USN

Eighth Semester B.E. Degree Examination, June/July 2018
Control Engineering

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1 a. Define control system. Compare open loop and closed loop control system with an example for each type. **(08 Marks)**

b. What are the ideal requirements of control system? **(06 Marks)**

c. Draw the block diagram of proportional plus integral plus derivative controller and state its characteristics. **(06 Marks)**

2 a. Write the differential equations governing the behaviour of the mechanical system shown in Fig.Q2(a). Also obtain the analogous electrical circuit based on force voltage analogy and loop equations.

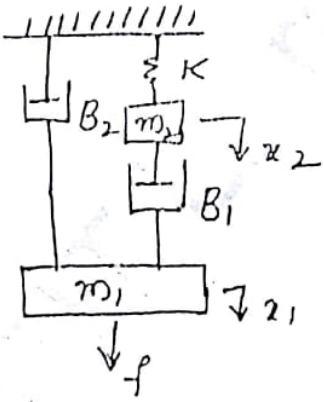

Fig.Q2(a) **(10 Marks)**

b. Obtain the transfer function of field controlled DC motor. **(10 Marks)**

3 a. Reduce the block diagram shown in Fig.Q3(a) and obtain the transfer function $\frac{C(s)}{R(s)}$.

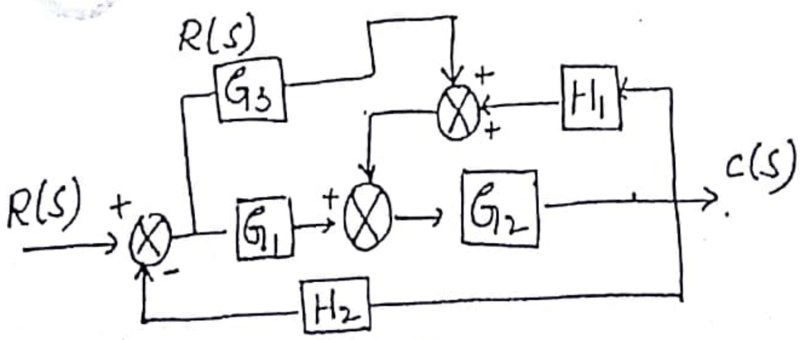

Fig.Q3(a) **(10 Marks)**

b. Find the transfer function by using Mason's gain formula for the signal flow graph shown in Fig.Q3(b).

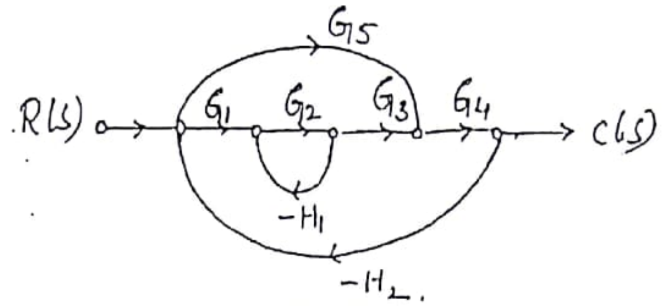

Fig.Q3(b) **(10 Marks)**

4 a. A units feedback system characterized by an open loop transfer function

$$G(s) = \frac{10}{s^2 + 2s + 6}$$

Determine the following, when the system is subjected to a unit step input:

i) Undamped natural frequently
ii) Damping ratio
iii) Peak overshoot
iv) Peak time
v) Settling time **(10 Marks)**

b. Explain Routh Hurwitz criterion for stability of a control system and examine the stability of $s^4 + 2s^3 + 3s^2 + 8s + 2 = 0$ using the same. **(10 Marks)**

PART – B

5 a. Sketch the polar plot for the transfer function $G(s) = \frac{10}{s(s+1)(s+2)}$. **(10 Marks)**

b. Plot the Nyquit diagram for the open loop transfer function

$$G(s)H(s) = \frac{12}{s(s+1)(s+2)}$$

and determine the nature of stability. **(10 Marks)**

6 Sketch the bode plot for a unity feed-back system, whose open loop transfer function is given by $G(s)H(s) = \frac{10}{s(1+s)(1+0.02s)}$, find:

i) Gain and phase cross over frequencies.
ii) Gain and phase margin.
iii) Stability of the closed loop system. **(20 Marks)**

7 Sketch the root locus plot for the system whose open loop transfer function is given by $G(s)H(s) = \frac{K}{s(s+2)(s^2+8s+20)}$. **(20 Marks)**

8 a. Explain the following:

i) Lead compensator
ii) Lag compensator **(10 Marks)**

b. Explain the series and feedback compensated system, with block diagrams. **(10 Marks)**

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Control Engineering

Sub code : 10ME82

Semester : 8th

Solution

Year : July 2018

Max. Marks : 100

1

(a) A control system is an arrangement of components interconnected in such a way so as to regulate, direct or command to obtain a certain objective.

	Open loop control system	Closed loop control system
1.	Output is neither measured nor compared with input	Output is measured and compared with input
2.	Feed back element is absent	Feed back element is present
3.	Simple to construct & economical	Complex in design and hence not economical
4.	More stable	Less stable
5.	Accuracy depends on the calibration and unreliable	More accurate due to feedback and reliable
6.	Highly sensitive to disturbances	Less sensitive to disturbances
E.g:	Automatic washing machine	E.g: Voltage stabilizer

Requirement of an ideal control system.

To achieve the required objective, a control system must satisfy the following requirements:

1. Stability: Stability in a control system implies that small changes in the system input, in initial conditions or in system parameters do not result in large changes in the system behaviour. An ideal control system is designed to be stable.

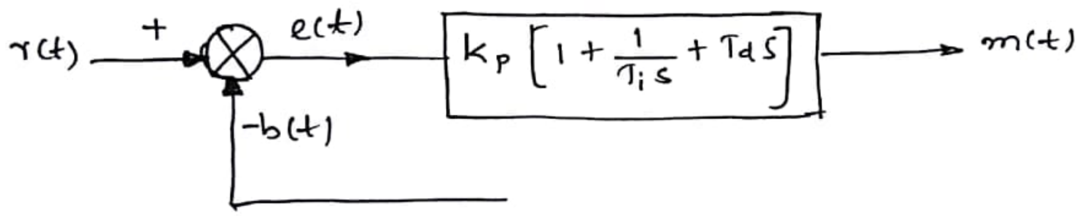
2. Sensitivity: An ideal control system should be insensitive to the variations in parameters of the system but it should be sensitive to input commands.

3. Speed: Speed of the control system means how fast the output of the system approaches the desired value. This is measured in terms of settling time and rise time.

4. Accuracy: Accuracy of the control system means how much the output of the control system is nearer to the input or desired value. An ideal control system must be highly accurate.

5. Disturbances: In the design of a control system, considerations should be given so that the system is insensitive to noise and disturbances but sensitive to input commands.

1\.
c. Proportional plus Integral plus Derivative controller



Characteristics of PID controller.

