Solutions for VTU examination question paper ①

June / July 2018. (2017-18, even sem).

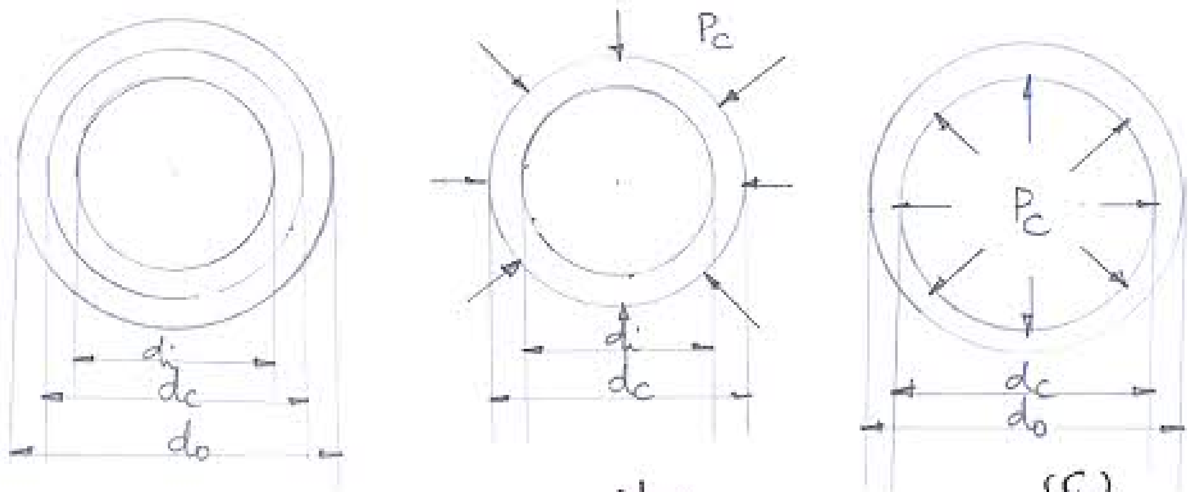
Sem : VI   Staff : RPR.

Sub : Design of Machine Elements II   Max. Marks : 80

Sub code : 15ME64.

1 (a) Explain the compounding in cylinders

Ans:

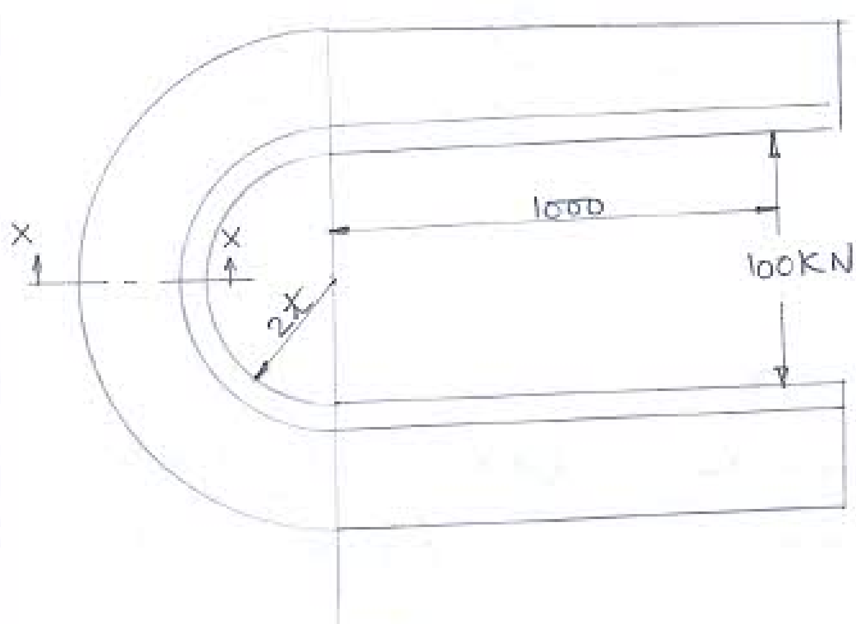


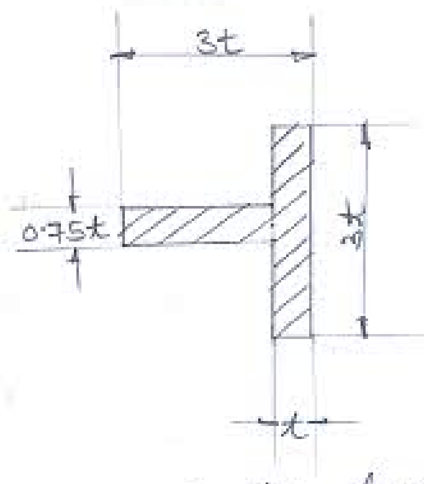
(a) Compound cylinder (b) Inner cylinder (or) tube (c) outer cylinder (or) Jacket.

A compound cylinder, consisting of a cylinder and a jacket is shown in figure (a). The inner diameter of jacket is slightly smaller than the outer diameter of cylinder (tube). when the jacket is heated, it expands sufficiently to move over the cylinder. As the jacket cools, it tends to contract onto the inner cylinder, which induces residual compr. stresses. There is a shrinkage pressure & contact pressure P_c between the inner cylinder and ② jacket. This pressure tends to contract the inner cylinder and expand the jacket as shown in figures (b) & (c).

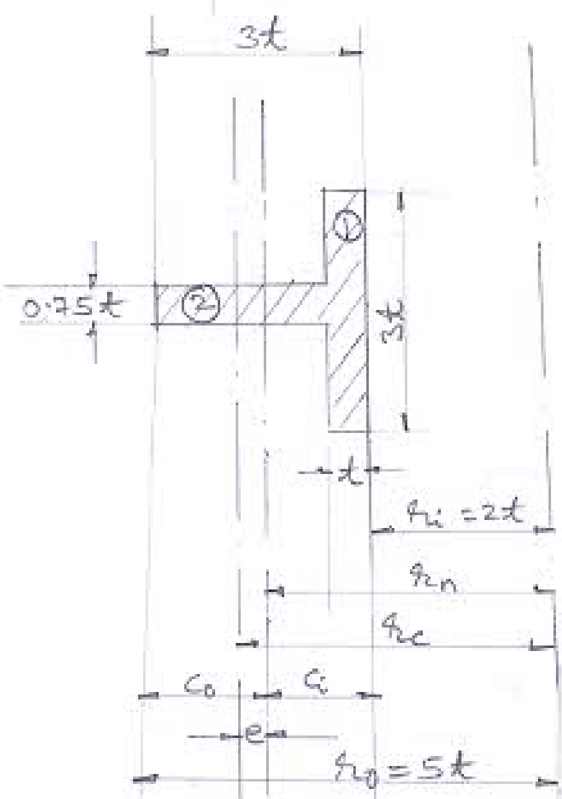
~~1(b) The C-frame of~~

1(b) The C-frame of a 1000 KN capacity press is shown in fig Q1(b). The material of the frame is grey cast iron FG 200 and the factor of safety is 3. Determine the dimensions of the frame. (3)





section at X-X



<u>data</u>

$F = 100\,KN$
$= 100 \times 10^3\,N$

Material : grey CI FG 200

$\sigma_y = 200\,MPa \left(\frac{T1.4}{P1.17}\right)$

$FS = 3.$

The max. tensile stress occurs at the innermost fibre of section X-X.

