CBCS Scheme

USN

15ME42

Fourth Semester B.E. Degree Examination, June/July 2018
Kinematics of Machines

Time: 3 hrs.
Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Define the following:
i) Link ii) Kinematic pairs iii) Kinematic chain
iv) Mechanism v) Structure vi) Degree of freedom **(06 Marks)**

b. Explain with neat sketch crank and slotted lever mechanism. **(05 Marks)**

c. Explain with neat sketch peaucellier mechanism. **(05 Marks)**

OR

2 a. Explain with neat sketch Ackerman steering mechanism. Mention condition for correct steering. **(08 Marks)**

b. Explain with neat sketch: i) Oldham's coupling ii) Pantograph. **(08 Marks)**

Module-2

3 The crank and connecting rod of a theoretical steam engine are 0.5 m and 2m long respectively. The crank makes 180 rpm in the clockwise direction. When it has turned 45° from the inner dead centre position, determine:
i) Velocity of piston
ii) Angular velocity of connecting rod
iii) Velocity of point E on the connecting rod 0.5 m from the crank end
iv) Velocities of rubbing at the pins of the crank shaft, crank and cross head when the diameter of their pins are 50 mm, 60 mm and 30 mm respectively
v) Position and linear velocity of any point G on the connecting rod which has the least velocity relative to crank shaft. **(16 Marks)**

OR

4 a. State and prove Aronhold Kennedy's theorem. **(04 Marks)**

b. In a slider crank mechanism, the length of crank and connecting rod are 125 mm and 500 mm respectively. The centre of gravity 'G' of the connecting rod is 275 mm from the slider. The crank speed is 600 rpm clockwise. The crank makes 45° from inner dead centre. Locate all the instantaneous centers and find velocity of slider, velocity of slider, velocity of point G and angular velocity of connecting rod. By Klein's construction, determine the acceleration of the slider and the point G. **(12 Marks)**

Module-3

5 The crank of an engine is 200 mm long and the ratio of connecting rod length to crank radius is 4. Determine the acceleration of piston when the crank has turned through 45° from the inner dead centre position and moving towards center at 240 rpm by complex algebra analysis. **(16 Marks)**

OR

6 a. Derive the expression for Freudenstein's equation for slider crank mechanism. **(12 Marks)**

b. Explain function generation for four bar mechanism. **(04 Marks)**

Module-4

7 a. Derive the equation for length of path of contact. **(08 Marks)**

b. A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 rpm. When the gear ratio is 1.75, find in order that the interference is just avoided:
   i) The addenda on pinion and gear wheel
   ii) Length of path of contact
   iii) The maximum velocity of sliding of teeth on either side of the pitch point. **(08 Marks)**

OR

8 a. Explain with neat sketch:
   i) Simple gear train
   ii) Compound gear train
   iii) Reverted gear train
   iv) Epicyclic gear train **(08 Marks)**

b. In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm of the gear train rotates at 150 rpm in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of the gear B. If the gear A instead of using fixed, makes 300 rpm in the clockwise direction, what will be the speed of gear B. Arrangement is shown in Fig.Q8(b).

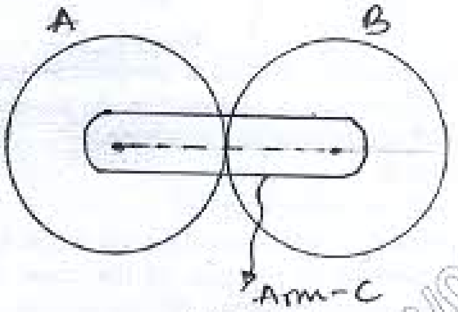


Fig.Q8(b) **(08 Marks)**

Module-5

9 A cam is to be designed for a knife edge follower with the following data, cam lift = 40 mm during 90° for cam rotation with simple harmonic motion, dwell for the next 30°, during the next 60° of cam rotation, the follower returns to its original position with simple harmonic motion, dwell during the remaining 180°. Draw the profile of the cam when the line of stroke of the follower passes through the axis of cam shaft. The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 rpm. Assume the direction of cam rotation is clockwise. **(16 Marks)**

OR

10 In a symmetrical tangent cam operating a roller follower, the least radius of the cam is 30 mm and roller radius is 17.5 mm. The angle of ascent is 75° and the total lift is 17.5 mm. The speed of the cam shaft is 600 rpm. Calculate:
   i) The principal dimensions of the cam.
   ii) The accelerations of the follower at the beginning of the lift, where straight flank merges into the circular nose and at the apex of the circular nose. Assume that there is no dwell between ascent and descent. **(16 Marks)**

* * * * *

Solution of : KINEMATICS OF MACHINES - 15ME42

Fourth Semester B.E. Degree Examination, June/July 2018

MODULE-1

1 a. (i) Link : Each part of a machine which moves relative to some other part is known as a link

(ii) Kinematic Pair : A kinematic pair is a joint of two links having relative motion between them.

(iii) Kinematic Chain : When the kinematic pair are coupled in such a way that the last link is joined to the first link to transmit definite motion is called as a kinematic chain.

(iv) Mechanism : When one of the links of a kinematic chain is fixed, it is called as a mechanism.

(v) Structure : Structure is an assemblage of a number of resistant bodies having no relative motion between them and is meant for carrying loads having straining actions.

(vi) Degree of Freedom : DoF of a pair is defined as the number of independant relative motions, both translational and rotational that a pair can have.

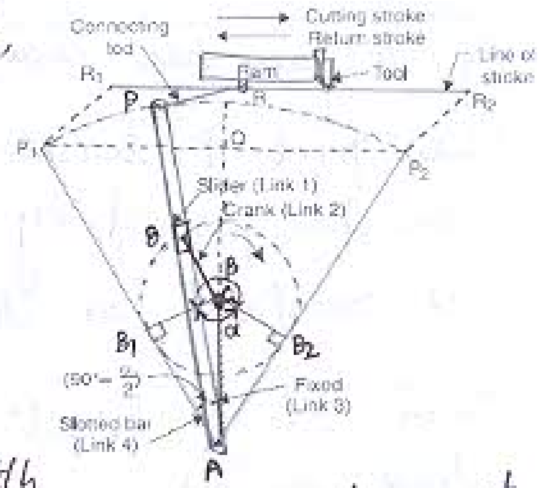
b. Crank and Slotted lever mechanism :⇒

- This mechanism is mostly used in shaping machines, slotting machines and in rotary internal combustion engines.

- In this mechanism, the link AC forming the turning pair is fixed. This is link 3.
- The link 3 corresponds to the connecting rod of a reciprocating steam engine.
- The driving crank CB revolves with uniform angular speed about the fixed centre C.
- A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A.
- A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R_1R_2. The line of stroke of the ram (i.e, R_1R_2) is perpendicular to AC produced.
- In the extreme positions, AP_1 and AP_2 are tangential to the cutting tool is at the end of the stroke.
- The forward or cutting stroke occurs when the crank rotates from the position CB_1 to CB_2 or through an angle 'β' in the clockwise direction.
- The return stroke occurs when the crank rotates from the position CB_2 to CB_1 or through an angle 'α' in the clockwise direction.
- Since, the crank has uniform angular speed,

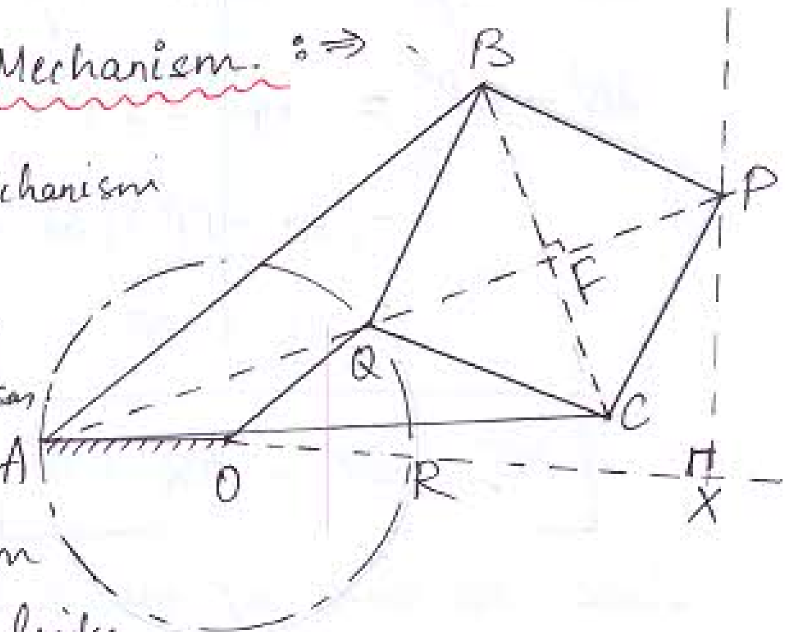
$$\Rightarrow \boxed{\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{\beta}{\alpha} = \frac{\beta}{360^\circ - \beta} = \frac{360^\circ - \alpha}{\alpha}}$$

- From the figure besides, its clear that the angle 'β', made by the forward or cutting stroke is greater than the angle 'α' described by the return stroke.