(i) It increases order of the system.

(ii) It increases TYPE of the system.

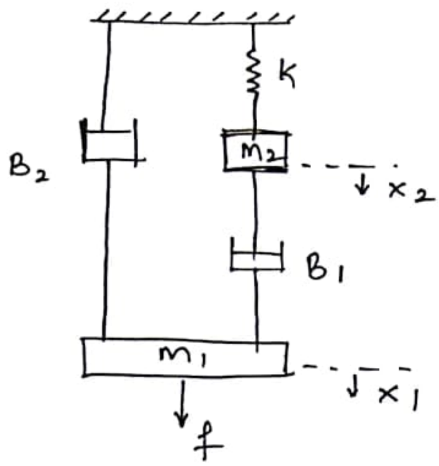
(iii) It improves steady state part of the response.

(iv) It reduces peak overshoot and settling time.

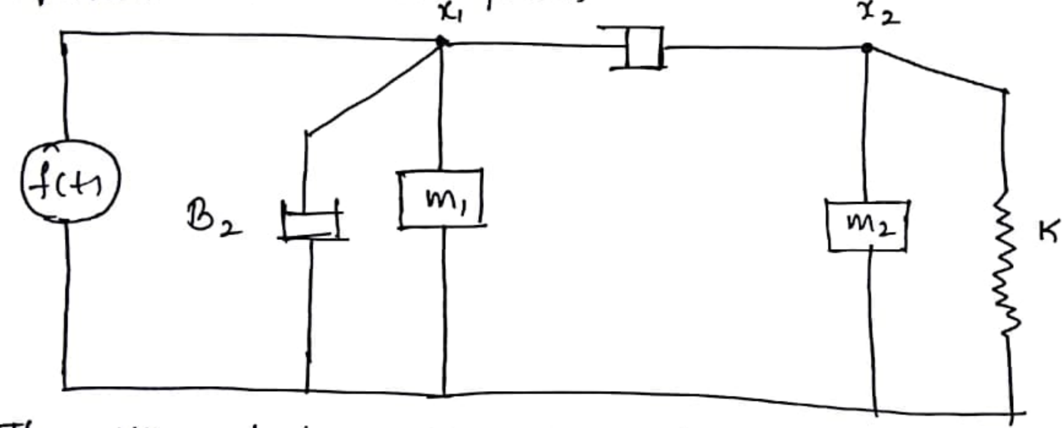
(v) It increases damping ratio.

2

a.



Equivalent mechanical system



The differential equation of equilibrium are:

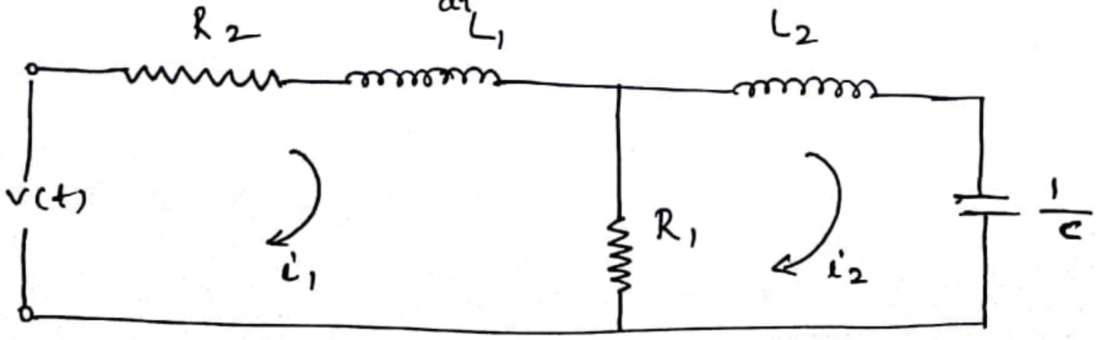
At node x_1: $f(t) = M_1 \ddot{x}_1 + B_2 \dot{x}_1 + B_1(\dot{x}_1 - \dot{x}_2)$ —(1)

F-V analogy: $F \to v$; $M \to L$; $B \to R$; $K \to \frac{1}{c}$; $x \to q$; $\dot{x} \to i$

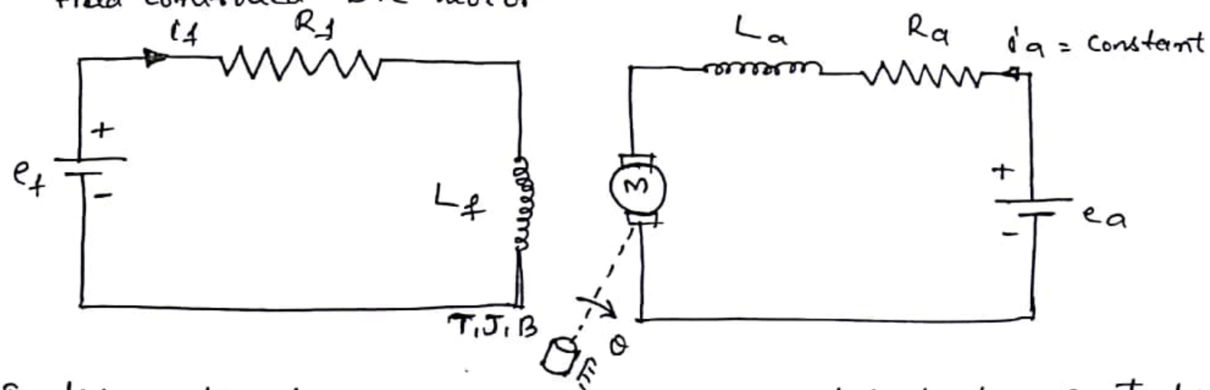
$$v(t) = L_1 \frac{di_1}{dt} + R_2 i_1 + R_1(i_1 - i_2) \quad \text{— (2)}$$

At node x_2: $0 = M_2 \ddot{x}_2 + B_1(\dot{x}_2 - \dot{x}_1) + K x_2$ —(3)

F-V analogy: $0 = L_2 \frac{di_2}{dt} + R_1(i_2 - i_1) + \frac{1}{c}\int i_2\, dt$ —(4)



6. Field controlled D.C motor



In this system, the armature current is maintained at a constant value and motor speed is controlled varying the field current.
The transfer function of the field controlled D.C motor can be found by dividing the system into three parts as the field circuit, D.C. motor and the mechanical system.
Hence differential equations are,