The combined stress at the inner most fibre is given by

$$\sigma_{ri} = \frac{F}{A} + \frac{M_b c_i}{A e r_i} \quad \text{—(1)}$$

Allow. tensile stress (σ_{ri})

$$\text{Allow stress } \sigma_{ri} = \frac{\sigma_y}{FS} = \frac{200}{3} = 66.67 \text{ N/mm}^2$$

r_c

$$r_c = r_i + \bar{x}$$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{(3t \times t)(0.5t) + (2t \times 0.75t)(2t)}{(3t \times t) + (2t \times 0.75t)}$$

$$= \frac{1.5t^3 + 3t^3}{3t^2 + 1.5t^2}$$

$$= \frac{4.5t^3}{4.5t^2}$$

$$= t$$

$$\therefore r_c = 2t + t$$
$$= 3t \text{ mm}$$

r_n

For I section

$$r_n = \frac{A}{b_i \ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 \ln\left(\frac{r_o - a_o}{r_i + a_i}\right) + b_o \ln\left(\frac{r_o}{r_o - a_o}\right)}$$

For T section $b_0 = 0$ & $a_0 = 0$.

$$\therefore r_n = \frac{A}{b_i \ln\left(\frac{r_i + a_i}{r_i}\right) + b_2 \ln\left(\frac{r_o}{r_i + a_i}\right)}.$$

here $A = (3t \times t) + (2t \times 0.75t)$
$= 3t^2 + 1.5t^2$
$= 4.5t^2$.

$b_i = 3t$

$a_i = t$

$b_2 = 0.75t$

$$\therefore r_n = \frac{4.5t^2}{3t \ln\left(\frac{2t + t}{2t}\right) + 0.75t \ln\left(\frac{5t}{2t + t}\right)}$$

$$= \frac{4.5t^2}{3t \ln 1.5 + 0.75t \ln\left(\frac{5}{3}\right)}$$

$$= \frac{4.5t^2}{(1.216t) + (0.383t)}$$

$$= (2.814t) \text{ mm}$$

Now $c_i = r_n - r_{ai}$
$= \cancel{3t - 2.814t}\ 2.814t - 2t$
$= \cancel{(0.186t)}\text{ mm}\ (0.814t)$ mm.

$e = r_c - r_n$
$= 3t - 2.814t$
$= (0.186t)$ mm.

M_b = B.M about C.G.

$= 100 \times 10^3 (1000 + x_c)$

$= 100 \times 10^3 (1000 + 3t)$

Sub in (1)

$$66.67 = \frac{100 \times 10^3}{4.5t^2} + \frac{100 \times 10^3 (1000+3t) \times (\overset{0.814t}{\cancel{0.186t}})}{4.5t^2 \times (0.186t) \times 2t}$$

$$= \frac{100 \times 10^3}{4.5t^2} + \frac{\overset{81.4 \times 10^6 t}{\cancel{18.6 \times 10^6 t}} \cancel{18,600t} + \overset{2,44,200}{\cancel{55,800}} t^2}{1.674t^4}$$

$$= \frac{(1,67,400t^2) + (366.3 \times 10^6 t) + (1.09 \times 10^6) t^2}{7.533t^4}$$

$\Rightarrow \quad 502.22t^4 = (1,67400)t^2 + (366.3 \times 10^6)t + (1.09 \times 10^6)t^2$

dividing with 't'

$502.22t^3 - 3086.29t - 729.36 \times 10^3 = 0.$

Solving $\quad x_1 = 101.38$ mm

x_2 = complex

x_3 = complex

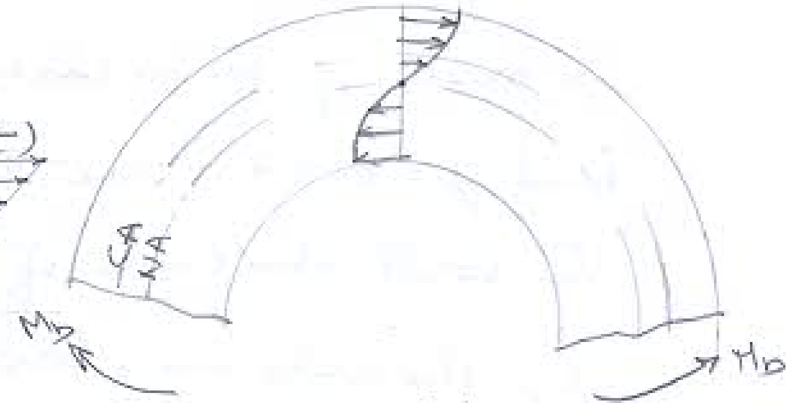
$\therefore \quad t = 101.38$ mm.

$\simeq 100$ mm.

2(a) Differentiate between a straight beam and a curved beam. (7)

Ans:

Straight beam



curved beam

Straight beam	curved beam
1. Radius of curvature is infinite.	1. Finite radius of curvature.
2. when subjected to BM, the centroidal & neutral axes coincide.	2. when subjected to BM, the neutral axis is shifted towards centre of curvature.
3. The bending stress distribution is linear	3. The bending stress distribution is hyperbolic.

2(b) The inner diameter of a cylinder is 250 mm. The pressure is limited to 15 MPa. The cylinder is made of plain carbon steel with $\sigma_{ut} = 340 \, N/mm^2$ and $\mu = 0.27$. Take the F.S as 5, and calculate the wall thickness if.

i) The ends are closed

ii) The ends are open.

Ans: data

$d_i = 250 \, mm$.

$P_i = 15 \, N/mm^2$

$\sigma_u = 340 \, N/mm^2$

$\nu = 0.27$ (poisson's ratio).

$FS \, (n) = 5$.

to find

h when (i) The cylinder ends are ~~open~~ closed

ii) the cylinder ends are open.

i) When the cylinder is made of ductile material & the ends are closed, clavarino's equation is used to find the cylinder wall thickness.

It is given by

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta' + (1-2\nu)P_i}{\sigma_\theta' - (1+\nu)P_i}} - 1\right]$$

here $\sigma_\theta' = \frac{\sigma_u}{FS} = \frac{340}{5} = 68 \, N/mm^2$.

$$\therefore\ h = \frac{250}{2}\left[\sqrt{\frac{68+(1-0.54)15}{68-(1+0.27)15}}-1\right]$$

$$= 29.62\,\text{mm say } 30\,\text{mm}.$$

ii) When the cylinder is made of ductile material and when the ends are open, ~~Bir~~ Birnie's equation is applicable. It is given by

$$h = \frac{d_i}{2}\left[\sqrt{\frac{\sigma_\theta' + (1-\nu)P_i}{\sigma_\theta' - (1+\nu)P_i}}-1\right]$$

$$= \frac{250}{2}\left[\sqrt{\frac{68+(1-0.27)15}{68-(1+0.27)15}}-1\right]$$

$$= \frac{250}{2}\left[\sqrt{\frac{68+10.95}{68-19.05}}-1\right]$$

$$= 33.74\,\text{mm say } 34\,\text{mm}.$$

5. It is required to design a pair of spur gears with 20° FDI teeth based on the Lewis equation. The velocity factor is to be used to account for dynamic load. The pinion shaft is connected to a 10 KW, 1440 rpm motor. The starting torque of motor is 150% of rated torque. The speed reduction is 4:1. The pinion as well as gear are made of plain carbon steel ($\sigma_0 = 200\ N/mm^2$). Take the no. of teeth on pinion = 18. Design the gears. Specify their dimensions & suggest suitable hardness. Assume carefully cut gears (class II).

data

Ans. $\alpha = 20°$ FDI

$P = 10\,KW$

$n_1 = 1440\ rpm$.