- Since the crank rotates with uniform angular speed, therefore the return stroke is completed within shorter time. Thus it is called quick return motion mechanism.

c.) Peaucellier's Mechanism :⇒



- Peaucellier's mechanism is an exact straight line motion mechanism.

- This mechanism consists of 8 links such that :-

$$AB = AC, \quad AO = OR, \quad QB = BP = PC = CQ.$$

- The pin Q is constrained to move along the circumference of a circle by means of the link OQ.

- The link OQ is the crank.
- The pins P and Q are on the opposite corners of a four bar chain which is a rhombus.
- The point 'P' traces a straight line path. This can be proved by showing the product $AQ \times AP$ as constant as the crank OQ rotates.

Join BC to bisect PQ at F, then from the Δ AFB, BFP, we have:

$AB^2 = AF^2 + FB^2$ and $BP^2 = BF^2 + FP^2$.

Substracting the above 2 eq^ns, we have :→

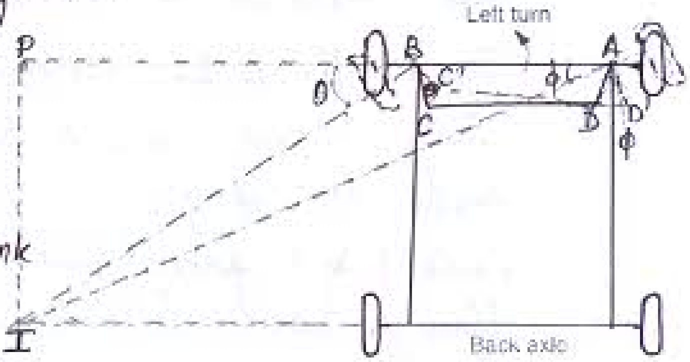
$$AB^2 - BP^2 = AF^2 - FP^2$$
$$= (AF - FP)(AF + FP).$$
$$= AQ \times AP.$$

$$\Rightarrow \boxed{AB^2 - BP^2 = AQ \times AP}$$

Since, AB and BP are links of constant length, the product of AP and AQ also remains constant. Therefore, the point 'P' traces out a straight path normal to AR produced.

2. a.) Ackerman Steering Gear Mechanism :⇒

- In Ackerman Steering gear mechanism, the whole of the mechanism is on the back of the front wheels.
- Ackerman steering gear consists of turning pairs only. Hence wear and tear is less compared to Davis steering gear.
- In Ackerman Steering gear, the mechanism ABCD is a four bar crank chain as shown in the figure.



- The shorter links BC and AD are of equal length and are connected by hinge joints with the front wheel axle.
- The longer links AB and CD are of unequal length.

* The following are the only three positions for correct steering :⇒

① When the vehicle moves along a straight path, the longer links AB and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle as shown by the firm lines in the above figure.

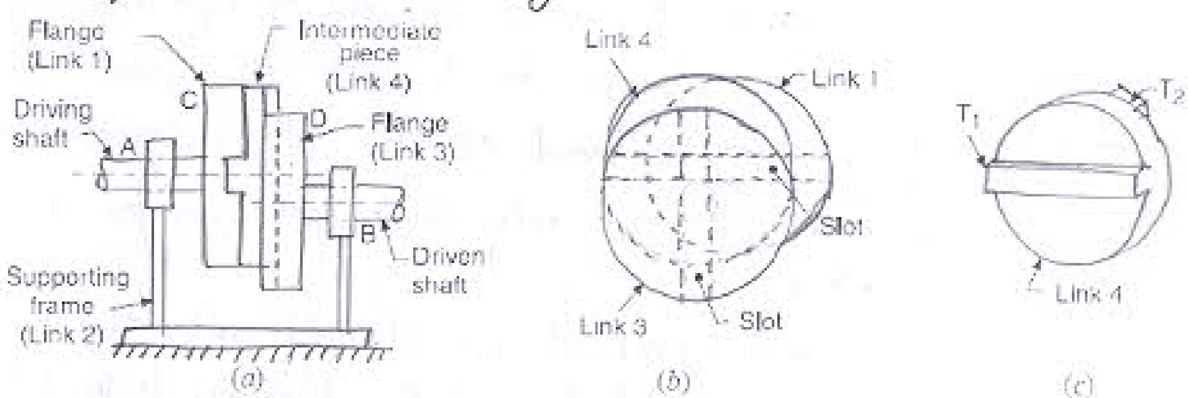
② When the vehicle is steering to the left, the position of the gear is shown by dotted lines in the figure. In this position, the lines of the front wheels axle intersect on the back axle at 'I' for correct steering.

③ When the vehicle is steering to the right, the similar position may be obtained.

→ **Condition for correct steering:** All the four wheels must turn about the same instantaneous centre.

b. (i) Oldham's Coupling :-

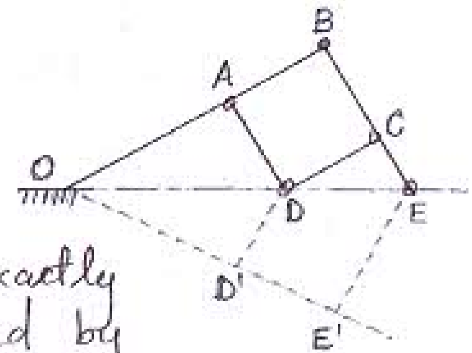
- An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart.
- The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed.
- This inversion is obtained by fixing link 2 as shown.
- The shafts to be connected have two flanges, link 1 and link 3 rigidly fastened at their ends by forging.
- Link 1 & 3 form turning pair with link 2. These flanges have diametrical slots cut in their inner faces as shown in figure 'b'.



- The intermediate piece (link 4) which is a circular disc, have two tongues or projections T_1 & T_2 on each face at right angles to each other as shown in fig. 'c'.
- The tongues on the link 4 closely fit into the slots in the two flanges (link 1 & 3).
- The link 4 can slide or reciprocate in the slots in the flanges.
- When the driving shaft A is rotated, the flange C (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange D (link 3) at the same angle and thus the shaft B rotates.

- Hence links 1, 3 and 4 have the same angular velocity at every instant.
- A little consideration will show that there is sliding motion between the link 4 and each of the other links 1 and 3.

(ii) Pantograph :⇒



- A pantograph is an instrument used to reproduce to an enlarged or reduced scale and as exactly as possible the path described by a given point.
- Hence, this mechanism finds its use in copying devices such as engraving or profiling machines.

Applications of Pantograph are as follows :→

1. Pantograph is used as geometrical instrument to produce geometrical figures and plane areas of irregular shapes such as maps, plans and drawings on enlarged or reduced scale.
2. Pantograph is used to guide cutting tools.
3. It is used as an indicator rig to reproduce the displacement of cross-head of reciprocating engine which gives the displacement of piston.
4. A modified form of pantograph is used to collect power at the top of an electric locomotive.
5. It is also used as an educational kit for kids.

MODULE - 2

3. Given :⇒

Crank ; $OB = 0.5m$.

Connecting rod , $BP = 2m$.

$N_{BO} = 180$ rpm (CW)

$\theta = 45°$.

$\omega_{BO} = \frac{2\pi N}{60} = \frac{2 \times \pi \times 180}{60} = 18.852$ rad/s .

$v_{BO} = v_B = \omega_{BO} \times OB$

$= 18.852 \times 0.5$

$\Rightarrow \boxed{v_{BO} = 9.426 \text{ m/s}}$.

(i) Velocity of Piston :⇒ .

Draw the velocity diagram by taking

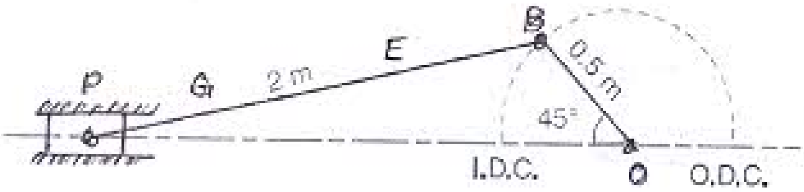
vector $ob = v_{BO} = v_B = 9.426$ m/s .

From velocity diagram, ,

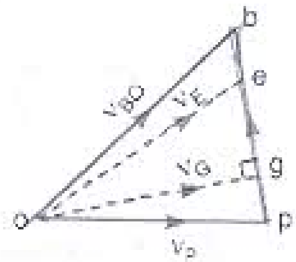
by measurement, we find the velocity of piston P as :→

v_P = vector $op = 8.15$ m/s .

$\Rightarrow \boxed{v_P = 8.15 \text{ m/s}}$ (Ans) .



(a) Space diagram.



(b) Velocity diagram

(ii) Angular velocity of Connecting Rod :→

- From velocity diagram, we find that the velocity of P with respect to B as :→

$$v_{PB} = \text{vector } bp = 6.8 \text{ m/s}.$$

∵ the length of the connecting rod PB = 2m,

∴ angular velocity of the connecting rod is :

$$\omega_{PB} = \frac{v_{PB}}{PB} = \frac{6.8}{2} = 3.4 \text{ rad/s (CCW)}.$$

$$\Rightarrow \boxed{\omega_{PB} = 3.4 \text{ rad/s (CCW)}} \text{ (Ans)}.$$

(iii). Velocity of E on the connecting rod :→

From velocity diagram ;

By measurement, we find that the velocity of point E ; v_E = vector oe = 8.5 m/s.

$$\Rightarrow \boxed{v_E = 8.5 \text{ m/s}}$$

$$\left(\because \frac{BE}{BP} = \frac{be}{bp} \Rightarrow be = \frac{BE \times bp}{BP} \right)$$

(iv) Velocity of rubbing :→

Given :→ diameter of crank-shaft pin at O

$d_O = 50mm = 0.05\,m$

diameter of crank-pin at B ;

$d_B = 60mm = 0.06\,m$.

and diameter of cross-head pin ;

$d_C = 30mm = 0.03\,m$.