For field circuit, $e_f(t) = R_f i_f(t) + L_f \frac{di_f}{dt}$

Taking Laplace on B.S

$E_f(s) = R_f I_f(s) + L_f s I_f(s)$

$E_f(s) = [R_f + L_f s] I_f(s) \quad -(1)$

For D.C motor $T(t) = K_T i_f \quad -(2)$

For mechanical system (Rotational) $J\ddot{\theta} + B\dot{\theta} = T(t)$

From eqn (2) we have, $J\ddot{\theta} + B\dot{\theta} = K_T i_f$

Taking laplace on both sides, we have

$Js^2\theta(s) + Bs\,\theta(s) = K_T I_f(s)$

$-K_T I_f(s) + [Js^2 + Bs]\theta(s) = 0$

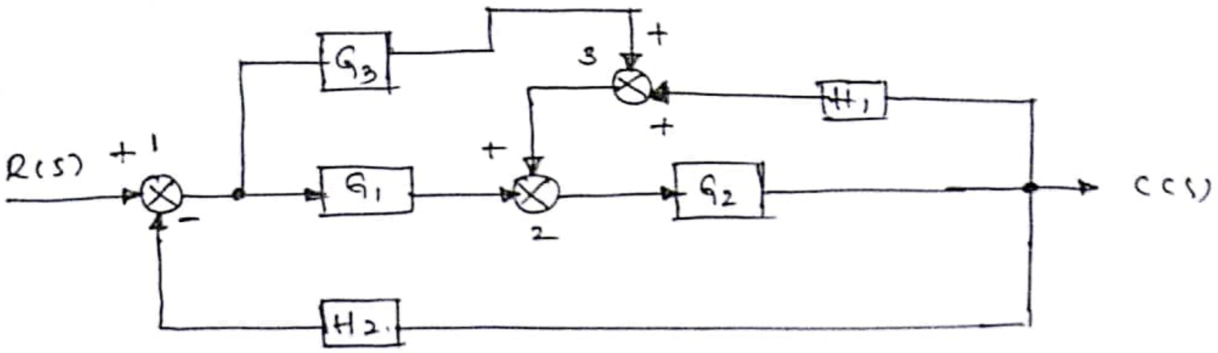
$$I_f(s) = \frac{(Js^2 + Bs)\theta(s)}{K_T} \quad -(3)$$

Substituting $I_f(s)$ in eqn (1), we get

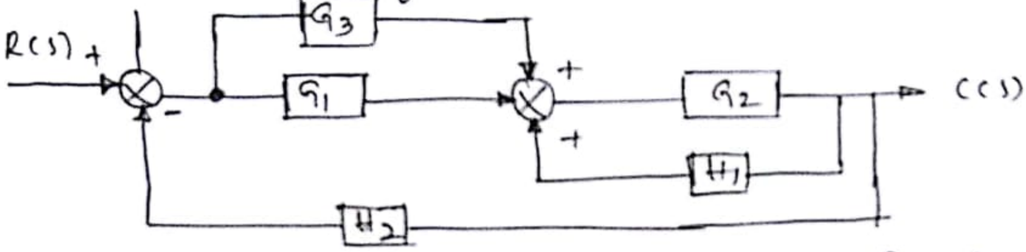
$$\therefore E_f(s) = \frac{(R_f + L_f s)(Js^2 + B(s))\,\theta(s)}{K_T}$$

Thus transfer function is $$\frac{\theta(s)}{E_f(s)} = \frac{K_T}{(R_f + L_f s)(Js^2 + B(s))\,\theta(s)}$$

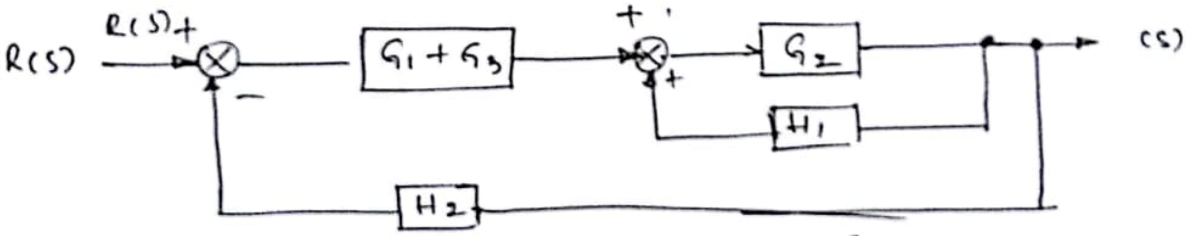
3
(a)



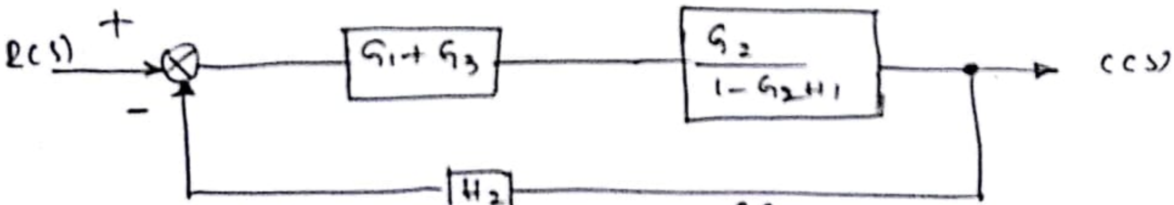
1. Combine two summing points (2) and (3)



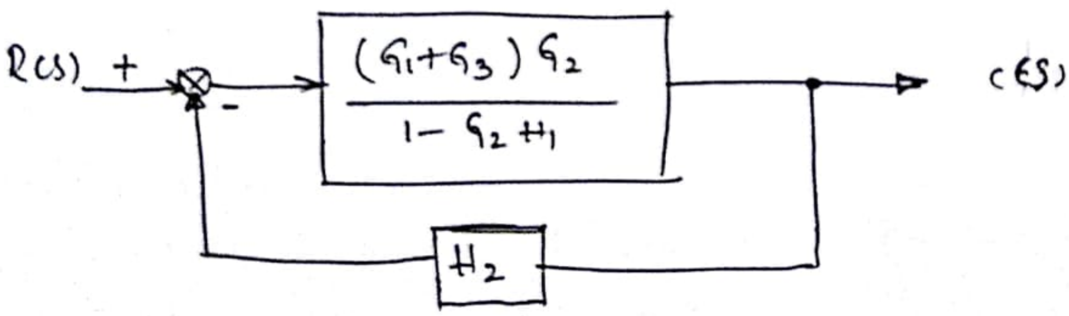
2. Combine blocks in parallel G_3 and G_1 $= G_1 + G_3$



3. Eliminate the minor feedback loop : $\dfrac{G_2}{1 - G_2 H_1}$



4. Combine blocks in series $= \dfrac{(G_1 + G_3)\, G_2}{(1 - G_2 H_1)}$



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5. Eliminate minor feedback loop.

$$\frac{\dfrac{(G_1+G_3)G_2}{1-G_2H_1}}{1+\dfrac{(G_1+G_3)G_2}{(1-G_2H_1)}\times H_2} = \frac{(G_1+G_3)G_2}{(1-G_2H_1)+(G_1+G_3)G_2H_2}$$