$C_S = 1.5$

$\frac{n_1}{n_2} = \frac{4}{1}$.

$\therefore \frac{1440}{n_2} = 4$

$\Rightarrow n_2 = \frac{1440}{4} = 360\ rpm$.

$\sigma_{01} = \sigma_{02} = 200\ N/mm^2$

$z_1 = 18$

$\therefore z_2 = 18 \times 4 = 72$.

1. Design tangential tooth load (F_t)

$$F_t = \frac{9550 \times P \times 1000}{n} \times \frac{C_s}{r}$$

$$= \frac{9550 \times 10 \times 1000}{1440} \times \frac{1.5}{\left(\frac{m \times 18}{2}\right)}$$

$$= \left(\frac{11,053.24}{m}\right)$$

2. module (m)

Sine both the pinion & gear are made of same meterial, pinion is weeker.

∴ Apply Lewis eqⁿ to pinion to solve for module.

$$F_t = \sigma_0 \cdot b \cdot y \cdot p \cdot K_v$$

$$= \sigma_{01} \cdot b \cdot y_1 \cdot p \cdot K_v \quad (\text{for pinion}).$$

Assume $b = 10m$.

$$y_1 = \left(0.154 - \frac{0.912}{18}\right)$$

$$= 0.103.$$

$$p = \pi m.$$

For carefully cut teeth, $K_v = \dfrac{4.5}{4.5 + v_m}$.

Where $v_m = \dfrac{\pi d n}{60,000} = \dfrac{\pi d_1 n_1}{60,000}$

$$= \frac{\pi (m \times 18) \times 1440}{60,000} = (1.357m) \text{ m/sec.}$$

$$\therefore K_v = \frac{4.5}{4.5 + 1.357m}$$

$$\therefore \frac{11,053}{m} = 200 \times (10m) \times (0.103) \times (\pi m) \times \frac{4.5}{4.5 + 1.35m}$$

$$= \frac{2912.25\, m^2}{(4.5 + 1.357m)}$$

$$\Rightarrow \quad 2912.25\, m^3 = 49738 + 14998\, m$$

$$\therefore \quad 2912.25m^3 - 14,998m - 49,738 = 0$$

Solving $\quad m_1 = 3.23$ mm

m_2 = complex

m_3 = complex.

Selecting std. module from T 23.3/p23.61

$m = 4$ mm.

3. pitch circle diameters (d_1, d_2)

$$d_1 = m z_1 = 4 \times 18 = 72 \text{ mm}$$

$$d_2 = 72 \times 4 = 288 \text{ mm}$$

4. Face width (b)

$b = 10m$

$= 40\,mm$

5. gear teeth proportions

$h_a = 1m$

$= 4\,mm$

$h_f = 1.25m$

$= 5\,mm$

$h = 2.25m$

$= 9\,mm$

$h' = 2m$

$= 8\,mm$

$c = \cancel{h_f - h_a}\ 0.25m$

$= 1\,mm$

6. check for dynamic load (F_d)

$$F_d = F_t + \frac{21 v_m (F_t + bc)}{21 v_m + \sqrt{F_t + bc}}$$

$\cancel{0.05 - 580}$

$\cancel{0.06 - ?}$

$\cancel{\frac{0.06}{0.05} \times 580}$

$\cancel{= 696\ N/mm}$

here $v_m = 1.35m$

$= 1.35 \times 4$

$= 5.4\ m/sec.$

For $m = 4\,mm$, allow error (f) $= 0.025\,mm$ (Fig 23.40(a) / p 23.34)

~~For $v_m = 5.4\,m/sec$, allow. error $f = 0.06\,mm$~~

(Fig 23.35 / p 23.35)

From T 23.32 / p 23.74,

$c = \cancel{696}\ 290\ N/mm.$

$\therefore \cancel{F_d} = \quad F_t = \dfrac{11,053.24}{4} \qquad b = 40\,mm$

$= 2763.31\ N.$

$$\therefore F_d = 2763.31 + \frac{21 \times 5.4 \left(2763.31 + \frac{40 \times \cancel{696}}{290}\right)}{21 \times 5.4 + \sqrt{2763.31 + \left(\frac{40 \times 696}{290}\right)}}$$

$$= 2763.31 + \frac{\cancel{347}\ \cancel{34.704 \times 10^6}\ \cancel{30,716.71}}{113.4 + \cancel{124.93}} \qquad \frac{1.628 \times 10^6}{233.24}$$

$= 9743\ N$

$= \cancel{2869.84\ N}\ \cancel{267.01\ KN}.\ \cancel{14.79}\ KN$

Endurance strength $F_y = \sigma_{ef} \cdot b \cdot y_1 \cdot m$
for pinion

Endurance limit for steel $(\sigma_{ef}) = 290\ MPa$
$\left(\dfrac{T\ 23.35}{P\ 23.76}\right)$

$Y_1 = \pi y_1$
$= \pi \times 0.103$
$= 0.323.$

$\therefore F_y = 290 \times 40 \times 0.323 \times 4$
$= 14,987.2\ N$
$= 14.98\ KN.$

Since $F_y > F_d$, the design is ~~not~~ safe.

7. check for wear load (F_w)

$F_w = d_1 \cdot b \cdot Q \cdot K$

where $Q = \dfrac{2z_2}{z_1 + z_2} = \dfrac{2 \times 72}{(18+72)} = 1.6.$

For safe design,

$F_w > F_d$.
$$\cancel{14.79\times10^3} \;\; \cancel{267.01\times10^3} \;\; 9743$$

$\therefore d_1 \cdot b \cdot Q \cdot K \geq \cancel{2869.84}$ $\cancel{267.01\times10^3}$ $\cancel{14.79\times10^3}$ 9743

$72\times40\times1.6\times K \geq \cancel{2869.84}$

$\therefore$

$\Rightarrow K \geq \cancel{0.622}\ 2.11$ MPa. $\cancel{57.94\text{ MPa}}$ 3.2

$\therefore$ BHN for pinion $= \cancel{450}\ 400$
" gear $= \cancel{450}\ 350$

T 23.37 B.
P 23.80

6. A pair of bevel gears with 20° pressure angle consists of a 20 teeth pinion meshing with a 30 teeth gear. The module is 4mm, while face width is 20mm. The material for pinion and gear is steel ($\sigma_0 = 250 N/mm^2$). The gear teeth are lapped and ground (class 3) and the surface hardness is 400BHN. The pinion rotates at 500 rpm and receives 2.5 KW power from an electric motor. The starting torque of motor is 150% of the rated torque. Det. the factor of safety against bending failure and against pitting failure. (16)

Ans: data

$\alpha = 20^\circ$

$$\left.\begin{array}{l} z_1 = 20 \\ z_2 = 30 \end{array}\right\} \Rightarrow i = \frac{z_2}{z_1} = \quad i = 1.5$$

$m = 4mm$

$b = 20mm$

$\sigma_{01} = \sigma_{02} = 250 N/mm^2$

BHN for pinion & gear $= 400$

$n_1 = 500$ rpm

$P = 2.5 KW$

$C_S = 1.5$

to find

Factor of safety

1) based on bending failure &

2) ,, pitting failure.