* The velocity of rubbing at the pin of crank-shaft :→

$$= \frac{d_O}{2} \times \omega_{BO} = \frac{0.05}{2} \times 18.85$$

$$= 0.47\,m/s \text{ (Ans)}$$

* The velocity of rubbing at the pin of crank :→

$$= \frac{d_B}{2}(\omega_{BO} + \omega_{PB})$$

$$= \frac{0.06}{2}(18.85 + 3.4)$$

$$= 0.6675\,m/s \text{ (Ans)}$$

* Velocity of rubbing at the pin of cross-head :⇒

$$= \frac{d_C}{2} \times \omega_{PB} = \frac{0.03}{2} \times 3.4$$

$$= 0.051\,m/s \text{ (Ans)}$$

(v) Position of and linear velocity of point G on the connecting rod :→

From the velocity diagram;

By measurement, we find :→

$$\text{vector } bg = 5 \text{ m/s}$$

The position of point G on the connecting rod is obtained as follows :→

$$\frac{bg}{bp} = \frac{BG}{BP} \Rightarrow BG = \frac{bg}{bp} \times BP$$

$$\Rightarrow \boxed{BG = 1.47 \text{m}} \text{ (Ans)}$$

By measurement, we find that the linear velocity of point G ;

$$v_G = \text{vector } og = 8 \text{ m/s}$$

$$\Rightarrow \boxed{v_G = 8 \text{ m/s}} \text{ (Ans)}.$$

4. a.) Aronhold Kennedy's Theorem :⇒

Statement : If three bodies moves relatively to each other, they have three instantaneous centres and lie on a straight line.

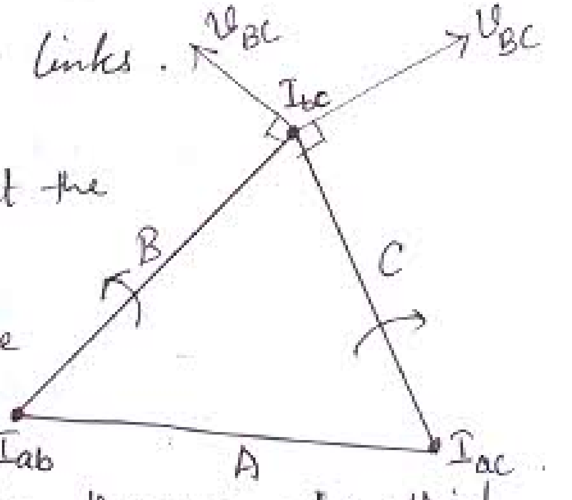
- Consider three kinematic links A, B and C having relative plane motion.
- The number of instantaneous centres (N) is

given by :→

$$N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2}$$

$$\Rightarrow N = 3.$$

where n = no. of of links.



- The two instantaneous centres at the pin joints of B with A, and C with A (i.e I_{ab} & I_{ac}) are the permanent instantaneous centres.
- According to Aronhold Kennedy's theorem, the third instantaneous centre I_{bc} must lie on the line joining I_{ab} and I_{ac}. In order to prove this, let us consider that the instantaneous centre I_{bc} lies outside the line joining I_{ab} and I_{ac} as shown in the figure.
- The point I_{bc} belongs to both the links B & C.
- Let us consider the point I_{bc} on the link B. Its velocity v_{BC} must be perpendicular to the line joining I_{ab} and I_{bc}.
- Now consider the point I_{bc} on the link C. Its velocity v_{BC} must be perpendicular to the line joining I_{ac} and I_{bc}.
- The velocity of the instantaneous centre is same whether it is regarded as a point on the first link or as a point on the second link.

- Therefore, the velocity of the point I_{bc} cannot be perpendicular to both lines $I_{ab} I_{bc}$ and $I_{ac} I_{bc}$ unless the point I_{bc} lies on the line joining the points I_{ab} and I_{ac}. Thus, the three instantaneous centres (I_{ab}, I_{ac} & I_{bc}) must lie on the same straight line.

The exact location of I_{bc} on line $I_{ab} I_{ac}$ depends upon the directions and magnitudes of the angular velocities of B and C relative to A.

b.) Given :→

Crank ⇒ $OC = 125mm = 0.125m$

Connecting rod ⇒ $PC = 500mm = 0.5m$.

$PG = 275mm = 0.275m$

$N_{OC} = 600 \text{ rpm (cw)}$.

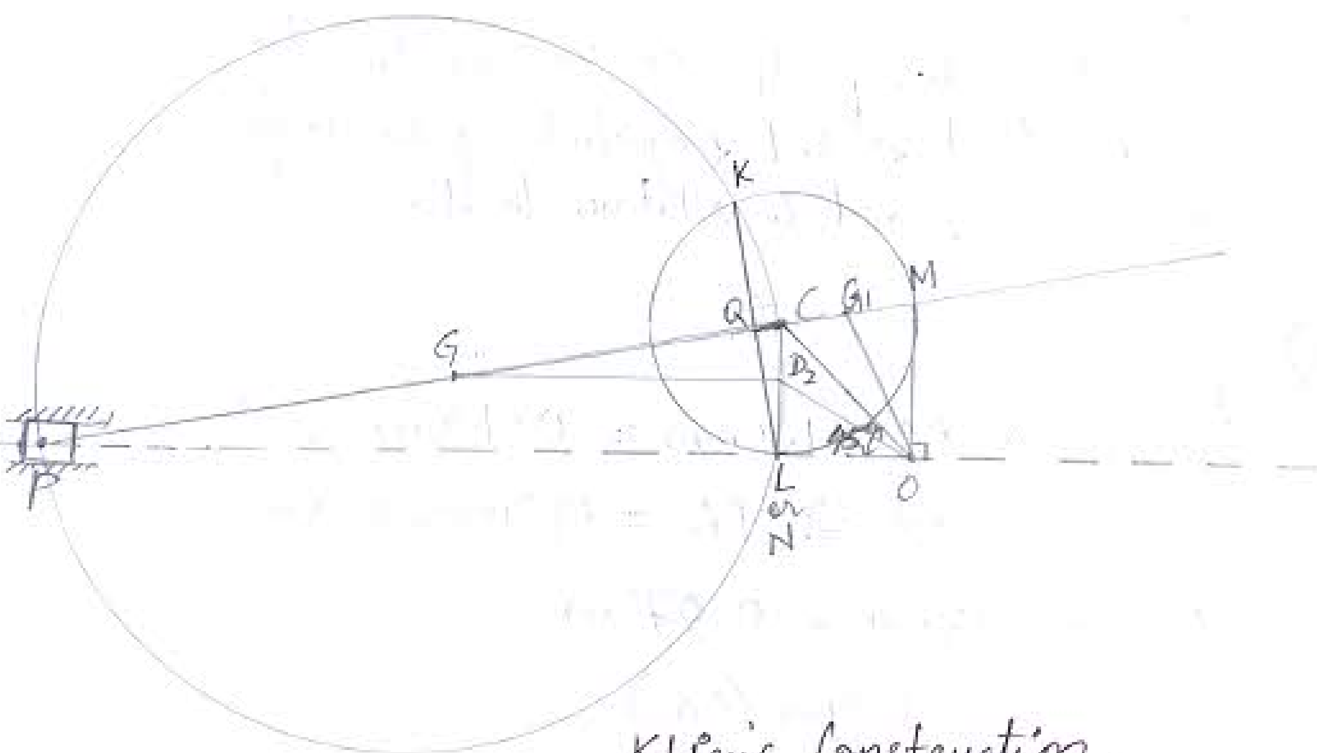
$\theta = 45°$.

$$\omega_{OC} = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 20\pi.$$

$$\Rightarrow \boxed{\omega_{OC} = 62.832 \text{ rad/s}}$$

Scale: ⇒

1 : 2.



Klien's Construction.

Δ OCM is Klien's Velocity diagram and Quadrilateral OCQN is Klien's Acceleration diagram.

Velocity of Slider or Piston: ⇒

$$v_p = \omega \times OM.$$

OM from the Klein's velocity diagram is measured as 2.1 cm.

Taking the scale and conversion;

$$OM = 0.105 \text{ mm}.$$

$\therefore v_p = \omega \times OM$

$= 62.832 \times 0.105$ m/s.

$\Rightarrow \boxed{v_P = 6.702 \text{ m/s}}$ ⇒ Velocity of Slider (Ans)

Velocity of Point G:→

$\frac{CG}{CP} = \frac{CG_1}{CM} = \frac{4.5}{10} = \frac{CG_1}{1.8} \Rightarrow CG_1 = 1.8$.