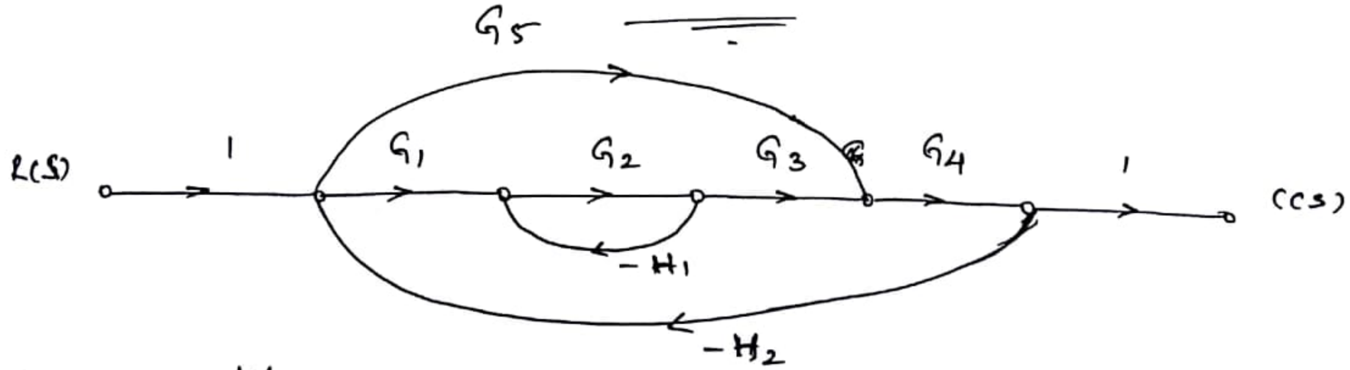
Thus

$$R(s) \longrightarrow \boxed{\frac{(G_1+G_3)G_2}{(1-G_2H_1)+(G_1+G_3)G_2H_2}} \longrightarrow C(s)$$

Overall transfer function is

$$\frac{C(s)}{R(s)} = \frac{G_1G_2+G_2G_3}{1-G_2H_1+G_1G_2H_2+G_2G_3H_2}$$

3.
b.



step 1: Identify no. of forward path and their gain

$K=2$ (No. of forward paths)

$$\left.\begin{aligned} P_1 &= 1\times G_1\times G_2\times G_3\times G_4\times 1 = G_1G_2G_3G_4 \\ P_2 &= 1\times G_5\times G_4\times 1 = G_4G_5 \end{aligned}\right\} \text{Forward path gains.}$$

step 2: Identify the individual loops and their loop gain.

Loop gains are

$$\left.\begin{aligned} L_1 &= G_2\times(-H_1) = -G_2H_1 \\ L_2 &= G_1\times G_2\times G_3\times G_4\times(-H_2) = -G_1G_2G_3G_4H_2 \\ L_3 &= G_5\times G_4\times(-H_2) = -G_4G_5H_2 \end{aligned}\right\} \text{Three individual loops}$$

Step 3: Find the combination of two non-touching loops.

1. Combination of two non-touching loops.

$$L_1 L_3 = (-G_2 H_1) \times (-G_4 G_5 H_2)$$

$$= \underline{G_2 G_4 G_5 H_1 H_2}$$

No other combination of two or more non-touching loops.

Step 4: Find the value of determinant Δ.

Δ = 1 − (Sum of individual loop gain) + (Sum of gain products of all combinations of two non touching loops) − (Sum of gain products of all combinations of three non touching loops) + - - - -

$$\therefore \quad \Delta = 1 - [L_1 + L_2] + [L_1 L_3]$$

$$= 1 - [G_1 G_2 G_3 G_4 + G_4 G_5] + [G_2 G_4 G_5 H_1 H_2] \quad - (1)$$

Step 5: Find the value of Δ_K

Consider P_1; $\Delta_1 = 1$ (Since all the loops are touching I forward path)

Consider P_2: $\Delta_2 = 1 - L_1$ (Except L_1, remaining loops are touc II forward path)

From Mason's gain formula:

Since $k = 2$ $$T = \frac{1}{\Delta} \sum_{k=1}^{2} P_K \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$\therefore$ Overall transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_4 G_5 (1 + G_2 H_1)}{1 - [G_1 G_2 G_3 G_4 + G_4 G_5] + [G_2 G_4 G_5 H_1 H_2]}$$

4

a).
$$G(s) = \frac{10}{s^2+2s+6}$$

The characteristic equation of the system is

$$1+G(s)H(s)=0$$

$$1+\frac{10}{s^2+2s+6}\times 1 = 0 \qquad \because H(s)=1$$

$$s^2+2s+6=0$$

i.e. $s^2+2s+16=0$: Comparing with $s^2+2\xi\omega_n s+\omega_n^2=0$

(i) Undamped natural frequency.

$\omega_n^2 = 16 \Longrightarrow \omega_n = 4$ rad/sec (Ans)

(ii) Damping ratio

$$2\xi\omega_n = 2 \Longrightarrow \xi = \frac{2}{2\omega_n} = \frac{2}{2\times 4}$$

$\therefore \xi = 0.25$ (Ans)

(iii) Peak overshoot, $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{\frac{-\pi\times 0.25}{\sqrt{1-0.25^2}}}$

$= 0.4443$ (Ans)

(iv) Peak time, $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{\pi}{4\sqrt{1-0.25^2}} = 0.8115$ sec (Ans)

(v) Settling time, $t_s = \dfrac{4}{\xi\omega_n} = \dfrac{4}{0.25\times 4} = 4$ sec (2% tolerance band)

(Ans)

b. $s^4 + 2s^3 + 3s^2 + 8s + 2 = 0.$

To examine stability, consider

The Routh's array for the given characteristic eqn

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 2 \\ s^3 & 2 & 8 & 0 \\ s^2 & -1 & 2 & 0 \\ s^1 & 12 & 0 & \\ s^0 & 2 & & \end{array}$$

For elements of s^2 row: $\frac{(2)(3) - 8(1)}{2} = -1$; $\frac{(2)(2) - (1)(0)}{2} = 2$

For elements of s^1 row: $\frac{(-1)(8) - (2)(2)}{-1} = 12$; $\frac{(-1)(0) - (0)(2)}{-1} = 0$

For elements of s^0 row: $\frac{(12)(2) - (-1)(0)}{12} = 2.$

Thus, from Routh's array it is clear that there are two sign changes in 1^{st} column of Routh's array. Hence system is unstable with two roots located in right half of s-plane.

