organization has large number of activities but it is interested in controlling a part of these activities to 7 in number. The following data is available for these activities.

Activity	Precedence	Time (days)		
		t_0	t_m	t_p
A	-	4	6	8
B	A	6	10	14
C	A	8	15	22
D	B	9	9	9
E	C	10	14	18
F	A	5	5	5
G	D, E, F	8	10	12

(i) Draw a PERT network for the activities.

(ii) Identify the critical path and its duration.

(iii) If the organization puts 47 days as dead line to complete, what is the probability of completion in 47 days. **(12 Marks)**

6 a. Define the term queue. State and explain the characteristics of queing system. **(08 Marks)**

b. Patrons arrive at a reception counter at an average inter arrival time of 2 minutes. The receptionist on duty takes an average of one minute per person. (Arrivals are as per exponential and services are as per Poisson distribution).

(i) What is the probability that a person will straight away meet the receptionist?

(ii) For what portion of the time the receptionist is busy?

(iii) What is the average queue length?

(iv) What is the average number of patrons in the system?

(v) What is the average waiting time of a patron?

(vi) What is the average time a patron spends in the system? **(12 Marks)**

7 a. Explain the following terms related to theory of games:

(i) Pay-off matrix.

(ii) Min.Max and Max.Min principle.

(iii) Dominance rule.

(iv) Pure and mixed strategies.

(v) Fair game. **(10 Marks)**

b. Use the dominance rule and solve the following game whose pay.off matrix for player A is:

		B: B_1	B_2	B_3
A	A_1	-4	6	3
	A_2	-3	-3	4
	A_3	2	-3	4

(10 Marks)

8 a. List out any four assumptions underlying sequencing problems. **(04 Marks)**

b. Consider the processing times (in minutes) of 5 jobs each of which must undergo through 2 machines M_1 and M_2 in the order M_1M_2.

Machine	Job J_1	J_2	J_3	J_4	J_5
M_1	5	1	9	3	10
M_2	2	6	7	8	4

Obtain the sequence for the jobs that minimizes the total elapsed time and also find the idle time of both the machines. **(08 Marks)**

c. There are five jobs, each of which is to be processed through machines A, B and C in the order CAB, processing time in hours is given below:

Job	Machine A	B	C
1	4	7	3
2	5	9	8
3	1	5	7
4	2	6	5
5	3	10	4

Determine the optimum sequence for the jobs and the total elapsed time. **(08 Marks)**

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Scheme & Solutions

Signature of Scrutinizer

Subject Title : Operations Research   Subject Code : 10ME74

Question Number	Solution	Marks Allocated
1 a)	Definition of the term Operations Research	02
	Historical background stating it's foundation during world war-II in military operations.	04
b)	Let x_1, x_2, x_3 be the number of acres of land in which tomatoes, lettuce and radishes are grown.	01
	Sales income : $2000x_1 \times 10 + 3000x_2 \times 7 + 1000x_3 \times 10$	
	Expenditure on the labour : $100(5x_1 + 6x_2 + 5x_3)$	03
	Farmer's net profit $Z_{max} = 19{,}500x_1 + 20{,}400x_2 + 9500x_3$	02
	Subject to	
	$x_1 + x_2 + x_3 \leq 100$ (land constraint)	02
	$5x_1 + 6x_2 + 5x_3 \leq 400$ (man-days constraint)	
	$x_1, x_2, x_3 \geq 0$	

Question Number	Solution	Marks Allocated
c)	i. Feasible Solution: Any solution of LPP which satisfies the constraints	02
	ii. Infeasible Solution: If an LPP doesn't get the common boundary satisfying all the constraints, its solution is infeasible.	02
	iii. Unbounded Solution: If an LPP has no limits on constraints then the solution is unbounded	02
2a)	Degeneracy: In the simplex table, if the tie exists between 2 or more basic variables to decide about the leaving variable, it is known as degeneracy. It may arise at the initial stage or at any stage during obtaining solution of LPP.	05
b)	$Z_{min} = -Max(Z) = -x_1 + 3x_2 - 2x_3$ $Z_{max} = -x_1 + 3x_2 - 2x_3$ Subject to: $3x_1 - x_2 + 2x_3 + u_1 = 7$ $-2x_1 + 4x_2 + u_2 = 12$ $-4x_1 + 3x_2 + 8x_3 + u_3 = 10$	02

Question Number	Solution	Marks Allocated
	I Simplex table	

Basis	C_B	x_1	x_2 (EV)	x_3	u_1	u_2	u_3	B	θ/ratio
		3	-1	2	1	0	0	7	-
u_1	0	-2	4	0	0	1	0	12	3 ← PR
LV ← u_2	0	-4	3	8	0	0	1	10	3.33
u_3	0								
C_j		-1	3	-2	0	0	0		
Z_j		0	0	0	0	0	0	0	
$C_j - Z_j$		-1	3 (PC)	-2	0	0	0		

05

04

Second Simplex Table

Third Simplex Table (Final) from which

$x_1 = 4, x_2 = 5, Z_{max} = 11$

$Z_{min} = -Z_{max} = -11$

04

3 a) It is a balanced, minimisation transportation problem.

i. Initial Basic feasible Solution by VAM:

A - II : 2 B - IV : 1
B - II : 1 C - I : 3
B - III : 4 C - IV : 4

06

ii. Optimality Check (using U-V method)