According to scale & measurement; $OG_1 = 0.105$m.

$\therefore v_G = \omega \times OG_1 = 62.832 \times 0.105$m.

$\Rightarrow \boxed{v_{G_1} = 6.6 \text{ m/s}}$ (Ans).

Angular Velocity of Connecting Rod:→

$v_{PC} = \omega \times CM$.

CM from the Klein's velocity diagram measured = 1.8cm.

Taking scale & conversion; $CM = 0.09$m.

$\therefore v_{PC} = \omega \times CM = 62.832 \times 0.09$

$\Rightarrow \boxed{v_{PC} = 5.65 \text{ m/s}}$ (Ans)

Acceleration of Slider/Piston:⇒

$a_p = \omega^2 \times NO$

According to measurement, scale and conversion;

$NO = 0.09$m.

$\therefore a_p = \omega^2 \times N_O = (63.832)^2 \times 0.09$

$\Rightarrow \boxed{a_p = 355.307 \text{ m/s}^2}$ (Ans).

Acceleration of Point G:→

Draw a line DD_2 $\|^{al}$ to PO which intersects CN at D_2.

According to measurement, scale & conversion;

$OD_2 = 0.1$m.

$\therefore a_D = \omega^2 \times OD_2 = (63.832)^2 \times 0.1 \Rightarrow \boxed{a_D = 407.45 \text{ m/s}^2}$

(Ans)

MODULE - 3

5. Given :→

AB = 200 mm = Crank = b.

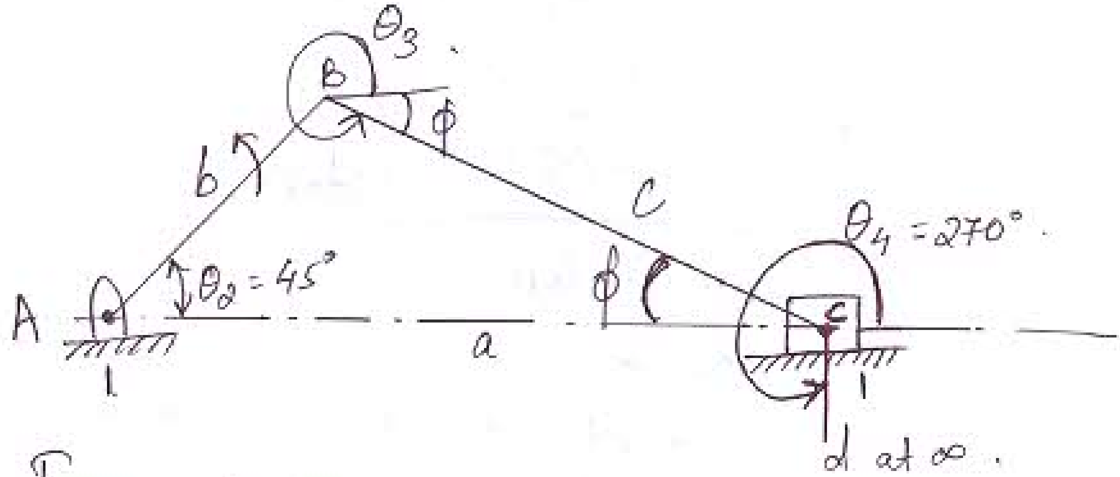
$$\frac{\text{Length of connecting rod}}{\text{Crank length}} = 4.$$

$\Rightarrow$ c = BC = 4 × 200 = 800 mm.

$\theta_2 = 45°$; $N_2 = 240$ rpm (ccw)

$$\therefore \omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 240}{60}$$

$\Rightarrow \omega_2 = 25.133$ rad/sec.



From Δ ABC.

$$\frac{\sin\phi}{b} = \frac{\sin\theta_2}{c}$$

$$\Rightarrow \frac{\sin\phi}{200} = \frac{\sin 45°}{800}$$

$$\Rightarrow \boxed{\phi = 10.182°}$$

$\therefore \theta_3 = 360^\circ - \phi = 360^\circ - 10.182$

$\Rightarrow \boxed{\theta_3 = 349.812^\circ}$

$$\omega_3 = -\frac{b\cos\theta_2}{c\cos\theta_3} \times \omega_2$$

$$= \frac{-200\cos 45}{800\cos 349.812} \times 25.133$$

$\Rightarrow \omega_3 = -4.514$ rad/sec

$$\alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 + \frac{b\omega_2^2\sin\theta_2 + c\omega_3^2\sin\theta_3}{c\cos\theta_3}$$

$$= 0 + \frac{200\times(25.133)^2\times\sin 45^\circ + 800(-4.514)^2\times\sin 349.812}{800\times\cos 349.812}$$

$\Rightarrow \boxed{\alpha_3 = 109.8 \text{ rad/sec}^2} \Rightarrow$ Angular acceleration of the connecting rod.

$\Rightarrow$ Real part of slider acceleration eq^n is :$\Rightarrow$

$$R_a = -b\left[\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2\right] - c\left[\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3\right]$$

$$= -200\left[(25.133)^2\times\cos 45^\circ + 0\right] - 800\left[(-4.514)^2\cos 349.812^\circ + 109.8\sin 349.812^\circ\right]$$

$\Rightarrow \boxed{R_a = -89838.22}$

$\Rightarrow$ Imaginary part of slider acceleration Eq^n is :→

$$I_a = b\left[\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2\right] + c\left[\alpha_3 \cos\theta_3 - \omega_3^2 \sin\theta_3\right]$$

$$= 200\left[0 - (25\cdot133)^2 \sin 45\right] + 800\left[109\cdot8 \times \cos 349\cdot812 - (-4\cdot514)^2 \sin 349\cdot812\right]$$

$$\Rightarrow \boxed{I_a = 0}$$

∴ Acceleration of the piston is given as :

$$A_p = \sqrt{R_a^2 + I_a^2}$$

$$= \sqrt{(-89838\cdot22)^2 - (0)^2}$$

$$= -89838\cdot22 \text{ mm/s}^2.$$

$$\Rightarrow \boxed{A_p = -89\cdot838 \text{ m/sec}^2}$$ (Ans).

6 a.) Freudenstein's Equation for slider Crank Mechanism is given below :⇒.

⇒ Freudenstein's Equation for Slider Crank Mechanism :

– A design problem where the link lengths of a slider crank mechanism must be determined so that the translation 'r' and the rotation ϕ are functionally related.

The desired relation is represented by $f(\phi, r) = 0$ as illustrated in fig.-1.

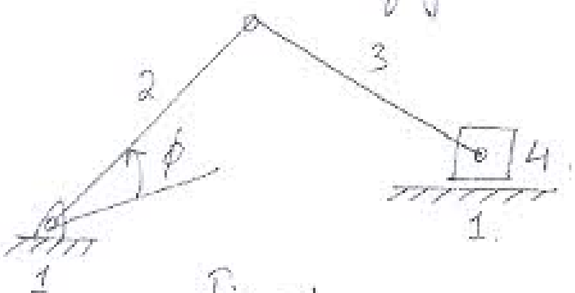


Fig.-1

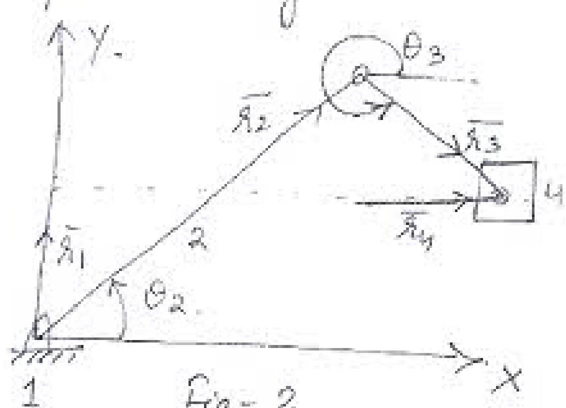


Fig-2.

Fig.-2 shows the slider crank mechanism and its vector loop.