1. Design tangential tooth load (F_t)

$$F_t = \frac{9550 \times P \times 1000}{n} \times \frac{C_s}{C_v}$$

Pitch angles & pitch cone radius

Assuming $\Sigma = 90^\circ$,

$$\tan \delta_1 = \frac{1}{i} = \frac{1}{1.5}$$

$$\Rightarrow \delta_1 = 33.69^\circ$$

$$\tan \delta_2 = i = 1.5$$

$$\Rightarrow \delta_2 = 56.3^\circ$$

pitch cone radius $R = \frac{m}{2}\sqrt{z_1^2 + z_2^2}$

$$= 4\sqrt{20^2 + 30^2}$$

$$= 144.22\ mm$$

$$d_1 = m z_1$$
$$= 4 \times 20$$
$$= 80\ mm$$

$$d_2 = m z_2$$
$$= 4 \times 30$$
$$= 120\ mm$$

Now $F_t = \frac{9550 \times 1000 \times 2.5}{500} \times \frac{1.5}{40}$

$= \cancel{1193.75\ N}$

$= 1790.62\ N$

(18)

2. Dynamic load (F_d)

$$F_d = F_t + \frac{21v(F_t + bc)}{21v + \sqrt{F_t + bc}}$$

$0.125 - 1450$

$0.15 - ?$

$\therefore \frac{0.15}{0.125} \times 1450$

$= 1740$ N/mm.

here $v = \frac{\pi d_1 n_1}{60{,}000} = \frac{\pi \times 80 \times 500}{60{,}000} = 2.09$ m/sec

for $v = 2.09$ m/sec, allow error $= 0.15$ mm.

$\therefore C = 1740$ N/mm (T 23.32 / P 23.74)

When $m = 4.0$ mm for class 3 gears, allow. error $(f) = 0.025$ mm (Fig 23.40(a) Page 23.34)

for $f = 0.025$ mm, $C = 290$ N/mm

$$\therefore F_d = 1790.62 + \frac{21 \times 2.09 \left[1790.62 + (20 \times \cancel{1740}\ 290)\right]}{21 \times 2.09 + \sqrt{1790.62 + (20 \times \cancel{1740}\ 290)}}$$

$$= 1790.62 + \frac{333.15 \times 10^3\ \ \cancel{1.605 \times 10^6}}{\cancel{236.17}\ 134.28}$$

$= \cancel{8615.47}$ N. 4271.62 N

3. Beam strength according to Lewis eqn

$$F_t = \sigma_0 \cdot b \cdot y \cdot m \left(\frac{R-b}{R}\right)$$

Since pinion is weaker,

$$F_t = \sigma_{01} \cdot b\, Y_1\, m \left(\frac{R-b}{R}\right)$$

$$Y_1 = \pi y_1$$

$$= \pi \left[0.154 - \frac{0.912}{20}\right]$$

$$= 0.340.$$

$$\therefore F_t = 250 \times 20 \times 0.340 \times 4 \left(\frac{144.22 - 20}{144.22}\right)$$

$$= 5857 \text{ N}$$

$$F_W = \frac{d_1 \cdot b \cdot Q \cdot K}{\cos\delta_1}$$

where $$Q = \frac{2 z_{v2}}{z_{v1} + z_{v2}}$$

$$z_{v1} = \frac{z_1}{\cos\delta_1} = \frac{20}{\cos 33.69^\circ} = 24.03$$

$$z_{v2} = \frac{z_2}{\cos\delta_2} = \frac{30}{\cos 56.3} = 54.06$$

$$\therefore Q = \frac{2 \times 54.06}{24.03 + 54.06} = 1.384$$

when BHN = 400 for both pinion & gear

$K = 2.5241$ (T 23.37B / P 23.80)

$$\therefore F_W = \frac{80 \times 20 \times 1.384 \times 2.5241}{\cos 33.69^\circ}$$

$$= 6717.57\ N$$

5. Factor of safety

F.S against bending failure $= \frac{5857}{4271.62} = 1.37$

F.S against pitting failure $= \frac{6717.57}{4271.62}$

$= 1.57$

3(a) Enumerate the objectives of chain lubrication.

Ans: 1. To reduce the wear of chain components

2. To protect the chain against rust and corrosion.

3. To carry away frictional heat

4. To prevent seizure of pins & bushes

5. To ~~chan~~ cushion shock loads & protect the chain.

3(b) A leather belt drive transmitting 15 KW power with the help of a flat belt made of leather of mass density 0.95 g/cc. The centre distance between the pulleys is twice the diameter of bigger pulley. The smaller pulley rotates at 1440 rpm and speed of bigger pulley is 480 rpm. The belt should operate at a velocity of 20 m/sec approximately and the stresses in the belt should not exceed 2.25 N/mm^2. The coeff. of friction is 0.35. The thickness of belt is 5 mm.

Calculate

i) The diameter of pulleys

ii) The length & width of belt

iii) The belt tensions.

<u>data</u>

$N = 15 KW$

$n_1 = 1400$ rpm

$n_2 = 480$ rpm

$C = 2D$

$v = 20 m/sec$

$\sigma_1 = 2.25 N/mm^2$

$\rho = 0.95 gm/cc$

$= \frac{0.95}{1000} \times 100^3$

$= 950 kg/m^3$

$\mu = 0.35$

$t = 5 mm$

<u>to find</u>

1) d, D

2) L & b

3) T_1 & T_2

Sp. wt $(\omega) = \rho g$

$= 950 \times 9.81$

$= 9319.5 \frac{N}{m^3}$

$= 9319.5 \times 10^{-9}$

$= 9.319 \times 10^{-6} N/mm^3$

<u>1) d, D</u>

we have $v = \frac{\pi (d+t) n_1}{60,000}$

$$20 = \frac{\pi (d+5) 1400}{60,000}$$

$\Rightarrow d = 267.8$ mm

≈ 270 mm

Now $n_1 d = n_2 D$

$1400 \times 270 = 480 \times D$

$\Rightarrow D = 787.5$ mm

≈ 790 mm

2) <u>L</u>

centre distance $C = 2 \times D$ (given)

$= 2 \times 790$

$= 1580$ mm

Now $L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_L + d\theta_S)$

$\theta_L = \pi + \left[2\sin^{-1}\left(\frac{D-d}{2C}\right)\right]\frac{\pi}{180}$

$= 3.47 \text{ rad}$

$\theta_S = \pi - \left[2\sin^{-1}\left(\frac{D-d}{2C}\right)\right]\frac{\pi}{180}$

$= 2.81 \text{ rad}.$

$\therefore L = \sqrt{4 \times 1580^2 - (790 - 270)^2}$

$+ \frac{1}{2}\left[(70 \times 3.47) + (270 \times 2.81)\right]$

$= 3116.92 + 1593.85$

$= 4710.77 \text{ mm}.$

b

we have, $\text{power transmitted}/\text{mm}^2 = \frac{(\sigma_1 - \sigma_c) K v}{1000} \text{ KW}$

where $\sigma_c = \frac{w}{g} v^2 \times 10^6 \text{ N/mm}^2$

$= \frac{9.319 \times 10^{-6}}{9810} \times 20^2 \times 10^6$

$= 0.38 \text{ N/mm}^2.$

$K = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \frac{e^{(0.35 \times 2.81)} - 1}{e^{(0.35 \times 2.81)}}$

$= 0.626.$

$\text{power}/\text{mm}^2 = \frac{(2.25 - 0.38) \times 0.628 \times 20}{1000}$

$= 0.023 \text{ KW}.$

Now $N = (\text{power}/mm^2)(b \times t)$

$\Rightarrow b = 130mm$.