04

Initial Basic Feasible Solution itself is Optimum with total transportation cost of Rs. 174 lakhs of rupees

02

Subject Title: Operations Research   Subject Code: 10ME74

Question Number	Solution	Marks Allocated
b)	Balancing the given problem	02
	Obtaining the allocations with modification of reduced matrix	03
	Optimal assignment $J_1 - M_4$, $J_2 - M_1$, $J_3 - M_2$, $J_4 - M_5$ and $J_5 - M_3$ with total time of 22 units.	03.
4 a)	The programming system where the results are only integers and not fractions	02
	It is a valuable tool in 'OR' having potential applications. These problems occur frequently in business and industry	03
b)	Ignoring the integer restriction. $x_1 = 9/2$, $x_2 = 7/2$, $Z_{max} = 63$	05
	Sub problem 1: $Z_{max} = 7x_1 + 9x_2$ s.t. $-x_1 + 3x_2 \leq 6$, $7x_1 + x_2 \leq 35$, $0 \leq x_1 \leq 4$ and $0 \leq x_2 \leq 7$	02
	Sub problem 2, Sub problem 3, Sub problem 4 (2 marks each)	08
	Optimal solution is $x_1 = 4$, $x_2 = 3$ and $Z_{max} = 55$ (Sub problem 1).	

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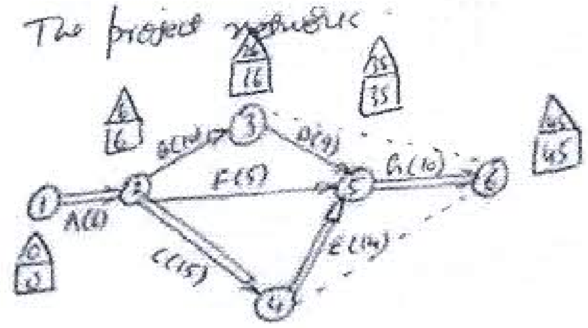
Question Number	Solution	Marks Allocated
5 a)	Mentioning of basic steps	01
	i. planning ii Scheduling iii Controlling	
	Brief explanation - 1 Mark each	03
b)	Meaning of Crashing of a project net-work	02
	Examples/ Situations where Crashing is essential	02
c)	i) The project network 	06
	ii Critical path: $A \rightarrow C \rightarrow E \rightarrow G$ T_E = project length: 45 days, $\sigma^2 = 8.9$	03
	iii probability of completing the project in 47 days $Z = \frac{T_S - T_E}{\sigma} = \frac{47-45}{2.98} = 0.67$ which the probability is 74%	03

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Question Number	Solution	Marks Allocated
6 a)	Definition/Meaning of Queue	02
	. Characteristics	
	i. Input pattern ii. Service Mechanism	02
	iii. Queue discipline iv. Customer's behaviour	04
b)	$\lambda = 30/hr.$, $\mu = 60/hr.$	
	i. $P_0 = 0.5$	
	ii. $\rho = \frac{\lambda}{\mu} = 0.5$	
	iii. $\frac{\lambda^2}{\mu(\mu-\lambda)} = 0.5$	
	iv. $\frac{\lambda}{\mu-\lambda} = 1$	
	v. $W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1}{60}$ hr. = 1 min	12
	vi. $W_s = \frac{1}{\mu-\lambda} = 2$ min	
	(2 Marks each)	
7 a)	i) Pay-off Matrix: The representation of gains and losses resulting from different courses of action of the players.	02
	ii) Min Max and Max Min principle: These are optimal strategies for both the players	02

Question Number	Solution	Marks Allocated
	for player 'A' tries to maximise his gains while player 'B' tries to minimise his losses. Thus, Max min is applicable for player 'A' and min Max is applicable for player 'B'.	
	iii Dominance rule: Concept of reducing the size of matrix row wise / column wise	02
	iv Pure and Mixed Strategies: Selection of a deterministic / firm strategies is pure strategy while if there is mix of strategies then it is mixed strategy game.	02
	v. Fair game: - If the value of game is zero then it is a fair game	02
	(2 Marks each)	
b)	By using dominance rule the reduced matrix is	
	B: I, II; A: I → −4, 6; III → 2, −3	04
	Best strategies for player 'A': $(\frac{1}{3}, 0, \frac{2}{3})$	02
	Best strategies for player 'B': $(\frac{3}{5}, \frac{2}{5}, 0)$	02

Reduced matrix:

		B: I	B: II
A	I	−4	6
	III	2	−3

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Question Number	Solution	Marks Allocated
	Value of game (for player 'A') = 0 V = 0 (It is a fair game)	02
8 a)	Basic assumptions such as, i No machine can process more than one operation at a time ii. All jobs are ready for processing iii Machines are having different capacity iv Transfer time of jobs is negligible	04
b)	Optimal sequence by Johnson's rule: 2 4 3 5 1	04
	Total elapsed time : 30 minutes	02
	Idle time for m/c 1 : 2 minutes	01
	Idle time for m/c 2 : 3 minutes.	01
c)	Given order is CAB Optimal sequence is J5 | J1 | J4 | J2 | J3	04
	Table showing In, out times on the machines C, A and B giving the total elapsed time of 44 hrs.	04
	END	

8/8

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