The vector loop eqⁿ is :⇒

$$\bar{r}_2 + \bar{r}_3 - \bar{r}_4 - \bar{r}_1 = 0 \quad \text{——— (1)}$$

Considering the links to be vectors, displacement along the x-axis is :→

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 - r_4 = 0$$

$$\Rightarrow \quad r_3 \cos\theta_3 = -r_2 \cos\theta_2 + r_4 \quad \text{——— (2)}$$

Squaring eqⁿ (2) ;

$$r_3^2 \cos^2\theta_3 = r_2^2 \cos^2\theta_2 + r_4^2 - 2 r_2 r_4 \cos\theta_2 \quad \text{——— (3)}$$

Displacement along y-axis is :-

$$r_2 \sin\theta_2 + r_3 \sin\theta_3 - r_1 = 0$$

$$\therefore r_3 \sin\theta_3 = -r_2 \sin\theta_2 + r_1 \quad \text{——— (4)}$$

Squaring Eq^n (4) :-

$$r_3^2 \sin^2\theta_3 = r_2^2 \sin^2\theta_2 + r_1^2 - 2r_1 r_2 \sin\theta_2 \quad \text{——— (5)}$$

To eliminate θ_3, add both sides of the eq^n (3) & (5) :→

$$\therefore r_3^2 \cos^2\theta_3 + r_3^2 \sin^2\theta_3 = r_2^2 \cos^2\theta_2 + r_4^2 - 2r_2 r_4 \cos\theta_2 + r_2^2 \sin^2\theta_2 + r_1^2 - 2r_1 r_2 \sin\theta_2$$

$$\Rightarrow r_3^2(\cos^2\theta_3 + \sin^2\theta_3) = r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_4^2 + r_1^2 - 2r_2 r_4 \cos\theta_2 - 2r_1 r_2 \sin\theta_2$$

$$\Rightarrow r_3^2 = r_2^2 + r_4^2 + r_1^2 - 2r_2 r_4 \cos\theta_2 - 2r_1 r_2 \sin\theta_2$$

$$\Rightarrow r_3^2 - r_2^2 - r_1^2 + 2r_2 r_4 \cos\theta_2 + 2r_1 r_2 \sin\theta_2 = r_4^2 \quad \text{——— (6)}$$

Let $S_1 = 2r_2$; $S_2 = 2r_1 r_2$; $S_3 = r_3^2 - r_2^2 - r_1^2$

Substituting these values in Eq^n (6) :→

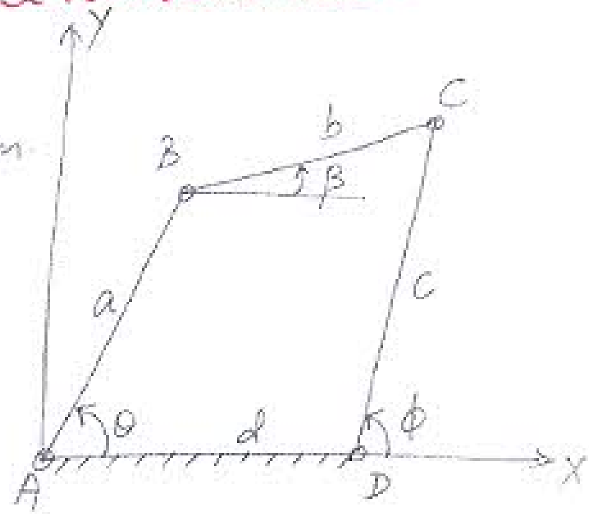
$$\boxed{S_1 r_4 \cos\theta_2 + S_2 \sin\theta_2 + S_3 = r_4^2} \quad \text{——— (7)}$$

Eq^n (7) is the Freudenstein's Eq^n for a slider crank mechanism.

- Unlike the four bar mechanism, function generating crank & slider mechanisms are not scale invariant because, unlike R_1, R_2 and R_3 in Eq^n (7) of four bar mechanism; S_1, S_2 and S_3 in Eq^n (7) of slider crank mechanism are not dimensionless ratios.

b.) FUNCTION GENERATION :→

⇒ Function Generation for four bar mechanism :→



- A four bar mechanism shown in the figure is in equilibrium.
- Let a, b, c and d be the magnitudes of the links AB, BC, CD and DA respectively.
- θ, β & ϕ are the angles of AB, BC and DC respectively with the x-axis.
- AD is the fixed link.
- AB and DC are the input and output links ~~to~~ ~~be~~ ~~vectors~~ respectively of the mechanism.

Considering the links to be vectors, displacement along the X-axis is ;

$$a\cos\theta + b\cos\beta - c\cos\phi - d = 0$$

$$\therefore b\cos\beta = -a\cos\theta + c\cos\phi + d$$

Squaring both sides ~~of the~~ above eqⁿ :⇒

$$b^2\cos^2\beta = a^2\cos^2\theta + c^2\cos^2\phi + d^2 - 2ac\cos\theta\cos\phi - 2ad\cos\theta + 2cd\cos\phi \quad \text{—— (1)}$$

Displacement along Y-axis :—

$$a\sin\theta + b\sin\beta - c\sin\phi = 0$$

$$\Rightarrow b\sin\beta = -a\sin\theta + c\sin\phi$$

Squaring both sides of the above eq^n :→

$$b^2 \sin^2 \beta = a^2 \sin^2\theta + c^2 \sin^2\phi - 2ac \sin\theta \sin\phi \quad \text{--- } (2)$$

Adding Eq^n (1) & Eq^n (2) :→

$$b^2 = c^2 + a^2 + d^2 - 2ac\cos\theta\cos\phi - 2ad\cos\theta + 2cd\cos\phi - 2ac\sin\theta\sin\phi$$

$$\Rightarrow a^2 - b^2 + c^2 + d^2 + 2cd\cos\phi - 2ad\cos\theta = 2ac(\cos\theta\cos\phi - \sin\theta\sin\phi)$$

Dividing both sides by $2ac$:-

$$\frac{a^2 - b^2 + c^2 + d^2}{2ac} + \frac{d}{a}\cos\phi - \frac{d}{c}\cos\theta = \cos(\theta - \phi) \quad \text{--- } (3)$$

Eq^n (3) is known as Freudenstein's Eq^n :→.

It can be written as :⇒

$$k_3 + k_1\cos\phi + k_2\cos\theta = \cos\theta - \phi \quad \text{--- } (5)$$

where; $k_3 = \dfrac{a^2 - b^2 + c^2 + d^2}{2ac}$; $k_1 = \dfrac{d}{a}$; $k_2 = -\dfrac{d}{c}$

Let the input and the output are related by some function such as $y = f(x)$.

For given positions :⇒

$\theta_1, \theta_2, \theta_3$ = Three positions of input link (given).

ϕ_1, ϕ_2, ϕ_3 = Three positions of output link (given).

- It is required to find the values of a, b, c & d to form a four bar mechanism giving the prescribed motions of the input and output links.

Eq^n (4) can be written as :⇒

$$k_1 \cos\phi_1 + k_2 \cos\theta_1 + k_3 = \cos(\theta_1 - \phi_1)$$
$$k_1 \cos\phi_2 + k_2 \cos\theta_2 + k_3 = \cos(\theta_2 - \phi_2)$$
$$k_1 \cos\phi_3 + k_2 \cos\theta_3 + k_3 = \cos(\theta_3 - \phi_3)$$

k_1, k_2 & k_3 can be evaluated by Gaussian elimination method or by the Cramer's rule :

$$\Delta = \begin{bmatrix} \cos\phi_1 & \cos\theta_1 & 1 \\ \cos\phi_2 & \cos\theta_2 & 1 \\ \cos\phi_2 & \cos\theta_3 & 1 \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \cos(\theta_1 - \phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2 - \phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3 - \phi_3) & \cos\theta_3 & 1 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} \cos\phi_1 & \cos(\theta_1 - \phi_1) & 1 \\ \cos\phi_2 & \cos(\theta_2 - \phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3 - \phi_3) & 1 \end{bmatrix}$$

$$\Delta_3 = \begin{bmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1 - \phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2 - \phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3 - \phi_3) \end{bmatrix}$$

k_1, k_2 and k_3 are given by :→

$$k_1 = \frac{\Delta_1}{\Delta}$$

$$k_2 = \frac{\Delta_2}{\Delta}$$

$$k_3 = \frac{\Delta_3}{\Delta}$$

Knowing k_1, k_2 & k_3, the values of a, b, c & d can be computed from the relations :→

$$k_1 = \frac{d}{a} \quad ; \quad k_2 = -\frac{d}{c} \quad ; \quad k_3 = \frac{a^2-b^2+c^2+d^2}{2ac}.$$

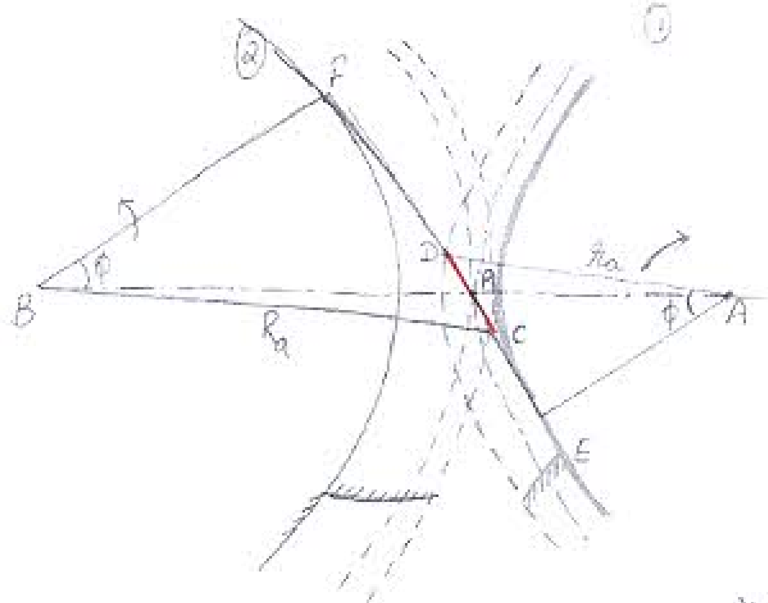
Value of either 'a' or 'd' can be assumed to be unity to get the proportionate values of other parameters.