3) <u>T_1, T_2</u>.

$T_1 = \sigma_1 \times (b \times t)$

$= 2.25 \times (130 \times 5)$

$= 1462.5N$.

$$\frac{\sigma_1 - \sigma_c}{\sigma_2 - \sigma_c} = e^{\mu\theta}$$

$$\frac{2.25 - 0.38}{\sigma_2 - 0.38} = 2.67.$$

$2.67(\sigma_2 - 0.38) = 1.87$

$\sigma_2 = 1.08 \ N/mm^2$

$\therefore T_2 = \sigma_2 \times (b \times t)$

$= 1.08 \times 130 \times 5$

$= \underline{702N}$.

4(a) Explain the advantages of regular-lay ropes.

(24)

Ans: 1. They have more structural stability

2. They have more resistance to crushing and distortion.

3. They have less tendency to rotate under load.

4. There is less possibility of kinking

5. They are easy in handling during installation.

4(b) Determine the percentage increase in power capacity made possible in changing over from flat belt to V-belt drive. The diameter of flat pulley is same as the pitch diameter of grooved pulley. The pulley rotates at the same speed of pulley. The co-eff of friction is 0.3 both for grooved pully. The co-eff of friction is 0.3 both for flat & V-belts. The groove angle for V-belt is 60°. The belts are of the same material and have the same c/s area. In each case the angle of wrap is 150°.

V- V-belt

F- Flat belt

Ans:

$d_F = d_V$

$n_F = n_V$

$\mu_F = \mu_V = 0.3.$

$2\alpha = 60^\circ$

$\omega_F = \omega_V$

$A_F = A_V$

$\theta = 150^\circ$

$= 150 \times \frac{\pi}{180}$

$= 2.19 \text{ rad}.$

power transmitted by flat / V-belt $= \frac{(T_1 - T_C)\,kv}{1000}$ kW

where $T_1 = \sigma_1 \times A$

& $T_C = \sigma_C \times A$.

Since both belts are made of same material and since the speeds are same, T_1 & T_C will be same for both the belts. Also v is same for both the belts. The only variable is k.

$$\therefore \quad \frac{P_V}{P_F} = \frac{k_V}{k_F}$$

where $k_V = \dfrac{e^{\frac{\mu\theta}{\sin\alpha}} - 1}{e^{\frac{\mu\theta}{\sin\alpha}}}$

& $k_F = \dfrac{e^{\mu\theta} - 1}{e^{\mu\theta}}$.

$$e^{\frac{\mu\theta}{\sin\alpha}} = e^{\frac{0.3 \times 2.62}{\sin 30^\circ}} = 4.816.$$

$$e^{\mu\theta} = e^{0.5 \times 2.62} = 2.19.$$

$$\therefore \quad k_V = \frac{4.816 - 1}{4.816} = 0.792$$

$$k_F = \frac{2.19 - 1}{2.19} = 0.543.$$

$$\therefore \quad \frac{P_V}{P_F} = \frac{0.792}{0.543} = 1.45.$$

$\therefore$ P_V is 45% greater than P_F.

ie power capacity of V-belt is 45% greater than that of flat belt.

4(c) A helical compression spring made of circular wire, is subjected to axial force, which varies from 2.5 KN to 3.5 KN. Over this range of force, the deflection of spring should be approximately 5 mm. The spring index can be taken as 5. The spring has squared and ground ends. The spring is made of patented ~~ed~~ cold drawn steel wire with ultimate tensile strength of 1050 N/mm² and modulus of rigidity of 81370 N/mm². The permissible shear stress for the spring wire should be taken as 50% of the ultimate tensile strength. Calculate

1) Wire diameter 2) Mean coil diameter 3) No. of active coils 4) Total no. of coils 5) Solid length of spring 6) Free length of spring 7) Required spring rate 8) Actual spring rate

Ans: data

$F_{max} = 3.5\,KN$

$F_{min} = 2.5\,KN$

$y = 5\,mm$

$C = \frac{D}{d} = 5$

no. of inactive coils $(n) = 2$

$\sigma_u = 1050$

$\tau = 0.5 \times 1050$
$= 525\,N/mm^2$

$G = 81370\,N/mm^2$

to find

1) d
2) D
3) i
4) i'
5) l_s
6) l_f
7) $(F_0)_{req}$
8) $(F_0)_{act}$

1. Spring wire diameter (d)

$$\tau = \frac{8FCk}{\pi d^2}$$

where $F = \cancel{35}\ F_{max} = 35KN$
$= 3500N.$

$C = 5.$

$$k = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{20-1}{20-4} + \frac{0.615}{4}$$

$$= 1.341$$

$$\therefore\ 525 = \frac{8 \times 3500 \times 5 \times 1.341}{\pi d^2}$$

$\Rightarrow d = 10.66\,mm$ say $11\,mm.$

2. Mean coil dia (D)

$$D = C \times d$$
$$= 5 \times 11$$
$$= 55\,mm.$$

3. No. of active coils (i)

$$y = \frac{8FD^3 i}{Gd^4}$$

$$5 = \frac{8(3500-2500) \times 55^3 \times i}{81,370 \times 11^4}$$

$\Rightarrow i = 4.47$ say $5.$

4. Total no. of coils (i')

$$i' = i + n$$
$$= 5 + 2$$
$$= 7$$

5. Solid length of spring (l_s)

$$l_s = (i+n)d$$
$$= (5+2) \times 11$$
$$= 77 \text{ mm}$$

6. Free length of spring (l_f)

$$l_f = (i+n)d + y_{max} + a$$

where $a = 0.25y$

$$\cancel{\therefore l_f = (5+2)11 + 5 + (0.25 \times 5)}$$
$$\cancel{= 83.25 \text{ mm}}$$

where
$$y_{max} = \frac{8 F_{max} D^3 i}{G d^4}$$
$$= \frac{8 \times 3500 \times 55^3 \times 5}{81,370 \times 11^4}$$
$$= 19.55 \text{ mm}$$

$$\therefore l_f = (5+2)11 + 19.55 + (0.25 \times 5)$$
$$= 97.8 \text{ mm}$$

7. Spring rate required $((F_0)_{req})$

$$(F_0)_{req} = \frac{F_{max} - F_{min}}{y} = \frac{3500 - 2500}{5}$$

$$= 200 \text{ N/mm}.$$

8. Actual spring rate $((F_0)_{act})$

$$(F_0)_{act} = \frac{F_{max}}{y_{max}} = \frac{Gd^4}{8D^3 i} \quad \left(\because y_{max} = \frac{8F_{max}D^3 i}{Gd^4}\right)$$

$$= \frac{81370 \times 11^4}{8 \times 55^3 \times 5}$$

$$= 179 \text{ N/mm}.$$

7. complete the design and determine the input power capacity of a worm gear speed reducer unit (31) composed of a hardened steel worm and a phosphor bronze gear having 20° stub involute teeth. The centre distance c is to be 200mm, The transmission ratio is 10 and the worm speed is 1750 rpm.