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5 (a) $$G(s) = \frac{10}{s(s+1)(s+2)}$$

Given transfer function is $G(s) = \frac{10}{s(s+1)(s+2)}$ Put $s = j\omega$, we get

$$G(j\omega) = \frac{10}{j\omega(1+j\omega)(2+j\omega)} \quad -(1)$$

Magnitude, $M(\omega) = |G(j\omega)| = \frac{10}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$

Phase angle, $\phi(\omega) = \angle G(j\omega) = -90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$

To find the intersection point

Separate the real and imaginary part of $G(j\omega)$ from eqn (1)

$$G(j\omega) = \frac{10}{(j\omega)(1+j\omega)(2+j\omega)} \times \frac{j(1-j\omega)(2-j\omega)}{j(1-j\omega)(2-j\omega)}$$

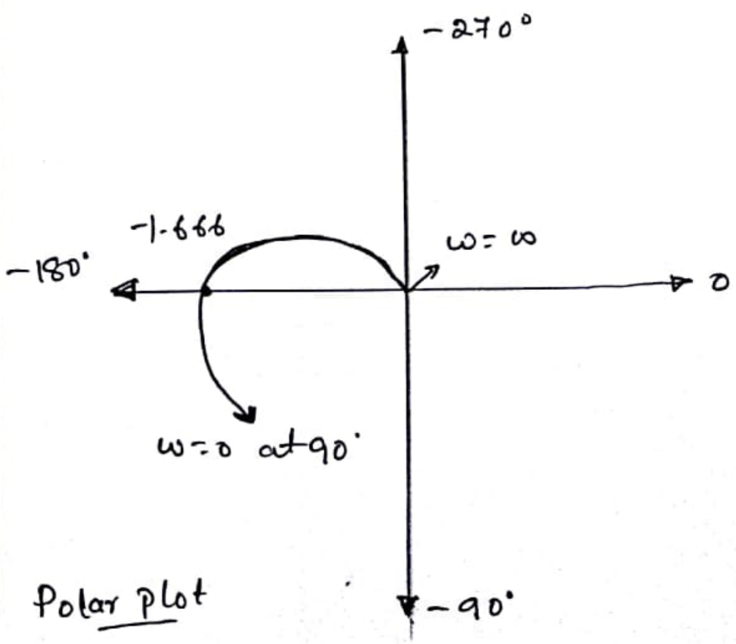
$$= \frac{30\omega + 10j\omega(2-\omega^2)}{-\omega(1+\omega^2)(4+\omega^2)} = \frac{-30}{(1+\omega^2)(4+\omega^2)} - j\frac{10(2-\omega^2)}{(1+\omega^2)(4+\omega^2)}$$

Equating imaginary part to zero, we get

$$\frac{10(2-\omega^2)}{(1+\omega^2)(4+\omega^2)} = 0 \;;\quad 10(2-\omega^2) = 0 \;;\; \therefore\; \omega = \sqrt{2}$$

Thus at $\omega = \sqrt{2} \implies M(\omega) = 1.666$ and $\phi = -180^\circ$

ω rad/sec	$M(\omega)$	$\phi(\omega)$
0	∞	-90°
1	3.156	-162°
$\sqrt{2}$	1.666	-180°
2	0.833	-198°
5	0.0666	-238°
∞	0	-270°



Polar plot

b. Given, $G(s)H(s) = \dfrac{12}{s(s+1)(s+2)}$ —(1)

Open loop poles are $s = 0, -1, -2$ since one pole is present at origin, Nyquist path includes a small circle around $s=0$ as shown in fig (a).

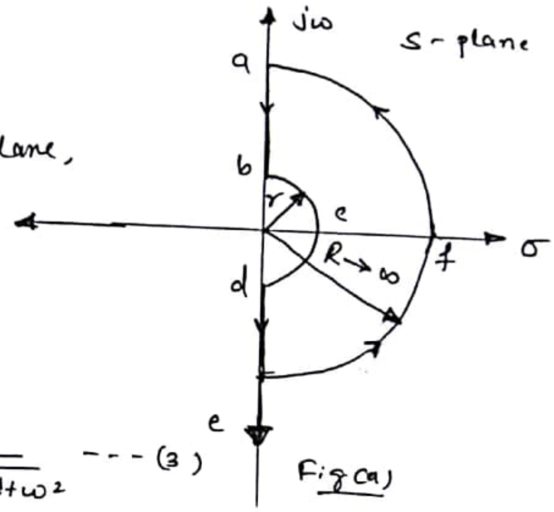


Fig (a)

For mapping from s-plane to G(s) H(s) plane, put $s = j\omega$ in $G(s)H(s)$, we get

$$G(j\omega)H(j\omega) = \frac{12}{(j\omega)(1+j\omega)(2+j\omega)} \quad \text{---(2)}$$

Thus,

Magnitude, $M(\omega) = |G(j\omega)H(j\omega)| = \dfrac{12}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$ --- (3)

Phase angle, $\phi(\omega) = \angle G(j\omega)H(j\omega) = -90° - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$ ---- (4)

To map section ab:

It is defined by $s = j\omega$, where ω varies from ∞ to 0

ω	M(ω)	φ(ω)
∞	0	−270°
0	∞	−90°

To map section bcd:

It is defined by $s = \lim_{r\to 0} re^{j\theta}$, where θ varies from 90° to −90° through 0°.

$$\therefore \lim_{r\to 0} G(re^{j\theta})H(re^{j\theta}) = \lim_{r\to 0}\left[\frac{12}{(re^{j\theta})(1+re^{j\theta})(2+re^{j\theta})}\right]$$

$$= \lim_{r\to 0}\left[\frac{12}{re^{j\theta}(1)(2)}\right] = \infty e^{-j\theta}$$

s-plane	G(s) H(s) plane
θ	$\phi = -\theta$
+90°	−90°
0	0
−90°	+90°

To map section de

It is defined by $s = j\omega$, where ω varies from 0 to $-\infty$. It is the mirror image of section ab.

To map section Efa

It is defined by $s = \lim_{R \to \infty} Re^{j\theta}$ where 'θ' varies from $-90°$ to $90°$ through $0°$

$$\lim_{R \to \infty} G(Re^{j\theta}) H(Re^{j\theta}) = \lim_{R \to \infty} \left[\frac{12}{Re^{j\theta}(1+Re^{j\theta})(2+Re^{j\theta})} \right]$$

$$= \lim_{R \to \infty} \left[\frac{12}{R^3 e^{j3\theta}} \right] = 0 e^{j3\theta}$$

θ	$\phi = -3\theta$
$-90°$	$270°$
$0°$	0
$90°$	$-270°$

To find intersection point with negative real axis.

Separate the real and imaginary part of $G(j\omega)H(j\omega)$, from eqn (2), we have,

$$G(j\omega)H(j\omega) = \frac{12}{(j\omega)(1+j\omega)(2+j\omega)} \times \frac{j(1-j\omega)(2-j\omega)}{j(1-j\omega)(2-j\omega)}$$

$$= \frac{36\omega + 12j(2-\omega^2)}{-\omega(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-36}{(1+\omega^2)(4+\omega^2)} - \frac{j12(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

Equating imaginary part to zero, we get

$$\frac{12(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)} = 0$$

$$12(2-\omega^2) = 0$$

$$\therefore \omega = \sqrt{2} = 1.414 \text{ rad/sec}$$

Thus at $\omega = 1.414$, from eqn (3)

Magnitude, $M(\omega) = 2$

Since plot encircle the point $(-1, j0)$ system is unstable

Nyquist plot of $G(s)H(s) = \frac{12}{s(s+1)(s+2)}$ is as shown in fig(b).