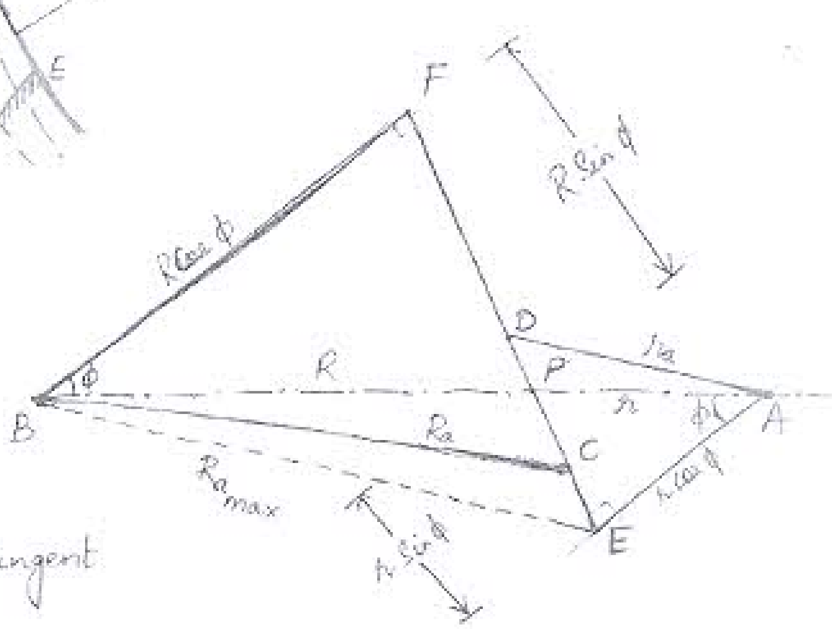
MODULE - 4

7. a.) Length of path of contact is the locus of the point of contact of the two mating teeth from the beginning of engagement to the end of the ~~engg~~ engagement.

In the below figure, CD is the length of path of contact.

LENGTH OF PATH OF CONTACT :⇒





- The pinion 1 is the driver & is rotating CW. The wheel 2 is driven in CCW direction.
- EF is their common tangent to the base circles
- Contact of the two teeth is made where the addendum circle of the gear meets the line of action EF, i.e, at C and is broken where the addendum circle of the pinion meets the line of action, i.e, at D. CD is the path of contact

Let r = Pitch circle radius of pinion
R = Pitch circle radius of gear.
r_a = Addendum circle radius of pinion.
R_a = Addendum circle radius of gear

Path of contact = Path of approach + path of recess

$\Rightarrow CD = CP + PD$

$= (CF - PF) + (ED - PE)$ ——— ①

In ΔBFC; $CB^2 = BF^2 + CF^2$

$\Rightarrow CF^2 = CB^2 - BF^2$

$= R_a^2 - R^2 \cos^2\phi$

$\Rightarrow \boxed{CF = \sqrt{R_a^2 - R^2\cos^2\phi}}$

In ΔAED; $AD^2 = ED^2 + EA^2$

$\Rightarrow ED^2 = AD^2 - EA^2$

$= r_a^2 - r^2 \cos^2\phi$

$\Rightarrow \boxed{ED = \sqrt{r_a^2 - r^2\cos^2\phi}}$

Putting the values in Eqⁿ ①;
we have

$CD = \left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] + \left[\sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi\right]$

Path of contact $\Rightarrow \boxed{CD = \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)(\sin\phi)}$

* Path of approach can be found if the dimensions of the driven wheel or gear are known. Similarily, the path of recess is known from the dimensions of the driving wheel or pinion.

b.) Given :⇒

$\phi = 16°$

$m = 6\,mm$

$t = 16$

$N_p = 240\,rpm$

$G = 1.75$

$$\omega_p = \frac{2\pi N_p}{60} = \frac{2\pi \times 240}{60} \Rightarrow \boxed{\omega_p = 25.136\,rad/s}$$

$$G = \frac{T}{t} \Rightarrow T = G.t = 1.75 \times 16$$

$$\Rightarrow \boxed{T = 28}$$

(i) Addenda on pinion and gear wheel :⇒

$$a_p = \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + \frac{28}{16}\left(\frac{28}{16} + 2\right)\sin^2 16°} - 1\right]$$

$$= 48\,(1.224 - 1) = 10.76\,mm$$

∴ $\boxed{\text{Addendum of the pinion} = 10.76\ mm}$ (Ans)

$$a_w = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6\times 28}{2}\left[\sqrt{1+\frac{16}{28}\left(\frac{16}{28}+2\right)\sin^2 16} - 1\right]$$

$$= 84\,(1.054-1) = 4.56\text{ mm}$$

$\therefore$ $\boxed{\text{Addendum of the wheel} = 4.56\text{ mm}}$

(ii) Length of Path of Contact :⇒

$$R = \frac{mT}{2} = \frac{6\times 28}{2} = 84\text{ mm}$$

$$r = \frac{mt}{2} = \frac{6\times 16}{2} = 48\text{ mm}$$

$$R_A = R + a_w = 84 + 10.76$$

$$\Rightarrow \boxed{R_A = 94.76\text{ mm}}$$

$$r_A = r + a_p = 48 + 4.56$$

$$\Rightarrow \boxed{r_A = 52.56\text{ mm}}$$

$\therefore$ Length of path of approach :⇒

$$CP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16^\circ} - 84\sin 16^\circ$$

$$= 49.6 - 23.15 = \underline{\underline{26.45\text{ mm}}}$$

∴ Length of path of recess :⇒

$$PD = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.56)^2 - (48)^2\cos^2 16^\circ} - 48\sin 16^\circ$$

$$= 25.17 - 13.23 = \underline{11.94\ mm}.$$

∴ Length of Path of Contact

$$CD = CP + PD$$

$$= 26.45 + 11.94$$

Path of Contact ⇒ $\boxed{CD = 38.39\ mm}$ Ans.

(iii) Maximum Velocity of Sliding :⇒

ω_w = Angular speed of gear wheel.

$$\frac{\omega_p}{\omega_w} = \frac{T}{t} = 1.75 = G$$

$$\Rightarrow \omega_2 = \frac{\omega_1}{G} = \frac{25.136}{1.75}$$

$$\Rightarrow \boxed{\omega_2 = 14.28\ rad/s}$$

∴ Maximum velocity of sliding of teeth on the left side of the pitch point, i.e; at point C;

$$= (\omega_p + \omega_w) \times CP$$

$= (25.136 + 14.28) \times 26.45$

$= 1043 \ mm/s$

$= \underline{1.043 \ m/s} \ (Ans).$

Maximum velocity of sliding of teeth on the right side of the pitch point, i.e at point D;

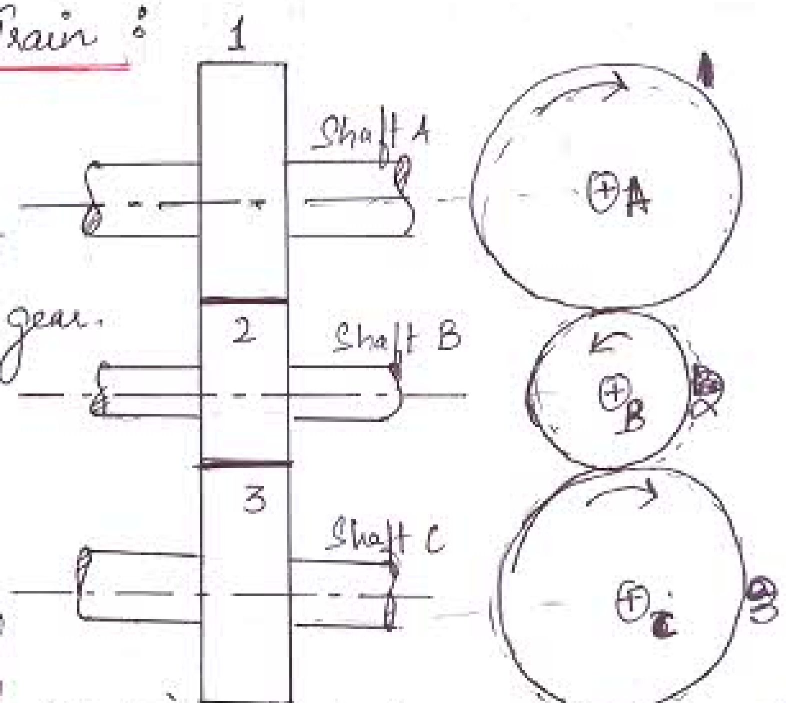
$= (\omega_p + \omega_w) \times PD$

$= (25.136 + 14.28) \times 11.94$

$= 471 \ mm/s$

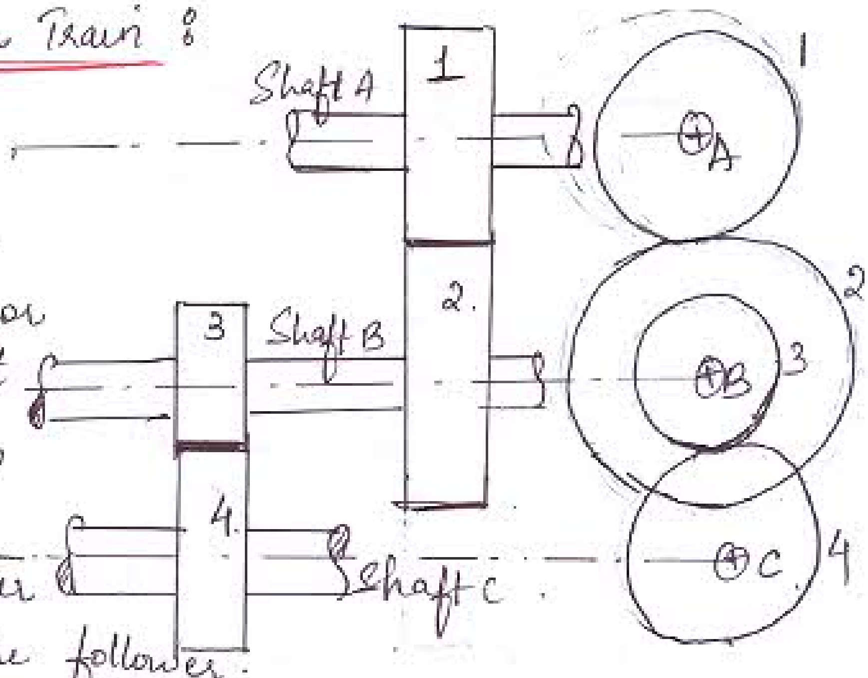
$= \underline{0.471 \ m/s} \ (Ans).$

8a.(i) Simple Gear Train :