Ans: <u>data</u>

<u>Materials</u>

worm : hardned steel

gear = phosphor bronze.

$\alpha = 20°$ stub

$a = 200$ mm

$i = 10$

$n_1 = 1750$ rpm.

$n_2 = \frac{1750}{10} = 175$ rpm.

<u>to find</u>

1) worm & worm gear dimensions

2) Input power capacity.

<u>1) worm & worm gear dimensions</u>

According to AGMA,

$d_1 = \frac{a^{0.875}}{\cancel{3.5}\ 1.5}$ ($\cancel{2.5196}$) 23.544 / 923.156

$= \cancel{69.87}$ 68.75 mm. ≃ 70 mm.

Since $a = \frac{d_1 + d_2}{2}$,

$$d_2 = (2\times200) - \cancel{68.75}\ 70$$
$$= \cancel{331.25\text{mm}}\ 330\text{mm}$$

But $d_1 = 3P_c$

$$= 3(\pi m)$$

$$\Rightarrow m = 7.42\text{mm}$$

$\therefore$ Select $m = 8\text{mm}$ (T23.3/P23.61)

Now $i = \dfrac{\pi d_2}{P_z} = \dfrac{\pi d_2}{P_x \cdot z_1} = \dfrac{\cancel{\pi} d_2}{(\cancel{\pi} m) z_1}$

$$\Rightarrow d_2 = i \cdot m \cdot z_1$$
$$= 10 \times 8 \times z_1$$
$$= 80 z_1$$

z_1	1	2	3	4	5
d_2	80	160	240	320	400

Since 320mm is closer to 330mm (which was calculated earlier), select $d_2 = 320$mm.

$$\therefore z_1 = 4$$

Since $d_2 = 320$,
using $a = \dfrac{d_1 + d_2}{2}$

$$200 = \frac{d_1 + 320}{2}$$

$$\Rightarrow d_1 = 80\text{mm}$$

Worm dimensions

No. of starts on worm $(z_1) = 4$.

PCD of worm $(d_1) = 80$ mm.

Face length $(L_1) = (4.5 + 0.02 z_1) P_x$ $\frac{T23.85}{P23.171}$

$= (4.5 + 0.02 \times 4) \pi \times 8$

$= 115$ mm.

depth of tooth $(h) = 0.623 \pi m$ (T23.85)

$= 15.65$ mm.

Addendum $(h_a) = 0.286 \pi m$

$= 7.18$ mm.

Worm gear dimensions

No. of teeth $(z_2) = z_1 \times i$

$= 4 \times 10$

$= 40$.

PCD of worm gear $(d_2) = 320$ mm.

Face width $(b) = 2.15 \pi m + 5.00$ (T23.85)

$= (2.15 \times \pi \times 8) + 5$

$= 59$ mm

Input power capacity

a) Input power based on strength (N_1)

From Lewis eqn,

$F_{t2} = \sigma_{02} \cdot m_n \cdot b \, y_2 C_v$ $\frac{23.567a}{P23.159}$

where $\sigma_{02} = 55\,N/mm^2$ (Wrongly given as 103.5 MPa in DHB)

$$\frac{T\ 23.96}{P\ 23.176}$$

$m_n = m_x \cos r$

$= 8 \cos 21.8^\circ$

$= 7.42\ mm$

$\tan r = \frac{P_z}{\pi d_1} = \frac{P_x \cdot z_1}{\pi d_1}$

$= \frac{\pi m_x z_1}{\pi d_1}$

$= \frac{\cancel{\pi} \times 8 \times 4}{\cancel{\pi} \times 80}$

$= 0.4$

$\Rightarrow r = 21.8^\circ$

$b = 59\ mm$ (already obtained)

$Y_2 = \pi y_2$

$= \pi \left[0.17 - \frac{0.95}{40} \right]$

$= 0.459$

$C_v = \frac{6}{6 + v_m}$ $\quad \frac{23.569b}{P23.159}$

where $v_m = \frac{\pi d_2 n_2}{60,000}$

$= \frac{\pi \times 320 \times 175}{60,000}$

$= 2.93\ m/sec$

$\therefore C_v = \frac{6}{6 + 2.93}$

$= 0.671$

$\therefore F_{t2} = 55 \times 7.42 \times 59 \times 0.459 \times 0.671$

$= 7415.72\ N$

Input power based on strength (N_1) is given by

$F_{t2} = \frac{9550\ N_1}{n_2} \times \frac{1}{(d_2/2)}$

$7415.72 = \frac{9550 \times N_1}{175} \times \frac{2}{320}$

$= 21.74\ kW$

b) Input power capacity based on wear (N_2)

wear load $F_W = d_2 \cdot b \cdot K \cdot \dfrac{23.572}{P\ 23.161}$

For hardened steel worm & phosphor bronze gear,

and $\gamma = 21.8°$,

load stress factor $K = 0.69 \left(\dfrac{T\ 23.100}{P\ 23.176} \right)$

$$\therefore F_W = 320 \times 59 \times 0.69$$
$$= 13{,}027\ N.$$

Now input power (N_2) based on wear is given by

$$N_2 = F_W \cdot v_g$$
$$= 13{,}027 \times \left(\frac{\pi d_2 n_2}{60{,}000} \right)$$
$$= 13{,}027 \times \frac{\pi \times 320 \times 175}{60{,}000}$$
$$= 38.197\ KW.$$

c) Input power based on heat dissipation (N_3)

Input power based on heat dissipation is given by (N_3)

$$N_3 = \frac{3650 \times a^{1.7}}{i+5}\ KW.$$ (where a is metres and N_3 is in KW)

$$= \frac{3650 \times (0.2)^{1.7}}{10+5}$$ (Not available in DHB)

$$= 15.7\ KW.$$

The least value of N_1, N_2 & N_3 is the permissible input power

$\therefore$ Input power capacity of the drive = 15.7 KW.

8(9) An automotive plate clutch consists of two pairs of contact surfaces with asbestos friction lining. The max. engine torque is 250 N-m. The co-eff. of friction is 0.35. The inner and outer diameters of friction lining are 175 mm and 250 mm respectively. The clamping force is provided by 9 springs, each compressed by 5mm to give a force of 800 N, when the clutch is new. (57)

1) what is the F.S with respect to slippage when the clutch is brand new?

2) what is the F.S w.r.t slippage after initial wear occurs?

3) How much wear of friction lining can take place before the clutch slips?

Ans: data

$i = 2$

$(M_t)_{max} = 250 \text{ N-m} = 250 \times 10^3 \text{ N-mm}$

$\mu = 0.35$

$D_1 = 175 \text{ mm}$

$D_2 = 250 \text{ mm}$

No. of springs = 9

deflection in each spring $(y) = 5 \text{ mm}$

to find

1) F.S w.r.t slippage when the clutch is brand new? (38)

2) F.S w.r.t slippage when the ~~clutch~~ initial wear occurs in clutch?

3) what is the wear, that takes place, before the clutch slips?