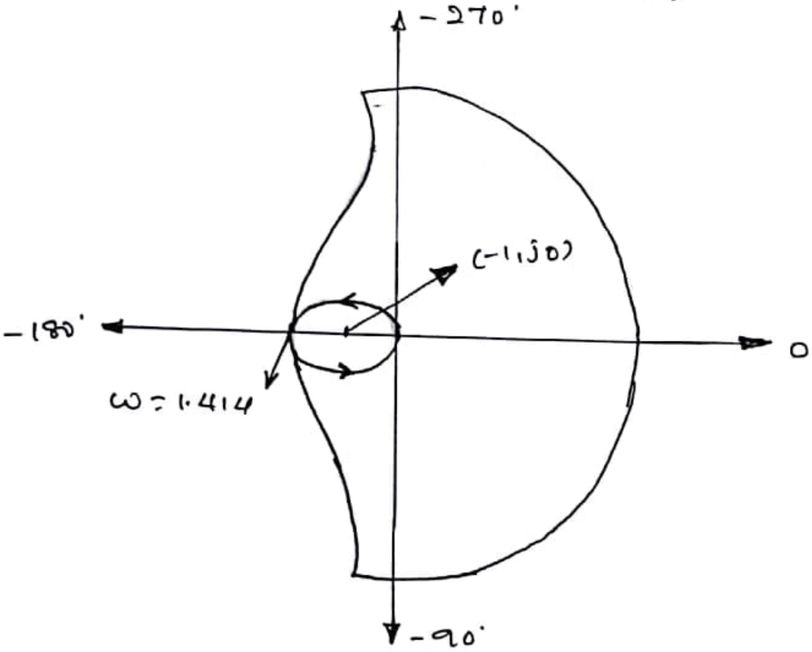


Fig (b)

6. step (1): Consider the given transfer function $G(s)H(s)$ and is in standard time constant form.

$$G(s)H(s) = \frac{10}{s(1+s)(1+0.02s)} \qquad - (1)$$

step (2): Obtain the expression for the phase angle ϕ to draw the phase angle plot. Put $s = j\omega$ in the equation (1)

$$G(j\omega)H(j\omega) = \frac{10 + j0}{(j\omega)(1+j\omega)(1+0.02j\omega)}$$

$$\therefore \text{Phase angle, } \phi = \frac{\tan^{-1}(0/10)}{\tan^{-1}(\frac{\omega}{0}) + \tan^{-1}(\frac{\omega}{1}) + \tan^{-1}(\frac{0.02\omega}{1})}$$

$$\phi = \tan^{-1}(0/10) - \tan^{-1}(\frac{\omega}{0}) - \tan^{-1}(\frac{\omega}{1}) + \tan^{-1}(\frac{0.02\omega}{1})$$

$$\phi = 0' - 90^\circ - \tan^{-1}(\omega) - \tan^{-1}(0.02\omega)$$

ω in rad/sec	Phase angle ϕ in degrees	ω in rad/sec	Phase angle ϕ in degrees
0.1	$-95.82'$	10	$-185.59'$
0.2	$-101.53'$	20	$-198.93'$
0.5	$-117.13'$	50	$-223.85'$
1	$-136.14'$	100	$-242.86'$
2	$-155.72'$	500	$-264.174'$
5	$-174.40'$	∞	$-270'$

Step (3): Compare $G(s)H(s)$ with the standard time constant form and obtain corner frequency of various factors.

standard time constant form is given by.

$$G(s)H(s) = \frac{k\,(1+T_a s)\,(1+T_b s) \cdots}{s^n\,(1+T_1 s)\,(1+T_2 s) \cdots}$$

Comparing with equation (1), we get

(i) System resultant gain, $K = 10$ with no corner frequency.

(ii) One pole at Origin, $\frac{1}{s}$ with no corner frequency.

(iii) Simple pole, $\frac{1}{(1+s)} \Longrightarrow T_1 = 1 \therefore \omega c_1 = \frac{1}{T_1} = 1$ rad/sec

(iv) Simple pole, $\frac{1}{(1+0.02s)} \Longrightarrow T_2 = 0.02$

$$\therefore \omega c_2 = \frac{1}{T_2} = 50 \text{ rad/sec}$$

$\therefore \omega c_1 = 1$ rad/sec and $\omega c_2 = 50$ rad/sec

Step 4: Prepare the asymptotic approximation table for the magnitude plot.

Factor	Contribution	Resultant slope	Corner frequency
System gain $K = 10$	$20 \log K = 20 \log 10$ $= 20\,dB$	–	–
1 pole at origin $(1/s)$	$-20\,dB/decade$ with intersection point on the 0 dB line at $\omega = 1\,rad/sec$	-20 Range $0 < \omega < 1$	–
Simple pole $\frac{1}{(1+s)}$	$-20\,dB/decade$	$-20-20$ $= -40$	$\omega_{c_1} = 1\,rad/sec$
Simple pole $\frac{1}{(1+0.02s)}$	$-20\,dB/decade$	$-40-20$ $= -60$	$\omega_{c_2} = 50\,rad/sec$

From the Bode plot

$\omega_{gc} = 3.4\,rad/sec$ – Gain cross over frequency.

Gain margin $= \underline{+18\,dB}$ (Ans)

$\omega_{pc} = 8.85\,rad/sec$ – Phase cross over frequency.

Phase margin $= \underline{+21^\circ}$ (Ans)

Since both gain margin and phase margin are positive, system is stable.

7. $$G(s)H(s) = \frac{K}{s(s+2)(s^2+8s+20)}$$

Step 1: Locate open loop poles (when $K=0$) and open loop zeros (when $K=\pm\infty$) on the s-plane

Open loop poles are at $s = 0, -2, -4+j2, -4-j2$ $\therefore P=4$

Open loop zeros NIL $\therefore Z=0$

Step 2: Determine the number of branches and its direction

No. of branches $N = P = 4$ $\because P > Z$

No. of branches terminating at infinity $= P-Z = 4-0 = 4$

$\therefore$ Starting points are $0, -2, -4+j2, -4-j2$ (open loop poles always)

Terminating points are $\infty, \infty, \infty, \infty$ (Since no zero exists)

Step 3: Determine the number of asymptotes and Angle of asymptotes

Number of branches, q = Number of branches approaching infinity

$= P-Z = 4-0 = 4$ branches

Angle of asymptotes, $\theta_A = \frac{(2q+1)180°}{(P-Z)}$

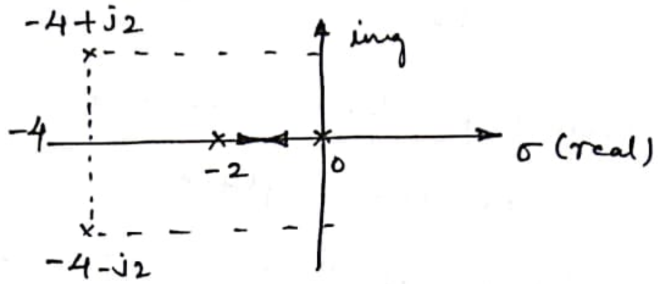
$\therefore \theta_{A1} = 45°$, $\theta_{A2} = 135°$, $\theta_{A3} = 225°$, $\theta_{A4} = 315°$

Step 4: Determine the centroid (σ_A)

Centroid, $\sigma_A = \frac{\Sigma \text{ Real part of open loop poles} - \Sigma \text{ Real part of open loop zero}}{(P-Z)}$

$$= \frac{0-2-4-4-0}{4-0} = -2.5$$

Step 5: Determine the segment of real axis which belongs to the root locus.