- In simple gear train, each shaft carries only one gear.
- All gears revolve about the fixed axis.
- It is a series of gears, capable of receiving and transmitting motion from one gear to another.

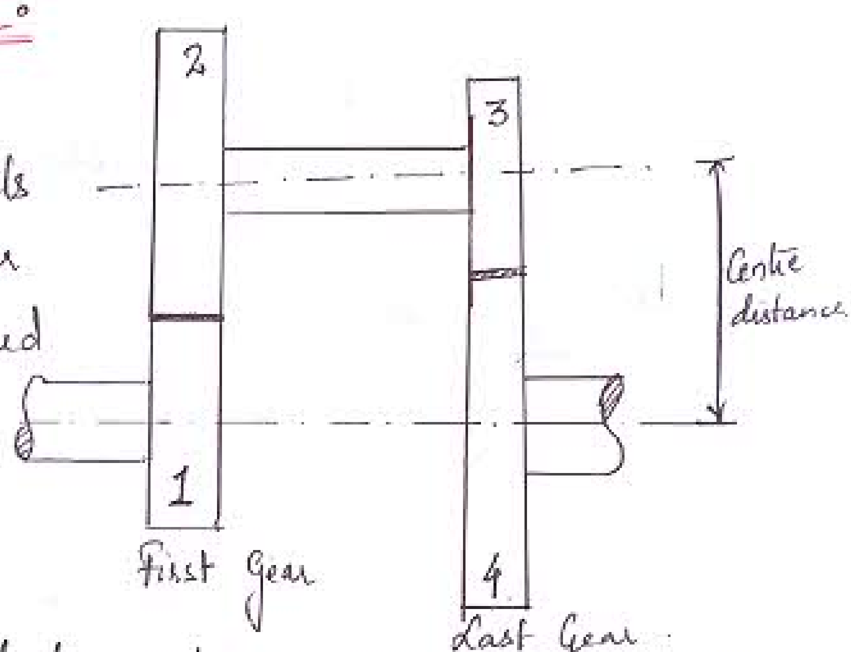
(ii) Compound Gear Train :



- In compound gears train, each shaft carries two or more gears except the first and last, one of which acts as the driver and the other the follower.
- All the gears revolves about a fixed axis.

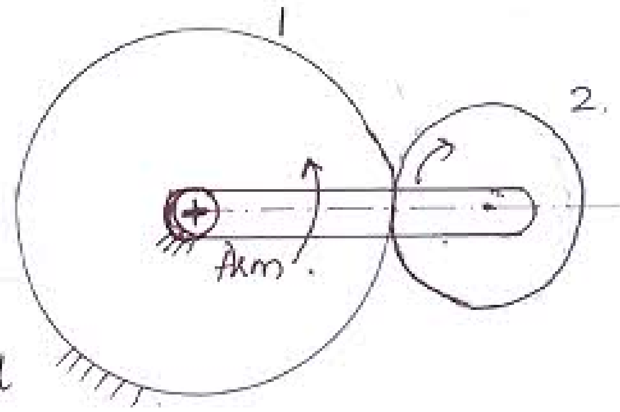
e.g. When a series of gears are connected in such a way that two or more gears rotate about an axis with the same angular velocity except the first and last gears.

(iii) Reverted Gear Train :



- If the axes of the first and last wheels of a compound gear coincide, it is called as a reverted gear train.
- Such type of gear train is used is clocks and in simple lathes where back gear is used to give a slow speed to the chuck.

(iv). Epicyclic Gear Train :

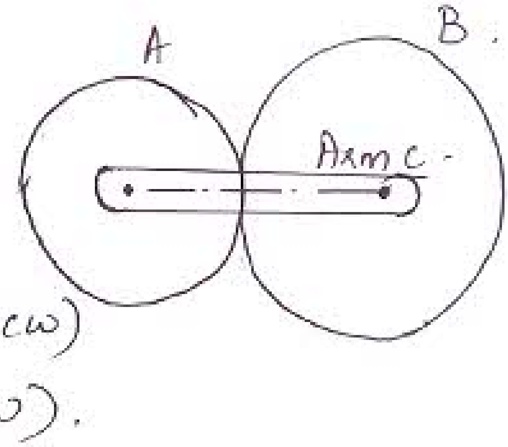


- In epicyclic gear train, axes of some gears (one or more) have relative motion with respect to other or relative to frame.
- The axes of rotation of all the wheels are not fixed.
- In the figure besides, the gear 2 revolves about its own axis as well as about the centre of the fixed gear 1.
- Epicyclic gear train is also called as planetary gear train.
- Large speed reductions are possible with epicyclic gears and if the fixed wheel is annular, a more compact unit is obtained.
- Important applications include in transmissions, computing devices, watches, etc.

b.) Given :→

$T_A = 36$

$T_B = 45.$



Case 1 :

$N_{arm} = 150$ rpm (ccw).

$N_A = 0.$

Case 2 : $N_{arm} = 150$ rpm (ccw)

$N_A = 300$ rpm (cw).

Tabular Method :→

Step No.	Conditions of Motion	Revolutions of elements		
		Arm C (N_C)	Gear A (N_A)	Gear B (N_B)
1.	Fix the arm 'c' and give +1 revolution to Gear A. for ccw.	0	+1	$-\frac{T_A}{T_B}$
2.	Give 'x' revolutions to Gear A with the arm 'c' fixed.	0	$+x$	$-\frac{T_A}{T_B}x$
3.	Add +y revolutions to all the elements	$+y$	$x+y$	$y-\frac{T_A}{T_B}x$

Case 1 :⇒ Gear A fixed.

$N_{arm} = 150 \text{ rpm} = N_C$

$\Rightarrow y = 150 \text{ rpm}$

Gear fixed $\Rightarrow N_A = 0$

$\Rightarrow x + y = 0$

$\Rightarrow x = -y \Rightarrow \boxed{x = -150 \text{ rpm}}$

$N_B = y - \frac{T_A}{T_B}x = 150 - (-150) \times \frac{36}{45}$

$\Rightarrow \boxed{N_B = 270 \text{ rpm (CCW)}}$

(Ans).

Case-2 :→ Gear A rotates at 300 rpm (CW).

$\therefore N_A = -300$ rpm. ($\because$ CCW is taken +ve)

$\Rightarrow x + y = -300$

$\Rightarrow x = -300 - y = -300 - 150$

$\Rightarrow \boxed{x = -450 \text{ rpm}}$

$\therefore N_B = y - \frac{T_A}{T_B} x$

$= 150 - \frac{36}{45} \times (-450)$

$\Rightarrow \boxed{N_B = 510 \text{ rpm (CCW)}}$

(Ans).

MODULE - 5

9. Given :→

$S = 40$ mm $= 0.04$ m.

