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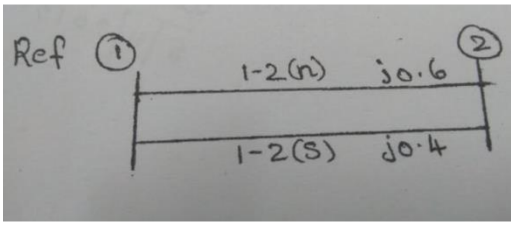
Internal Assesment Test - II

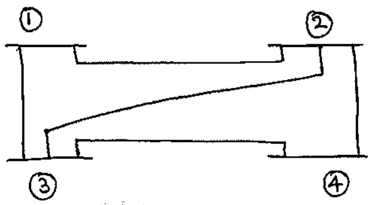
Sub:	Power System Analysis II						Code:	15EE71
Date:	05/11/2019	Duration:	90 mins	Max Marks:	50	Sem: 7	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1a	Compare NR and GS method for load flow analysis	[4]	CO2	L3
1b	Form Zbus using step by step building algorithm of the system shown in fig. Take element connected between 1-2 (s) as LINK.	[6]	CO5	L3



		Marks	CO	RBT
2	Obtain the Gauss –Seidal load flow solution at the end of first iteration for the power system shown in fig .Assume flat start for bus voltages V_3 and V_4. Given :0.2<=Q2<=1.0	[10]	CO1	L3



Line Dat

SB	EB	R(p.u)	X(p.u)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Bus Data

Bus No.	P_i	Q_i	V_i
1	-	-	$1.04\angle 0°$
2	0.5	-	1.04
3	-1.0	0.5	-
4	-0.3	-0.1	-

		Marks	CO	RBT
3	For the positive sequence network data shown in table below, obtain Z bus by building procedure.	[10]	CO5	L3

#	P-Q(nodes)	Pos.Seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

		Marks	CO	RBT
4	Derive the algorithm for the formation of bus impedance matrix Zbus for a single phase system when a link element is added to the partial network.	[10]	CO5	L2

5	Deduce the fast decoupled load flow model clearly stating all the assumptions made	[10]	CO2	L2
6	With the help of necessary equation explain how load flow analysis is conducted in NR method	[10]	CO2	L2
7	With the help of flow chart , explain the load flow study procedure with expressions as per Gauss Seidal method for power system having all types of buses	[10]	CO2	L2
8	Derive the algorithm for the formation of bus impedance matrix Zbus for a single phase system when a branch element is added to the partial network	[10]	CO5	L2

Solution

1a

18.6 Comparison of Solution Methods

Since the Gauss-Seidel is undoubtedly superior to Gauss method, the comparison is restricted only between G-S method and Newton-Raphson method and that too when Y bus matrix is used for problem formulation. From the view point of computer memory requirements, polar coordinates are preferred for solùtion based on N-R method and rectangular coordinates for the G-S method.

The time taken to perform one iteration of the computation is relatively smaller in case of G-S method as compared to N-R method but the number of iterations required by G-S method for a particular system are greater as compared to N-R method and they increase with the increase in the size of the system. In case of N-R method the number of iterations is more or less independent of the size of the system and vary between 3 to 5 iterations. The convergence characteristics of N-R method are not affected by the selection of a slack bus whereas that of G-S method is sometimes very seriously affected and the selection of a particular bus may result in poor convergence.

The main advantage of G-S method as compared to N-R method is its ease in programming and most efficient use of core memory. Nevertheless, for large power systems N-R method is found to be more efficient and practical from the view point of computational time and convergence characteristics. Even though N-R method can solve most of the practical problems, it may fail in respect of some ill-conditioned problem where other advanced mathematical programming techniques like the non-linear programming techniques can be used. For the readers to have an approximate idea of the computation time taken by N-R method in solving the load flow problem is that, IBM 360/PS 44 system takes less than 10 seconds to obtain a load flow solution of a 14 bus system starting with a flat voltage solution of $(1 + j0.0)$. This includes the formulation of nodal admittance matrix and its storage time.