1) F.S when the clutch is brand new

when clutch is brand new, uniform pressure theory is applicaple

$M_t = \frac{1}{2} \mu F_a \cdot D_m \cdot i$

~~where~~ $\cancel{F_a = \frac{\pi}{4}(D_2^2 - D_1^2) p}$

here $F_a = 9 \times 800$

$= 7200$ N.

and $D_m = \frac{2}{3}\left(\frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}\right)$

$= \frac{2}{3}\left(\frac{250^3 - 175^3}{250^2 - 175^2}\right)$

$= 214.70$ mm

$\therefore M_t = \frac{1}{2} \times 0.35 \times 7200 \times 214.70 \times 2$

$= 5,41,044$ N-mm

$\therefore F S = \frac{5,41,044}{250 \times 10^3}$

$= 2.16$.

2) F.S. when the clutch starts to wear out

In this case, uniform wear theory is applicable.

$$M_t = \frac{1}{2}\,\mu F_a \cdot D_m \cdot i$$

where $D_m = \dfrac{D_1 + D_2}{2}$

$$= \frac{175 + 250}{2}$$

$$= 212.5\ \text{mm}$$

& $F_a = 7200\ \text{N}$

$$\therefore\ M_t = \frac{1}{2} \times 0.35 \times 7200 \times 212.5 \times 2$$

$$= 5{,}35{,}500\ \text{N}.$$

$$\therefore\ F.S. = \frac{5{,}35{,}500}{250 \times 10^3}$$

$$= 2.14$$

3) wear of friction lining before slipping

The axial force is provided by the springs in a clutch and is responsible for transmitting the required torque. The clutch slips when ~~there is no sufficient~~ ~~spring force to transmit the required torque~~ the axial force provided by the springs is insufficient.

Now the axial force provided by the springs to transmit 250 N-m of torque is given by

$$M_t = \frac{1}{2}\,\mu F_a \cdot D_m \cdot i$$

where $D_m = \left(\dfrac{D_1 + D_2}{2}\right)$ for uniform wear.

$$250 \times 10^3 = \frac{1}{2} \times 0.35 \times F_a \times \left(\frac{250+175}{\cancel{2}}\right) \times \cancel{2}$$

$$\Rightarrow F_a = 3361.74 \text{ N.}$$

Since this force is provided by 9 springs, the axial force provided by each spring

$$F_a^{'} = \frac{3361.74}{9} = 373.48.$$

∴ The clutch starts slipping when the axial force provided by a springs falls below the value of 373.48 N.

In a helical compr. spring $F \propto y$

∴ when the compression of spring reduces, the force provided by spring also reduces. when there is wear in friction lining, the compression of spring reduces and therefore the spring force of a spring is also reduces. But the stiffness of spring is always constant.

Let 'x' mm be the wear in friction lining.

Since 5 mm is the initial compression in the spring, the compression of spring after the friction lining wears $= (5-x)$ mm

$$\therefore \text{stiffness of spring} = \frac{373.48}{(5-x)} \quad -(1)$$

when the initial compression is 5 mm, each spring provides a load of 800 N.

$\therefore$ Stiffness of spring $= \frac{800}{5} = 160$ N/mm — (2)

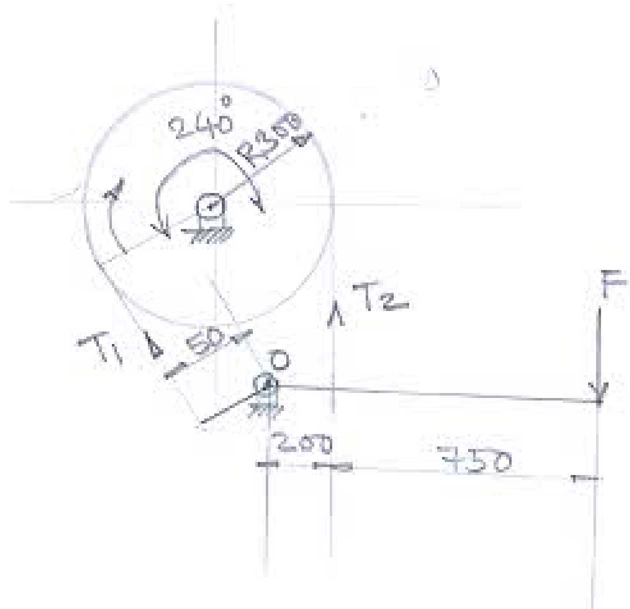
equating (1) & (2)

$$\frac{373.48}{5-x} = 160$$

$\Rightarrow x = 2.66$ mm.

8(b) A differential band brake is shown in figure. The width and thickness of steel band are 100 mm and 3 mm respectively and the max. tensile stress in the band is 50 N/mm². The co-eff. of friction between the friction lining and brake drum is 0.25. Calculate (i) the tensions in the band ii) the actuating force and iii) the torque capacity of the brake. Find out whether the brake is self locking. (42)

Ans:



<u>data</u>

$\omega = 100$ mm

$h = 3$ mm

$\sigma_d = 50$ MPa

$\mu = 0.25$

$\theta = 240^\circ$
$= 4.18$ rad.

<u>to find</u>

1) T_1 & T_2
2) F
3) M_t
4) whether the brake is self locking.

1. T_1 & T_2

$$\frac{T_1}{T_2} = e^{\mu\theta}$$
$$= e^{(0.25 \times 4.19)}$$
$$= 2.85$$

Also $\omega = \dfrac{T_1}{h\,\sigma_d}$

$$100 = \frac{T_1}{3 \times 50}$$
$$\Rightarrow T_1 = 15{,}000 \text{ N}$$
$$\therefore \quad T_2 = \frac{15{,}000}{2.85} = 5263 \text{N}.$$

2. F

Taking moments about O,

$$(T_1 \times 50) + F(950) = (T_2 \times 200)$$
$$(15{,}000 \times 50) + (F \times 950) = (5263 \times 200)$$
$$\Rightarrow F = 318.52$$

3. M_t

$$M_t = (T_1 - T_2)\,r$$
$$= (15{,}000 - 5263) \times \overset{300}{\cancel{450}}$$
$$= 2.92 \times 10^6 \text{ N-mm}$$
$$= 2.92 \text{ KN-m}$$

4. Self locking

For the brake to be self locking F should be equal to zero. As per the expression in 2, F is not zero. ∴ The brake is not self locking. (44)

9(a) List the applications of anti-friction bearings.

Ans- Applications

1. Machine tool spindles
2. Automobile front and rear axles.
3. gear boxes.
4. small size electric motors.
5. rope sheaves, crane hooks and hoisting drums.

9(b) A 75mm long full journal bearing of diameter 75mm supports a load of 12KN on a journal turning at 1800 rpm. Assuming a $\frac{r}{c}$ ratio of 1000, and an oil having viscosity of 0.01 kg/ms at the operating temperature, determine the co-eff. of friction by using i) MC Kee's equation ii) Raimondi & Boyd curve and iii) determine the amount of heat generated using co-eff of friction as calculated by MC Kee equation.