1. For s, between -2 and 0; $P_r + Z_r = 1 + 0 = 1$ (odd) RL exist
2. For s, between $-\infty$ and -2; $P_r + Z_r = 2 + 0 = 2$ (even) RL doesn't exist

Step 6: Determine the breakaway point

Characteristic eqn is given by $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+2)(s^2+8s+20)} = 0$$

$$s(s^3 + 10s^2 + 36s + 40) + K = 0$$

$$\therefore K = -s^4 - 10s^3 - 36s^2 - 40s \quad \text{— (1)}$$

Differentiating w.r.t 's' and equating to zero, we get

$$\frac{dK}{ds} = -4s^3 - 30s^2 - 72s - 40 = 0$$

$$\therefore s^3 + 7.5s^2 + 18s + 10 = 0$$

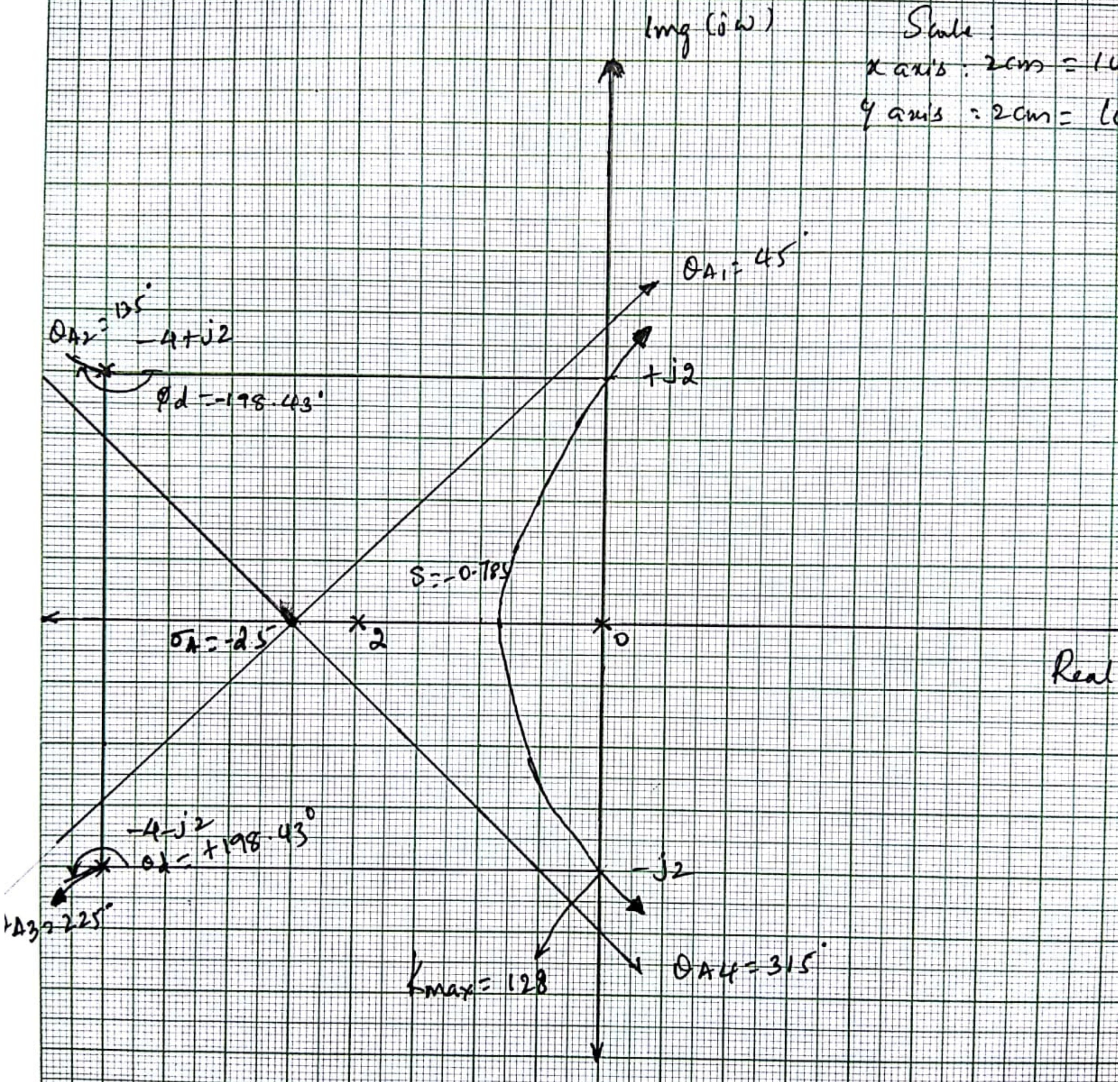
$\therefore$ Roots are at $s = -0.785, -3.357 + j1.205, -3.357 - j1.205$

Since -0.785 lies between 0 and -2 it is a valid breakaway point.

Step 7: Determine the intersection point with imaginary axis

Characteristic eqn is $s^4 + 10s^3 + 36s^2 + 40s + K = 0$ From eqn (1)

7.



$$G(s)H(s) = \frac{k}{s(s+2)(s^2+8s+20)}$$

Routh's array is

s^4	1	36	K
s^3	10	40	0
s^2	32	K	
s^1	$\frac{1280-10K}{32}$	0	
s^0	K		

The roots of the characteristic equation may lie on the imaginary ($j\omega$) axis if the elements in s^1 row are all zero.

$$\frac{1280-10K}{32} = 0$$

$$\therefore \quad \underline{K = 128}$$

Intersection points on the imaginary axis that corresponds to K = 128 are determined from the auxiliary eqn

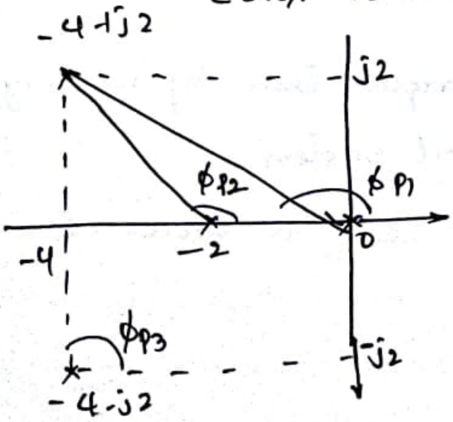
$$A(s) = 32s^2 + K = 0$$

$$s^2 \Big|_{K=128} = -4$$

$$s = \pm j2$$

step 8: Determine the angle of departure.