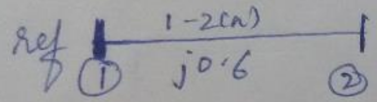
1b

→ easy to program	→ difficult to program.
→ efficient for core memory storage.	→ not efficient for core memory storage.

1. (b) **Step 1**

Addition of a node ② (bus) to ref node ① (bus)

↳ new branch is added.



$Z_{bus} = [j0.6]_{1\times1}$

branches

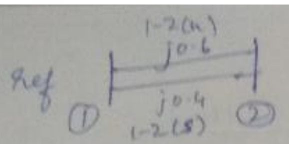
$[Z_{qi} = Z_{pi}, Z_{qi} = 0$ (ref node)

$, Z_{qq} = z_{pq,pq}]$ (ref node)

$Z_{qq} = Z_{pi} + z_{pq,pq}$

Step 2

Addition of a link between ① & ②



$Z_{bus} = \begin{bmatrix} j0.6 & Z_{1l} \\ Z_{l1} & Z_{ll} \end{bmatrix}_{2\times 2}$

6

2.

$Z_{li} = Z_{pi} - Z_{qi} \; ; \; Z_{ll} = Z_{pl} - Z_{ql} + z_{pq,pq}$ (links)

$Z_{l1} = Z_{1l} = -Z_{qi}$ (for ref node) $\left.\begin{matrix} p=0 \\ q=1 \end{matrix}\right\}$

(bilateral symmetry) $= -Z_{11} = \underline{\underline{-j0.6}}$

$(i=1)$

$Z_{ll} = -Z_{ql} + z_{pq,pq}$ (ref node)

$= -Z_{1l} + j0.4$

$= -(-j0.6) + j0.4 = j0.6 + j0.4 = \underline{\underline{j1}}$

$\therefore Z_{bus} = \begin{bmatrix} j0.6 & -j0.6 \\ -j0.6 & j1 \end{bmatrix}_{2\times 2}$

eliminating l^{th} elements (link)

$Z_{ij \, (\text{modified})} = Z_{ij \, (\text{old})} - \dfrac{Z_{il} \cdot Z_{lj}}{Z_{ll}}$

$Z_{11 \, (\text{new})} = Z_{1 \, (\text{old})} - \dfrac{Z_{1l} \cdot Z_{l1}}{Z_{ll}} = j0.6 - \dfrac{(-j0.6)(-j0.6)}{j1}$

$= \underline{\underline{j0.24}}$

$Z_{bus} = [j0.24]_{1\times1}$

2

$Y_{22,\text{ new}} = Y_{22,\text{ old}}$ + (

Modified Y_{BUS} is written below

$$Y_{BUS} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

In Example 6.4, let bus 2 be a PV bus now with $|V_2| = 1.04$ pu. Once ... assuming a flat voltage start, find Q_2, δ_2, V_3, V_4 at the end of the first iteration.

Given: $0.2 \leq Q_2 \leq 1$.

From Eq. (6.5), we get (*Note* $\delta_2^0 = 0$, i.e. $V_2^0 = 1.04 + j0$)

$$Q_2^1 = -\text{Im}\,\{(V_2^0)^* Y_{21} V_1 + (V_2^0)^* [Y_{22} V_2^0 + Y_{23} V_3^0 + Y_{24} V_4^0]\}$$

$$= -\text{Im}\,\{1.04\,(-2 + j6)\,1.04 + 1.04\,[(3.666 - j11)\,1.04 + (-0.666 + j2) + (-1 + \ldots$$

$$= -\text{Im}\,\{-0.0693 - j0.2079\} = 0.2079 \text{ pu}$$

$$\therefore Q_2^1 = 0.2079 \text{ pu}$$

From Eq. (6.51)

$$\delta_2^1 = \angle\left\{\frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2^1}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]\right\}$$

$$= \angle\left\{\frac{1}{3.666 - j11}\left[\frac{0.5 - j0.2079}{1.04 - j0} - (-2 + j6)(1.04 + j0) - (-0.666 + j2)(1 + j0) - (-1 + j3)(1 + j0)\right]\right\}$$