Ans: data

$L = 75mm$
$= 0.075m$

$d = 75$
$= 0.075m$

$W = 12KN$
$= 12 \times 10^3 N$

$\frac{1}{\psi} = \frac{d}{c} = 1000.$

$\therefore \psi = \frac{c}{d} = 0.001$

$\eta = 0.01 \, kg/m\,sec$
$= 0.01 \, Pa\text{-}sec.$

$n = \cancel{600}\ 1800 \, rpm$
$\therefore n' = \frac{1800}{60}$
$= 30 \, rps.$

$$1\,pa\text{-}sec = \frac{1\,N\text{-}sec}{m^2} = 1\left(\frac{kg\text{-}m}{sec^2}\right)\times\frac{sec}{m^2}$$

$$= 1\,kg/m\text{-}sec.$$

to find

1) μ according to Mc Kee's equ^n

2) μ ,, ,, Boyd & Raimondi curve

3) Hg.

1) μ according to McKee's eq^n

$$\mu = K_a\left(\frac{\eta n'}{p}\right)\left(\frac{1}{\psi}\right)10^{-10} + \Delta\mu.$$

here $p = \frac{W}{Ld}$.

$$= \frac{12\times10^3}{0.075\times0.075}$$

$$= 2.13\times10^6\ N/m^2$$

$K_a = 1.95\times10^{11}$ for full journal bearing

$\Delta\mu = 0.002$

$$\therefore\ \mu = 1.95\times10^{11}\left[\frac{0.01\times30}{2.13\times10^6}\right]\times1000\times10^{-10} + 0.002$$

$$= 4.74\times10^{-3}.$$

2) μ according to Boyd & Raimondi curve

Bearing characteristic number

$$S = \frac{\eta n'}{p}\times\frac{1}{\psi^2}$$

$$= \frac{0.01 \times \cancel{2}30}{2.13 \times 10^6} \times \frac{1}{0.001^2}$$

$$= 0.140$$

From fig 24.20/p24.26, (Boyd & Raimondi curve)

for $S = 0.140$ & $\frac{L}{d} = \frac{0.075}{0.075} = 1$,

co-eff. of friction variable $\left(\frac{\mu}{\psi}\right) = 3.4$

$$\therefore \frac{\mu}{0.001} = 3.4$$

$$\therefore \mu = 3.4 \times 10^{-3}$$

3) <u>Hg</u>

$$Hg = \mu (PLd) v$$

$$= 4.74 \times 10^{-3} \left(12 \times 10^3\right) \times 7.06$$

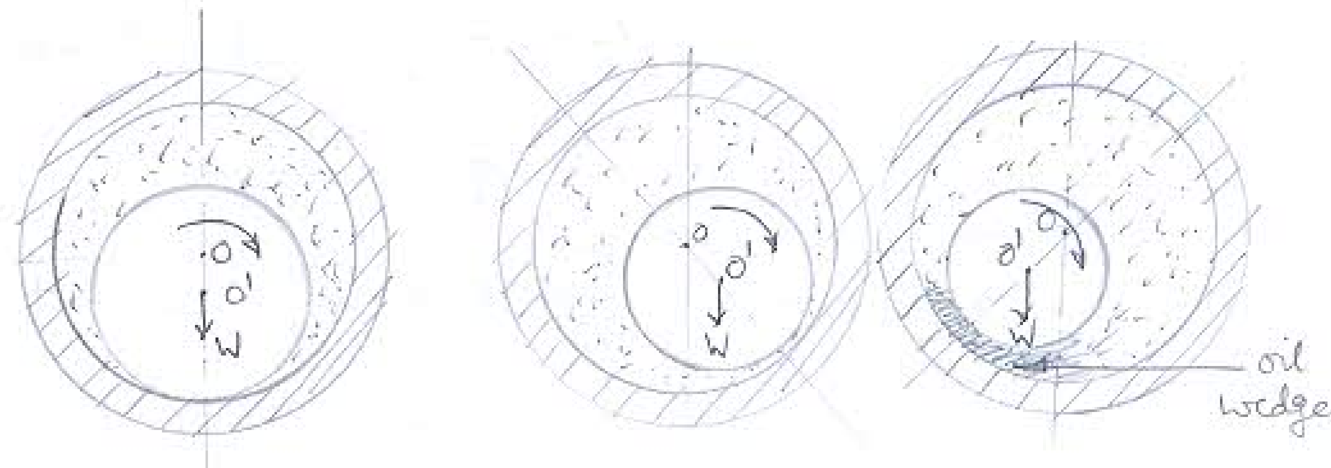
$$= \underline{401.57 W}$$

$$v = \frac{\pi d n}{60}$$

$$= \frac{\pi \times 0.075 \times 1800}{60}$$

$$= 7.06 \text{ m/sec}$$

10(a) Define hydrodynamic lubrication. Explain the principle of hydrodynamic lubrication.



(a) (b) (c)

Fig (a) : Journal at rest.
Fig (b) : Journal starts to rotate
Fig (c) : Journal at full speed.

Hydrodynamic lubrication is defined as a system of lubrication in which ~~the~~ a load bearing fluid film is created due to the relative motion between sliding surfaces.

The principle of hydrodynamic lubrication is shown in the above figures. Initially when the shaft is at rest, (Fig. a), it sinks to the bottom of clearance space under the action of load W. The surface of journal is in contact with that of bearing when it is at rest. As the journal starts to rotate (Fig b), it climbs the bearing surface and as the speed increases, it forces the fluid into the wedge shaped region (Fig c). As more and more fluid is forced into wedge shaped clearance space, pressure is generated within the system. This pressure supports the external load W. This principle is known as hydrodynamic lubrication.

10(b) A single row deep groove ball bearing is subjected to a radial force of 7 KN and a thrust force of 2.2 KN. The shaft rotates at 1200 rpm. The expected life L_{10h} of the bearing is 20,000h. The minimum acceptable diameter of shaft is 75mm. Select a suitable ball bearing for this application. Take X = 0.56 and Y = 1.8.

Ans: <u>data</u>

$F_r = 7KN = 7000N$.

$F_a = 2200 N$

$n = 1200$ rpm

$L_{10h} = 20,000$ hrs

$\left.\begin{matrix} X = 0.56 \\ Y = 1.8 \end{matrix}\right\}$ (given)

$d = 75$ mm.

The ~~static~~ dynamic equivalent load (F) $= X \cdot F_r + Y \cdot F_a$

24.177b ~~24.162~~ / ~~p24.59~~ p24.62

$$\therefore F = (0.56 \times 7000) + (1.8 \times 2200)$$

$$= 7880\ N.$$

The relation between life in hours (L_h) and life in millions of revolutions is given by

$$L_h = \frac{10^6 L}{60n}$$

24.178b / p24.62

$$20{,}000 = \frac{10^6 \times L}{60 \times 1200}$$

$$\therefore L = 1440 \text{ million rev.}$$

Now $L = \left(\frac{C}{F}\right)^m$ 24.175 / p 24.61

Where $m = 3$ for ball bearing (page 24.62)

$$\therefore 1440 = \left(\frac{C}{7880}\right)^3$$

$$\Rightarrow \cancel{C = (1440 \times 7880)^{\frac{1}{3}}}$$

$$\Rightarrow (1440)^{\frac{1}{3}} = \frac{C}{7880}$$

$$C = 7880 \times (1440)^{\frac{1}{3}}$$

$$= 88{,}984\ N.$$

Select FAG/SKF 6315 bearing from

T 24.61/P 24.85 for $d = 75$ mm & $C = 88,984$

Now for the bearing selected

$C = 76,500$

& $C_0 = 1,14,000$