Consider the complex pole $(-4+j2)$ join remaining poles and zeros to it.



$\phi_{p1} = 180 - \tan^{-1} 2/4 = 153.43^\circ$

$\phi_{p2} = 180 - \tan^{-1} 2/2 = 135^\circ$

$\phi_{p3} = 90^\circ$

$\therefore \Sigma\phi_p = 153.43^\circ + 135^\circ + 90^\circ = 378.43^\circ$

$\Sigma\phi_z = 0.$

$\therefore \phi = \Sigma\phi_p - \Sigma\phi_z = 378.43^\circ$

Angle of departure, $\phi_d = 180^\circ - \phi$

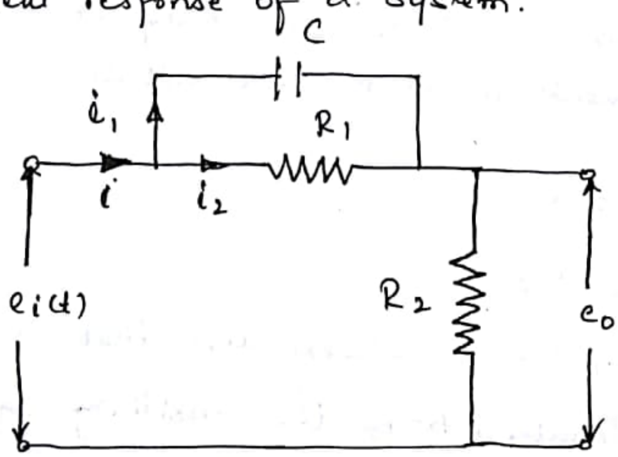
$= 180^\circ - 378.43^\circ$

$= -198.43$ at $-4+j2$

$= +198.43$ at $-4-j2$

8

a (i) Lead compensator: In lead compensator, the output voltage leads the input voltage wave. A practical lead compensator is one that produces effect of adding zero to the system. The lead compensator has an advantage of improving the transient response of a system.



Lead network

Important characteristics:

(1) It improves transient response of a system.

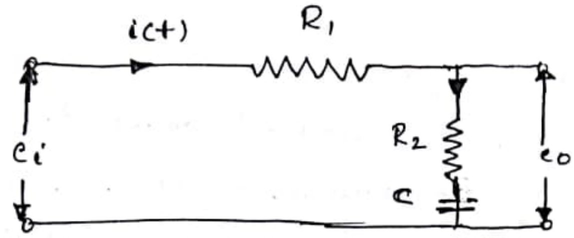
(2) It increases the damping due to addition of dominant zero and a pole.

(3) Due to increased damping, improvement in the settling time, less rise time, less overshoot.

(4) It improves the gain and phase margin there by improving the relative stability of the control system.

(5) It makes the system response faster due to increased bandwidth.

(ii) Lag compensator: In lag compensator, the output voltage lags the input voltage wave. A practical lag compensator is one that has a simple pole and a simple zero in the left-half of S-plane with a pole to the right of the zero i.e pole is nearer to the origin than the zero.



Characteristics of lag compensator:

(1) For $\beta > 1$, pole is always located to right of the zero. Therefore pole is nearer to the origin than zero.

(2) It improves steady-state performance of the control system.

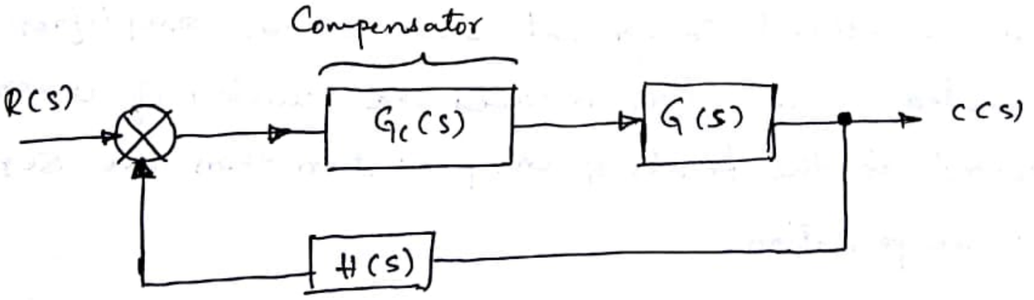
(3) It prolongs the transient response of the system.

(4) It makes system very sensitive to parameter variations.

(5) It increases rise time and settling time.

8.
(b)

(i) Series compensated system:

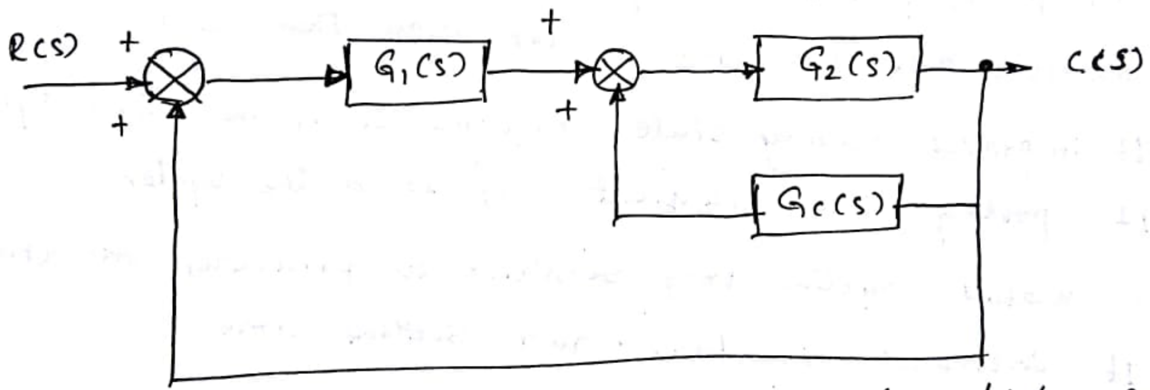


In series compensated system, an external device known as compensator, which is having transfer function $G_c(S)$ is introduced in series with the feed forward transfer function G(S) of the plant. Therefore this type of arrangement is known as feed forward compensation.

In this system, signal flows from lower energy level to the higher energy level, due to this gain of the system is reduced. To increase the gain and also to provide the isolation, an additional component such as amplifier is required. Thus, thus increase the number of components required in this system than the feedback compensation.

(ii) Feedback compensated system:

In feedback compensation, an external device known as compensator which is having transfer function $G_c(s)$ is introduced in the feedback path. Therefore this type of arrangement is known as feedback compensation.



In this type of arrangement signal flows from higher energy level to the lower energy level, due to this requirement of an additional component such as an amplifier is eliminated. Thus, this reduces the number of components required in the feedback compensation than the series - compensation.