$\theta_a = 90^\circ = 1.571$ rad ($\because 90^\circ = \pi/2$ rad)

$D_1 = 30^\circ$

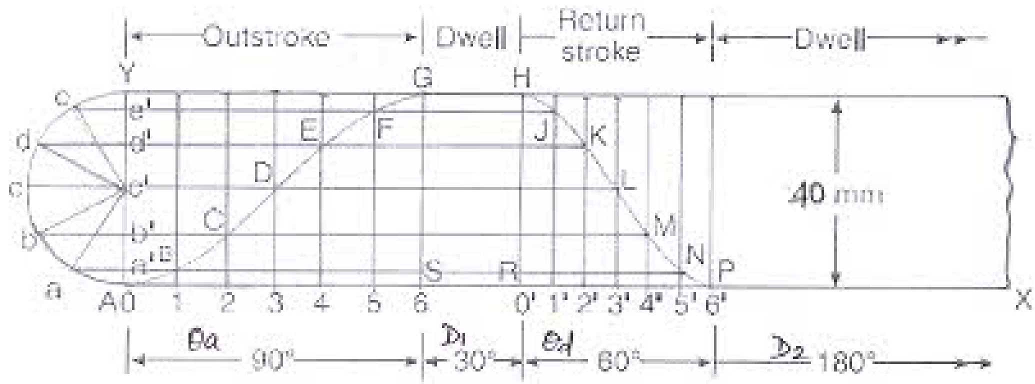
$\theta_d = 60^\circ = 1.047$ rad. ($\because 60^\circ = \pi/3$ rad)

$D_2 = 180^\circ$

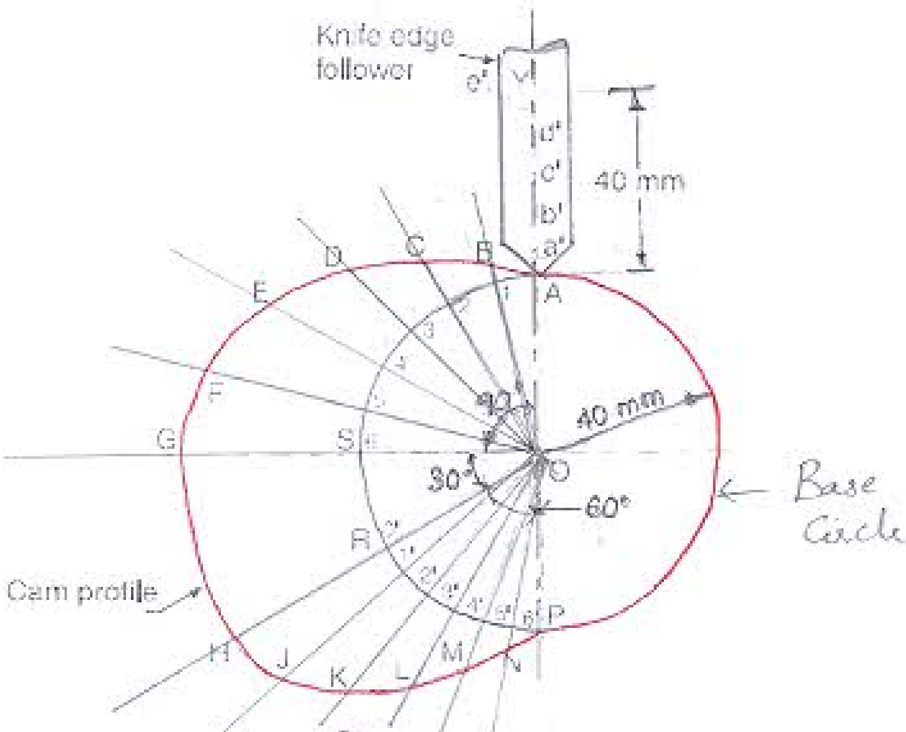
$N = 240$ rpm.

$r_{base\ circle} = 40\ mm$.

Inline Knife edge follower.



Displacement Diagram.



Cam Profile

* Maximum velocity of follower during ascent & descent

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$$

$$\Rightarrow \boxed{\omega = 25.14\ rad/s}$$

During ascent :→

$$\text{Maximum Velocity} = v_a = \frac{\pi \omega S}{2\theta_a} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571}.$$

$$\Rightarrow \boxed{v_a = 1\,m/s.} \text{ (Ans).}$$

During descent :→.

$$\text{Maximum Velocity} = v_d = \frac{\pi \omega S}{2\theta_d} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047}$$

$$\Rightarrow \boxed{v_d = 1.51\,m/s.} \text{ (Ans).}$$

* Maximum acceleration of follower during ascent & descent :

During ascent :⇒

$$\text{Maximum Acceleration} = a_a = \frac{\pi^2 \omega^2 S}{2\theta_a^2} = \frac{\pi^2 \times (25.14)^2 \times 0.04}{2 \times (1.571)^2}$$

$$\Rightarrow \boxed{a_a = 50.6\,m/s^2} \text{ (Ans).}$$

During descent :→

$$\text{Maximum Acceleration} = a_d = \frac{\pi^2 \omega^2 S}{2\theta_d^2} = \frac{\pi^2 \times (25.14)^2 \times 0.04}{2 \times (1.047)^2}$$

$$\Rightarrow \boxed{a_d = 113.8\,m/s^2} \text{ (Ans)}$$

10. Given :⇒

$r_1 = 30mm$

$r_2 = 17.5mm$

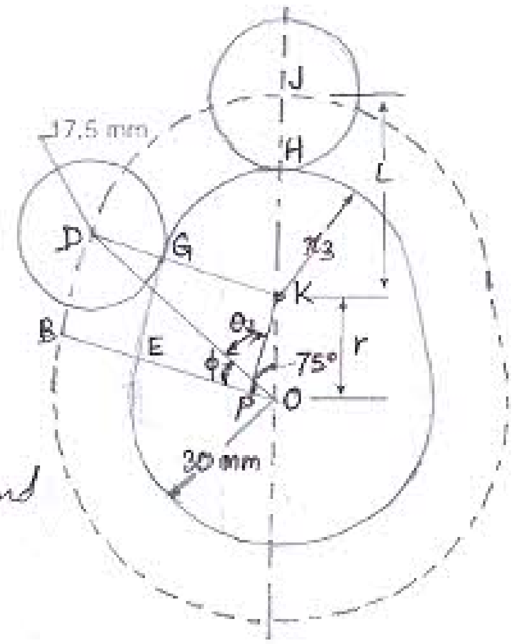
$\alpha = 75°$

Total lift $= 17.5mm$

$N = 600 rpm$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60}$

$\Rightarrow \omega = 62.84 rad/s.$



(i) Principal dimensions of the Cam :⇒

Let $r = OK$ = Distance between cam centre and nose centre.

r_3 = Nose radius

ϕ = Angle of contact of cam with straight flanks.

From the geometry of the figure given above :→

$r + r_3 = r_1 +$ Total lift

$= 30 + 17.5 = 47.5 mm$

$\Rightarrow r = 47.5 - r_3$ ——— (1).

Also $OE = OP + PE$

$\Rightarrow r_1 = OP + r_3$

$OP = r_1 - r_3 = 30 - r_3 \Rightarrow OP = 30 - r_3$ ——— (2).

Now from $\triangle$ OKP ;

$$OP = OK \times \cos\alpha .$$

$$\Rightarrow 30 - r_3 = (47.5 - r_3)\cos 75^\circ$$

$$= (47.5 - r_3) \times 0.2588$$

$$= 12.3 - 0.2588\, r_3 .$$

$$\Rightarrow \boxed{r_3 = 23.88\, mm} \text{ (Ans).}$$

$$r = OK = 47.5 - r_3 = 47.5 - 23.88$$

$$\Rightarrow \boxed{r = 23.62\, mm} \text{ (Ans)}$$

Again from $\triangle$ ODB :

$$\tan\phi = \frac{DB}{OB} = \frac{KP}{OB} = \frac{OK \sin\alpha}{r_1 + r_2} .$$

$$= \frac{23.62 \times \sin 75^\circ}{30 + 17.5} = 0.4803$$

$$\Rightarrow \boxed{\phi = 25.6^\circ} \text{ (Ans).}$$

(ii) Acceleration of the follower at the beginning of the lift :→

We know that acceleration of the follower at the beginning of the lift, i.e, when the roller has contact E on the straight flank :

$$a_{min} = \omega^2 (r_1 + r_2) = (62.84)^2 \times (30 + 17.5)$$

$$= 187600\, mm/s^2 \Rightarrow \boxed{a_{min} = 187.6\, m/s^2} \text{ Ans}$$

Acceleration of the follower where straight flank merges into a circular nose :→

We know that acceleration of the follower where straight flank merges into a circular nose, i.e; when the roller just leaves contact at G;

$$a_{max} = \omega^2(r_1 + r_2)\left[\frac{2 - \cos^2\phi}{\cos^3\phi}\right]$$

$$= (62.84)^2 (30 + 17.5)\left[\frac{2 - \cos^2 25.6^\circ}{\cos^3 25.6^\circ}\right]$$

$$= 303800 \text{ mm/s}^2$$

$$\Rightarrow \boxed{a_{max} = 303.8 \text{ m/s}^2} \text{ (Ans)}$$

Acceleration of the follower at the apex of the circular nose :→

We know that acceleration of the follower for contact with the circular nose :

$$a = \omega^2 \cdot r\left[\cos\theta_1 + \frac{L^2 \cdot r\cos 2\theta_1 + r^3\sin^4\theta_1}{(L^2 - r^2\sin^2\theta_1)^{3/2}}\right]$$

Since θ_1 is measured from the top position of the follower, therefore for the follower to have contact at the apex of the circular nose (i.e at point H);

$$\theta_1 = 0.$$

∴ Acceleration of the follower at the apex of the circular nose :→

$$a = \omega^2 \cdot r\left(1 + \frac{L^2 \cdot r}{L^3}\right)$$

$$\Rightarrow a = \omega^2 . r \left(1 + \frac{r}{L}\right)$$

$$= \omega^2 . r \left(1 + \frac{r}{l_2 + r_3}\right)$$

$$= (62.84)^2 \times 23.62 \left(1 + \frac{23.62}{17.5 + 23.88}\right).$$

$$= 146530 mm/s^2$$

$$\Rightarrow \boxed{a = 146.53 m/s^2} \quad \text{(Ans)}.$$