$$= \angle\left(\frac{4.2267 - j11.439}{3.666 - j11}\right) = \angle(1.0512 + j0.0339)$$

or $\delta_2^1 = 1.84658° = 0.032$ rad

$$\therefore V_2^1 = 1.04\,(\cos\delta_2^1 + j\sin\delta_2^1)$$

$$= 1.04\,(0.99948 + j0.0322)$$

$$= 1.03946 + j0.03351$$

$$V_3^1 = \frac{1}{Y_{33}}\left\{\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right\}$$

$$= \frac{1}{3.666 - j11}\left[\frac{-1 - j0.5}{(1 - j0)} - (-1 + j3)\,1.04 - (-0.666 + j2)(1.03946 + j0.03351) - (-2 + j6)\right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4^1 = \frac{1}{Y_{44}}\left\{\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1 - Y_{42}V_2^1 - Y_{43}V_3^1\right\}$$

$$= \frac{1}{3 - j9}\left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0394 + j0.0335) - (-2 + j6)(1.0317 - j0.08937)\right]$$

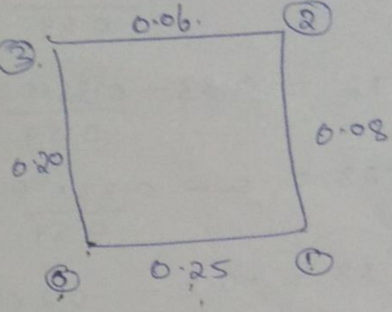
$$= \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031$$

Now, suppose the permissible limits on Q_2 (reactive power injection) are revised as follows:

3

(Q) Zbus building Algorithm.

	Pq	+ve reactance sequence in Pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06



① Add branch ⓪ to ① $Z_{bus} = 1\,[0.25]$

② Add branch ① to ② Z_{bus}

	1	2
1	0.25	0.25
2	0.25	0.33

③ Add branch ⓪ to ③ Z_{bus}

	1	2	3
1	0.25	0.25	0
2	0.25	0.33	0
3	0	0	0.2

$Z_{31} = Z_{pi} = 0$

$Z_{32} =$

Z_{33}

④ Add link ② & ③.

$Z_{qi} = Z_{pi} - Z_{qi}$

$Z_{ll} = Z_{pl} - Z_{ql} + z_{pqpq}$

$Z_{bus} =$

	1	2	3	l
1	0.25	0.25	0	0.25
2	0.25	0.33	0	0.33
3	0	0	0.2	-0.2
l	0.25	0.33	-0.2	0.59

$Z_{l1} = Z_{21} - Z_{31} = 0.25 - 0 = 0.25$

$Z_{l2} = Z_{22} - Z_{32} = 0.33 - 0 = 0.33$

$Z_{l3} = Z_{23} - Z_{33} = 0 - 0.2 = -0.2$

$Z_{ll} = Z_{2l} - Z_{3l} + z_{pqpq} = 0.33 + 0.2 + 0.06 = 0.59$

$Z_{12} = 0.25 - \frac{0.25 \times 0.33}{0.59} = 0.110$

$Z_{11} = 0.25 - \frac{0.25 \times 0.25}{0.59} = 0.144$

$$Z_{13} = 0 - \frac{0.25 \times -0.2}{0.59} = 0.084$$

$$Z_{22} = 0.33 - \frac{0.33 \times 0.33}{0.59} = 0.145$$

$$Z_{23} = 0 - \frac{0.33 \times -0.2}{0.59} = 0.112$$

$$Z_{33} = -0.2 - \frac{-0.2 \times -0.2}{0.59} = \cancel{0.268}\ .132$$

$$Z_{bus} = \begin{bmatrix} 0.114 & 0.11 & 0.084 \\ 0.11 & 0.145 & 0.112 \\ 0.08 & 0.112 & \cancel{0.268}\ .132 \end{bmatrix}$$

4

Addition of a link

If the added element p–q is a link, the procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage source e_l as shown in Fig. 4.4. This creates a

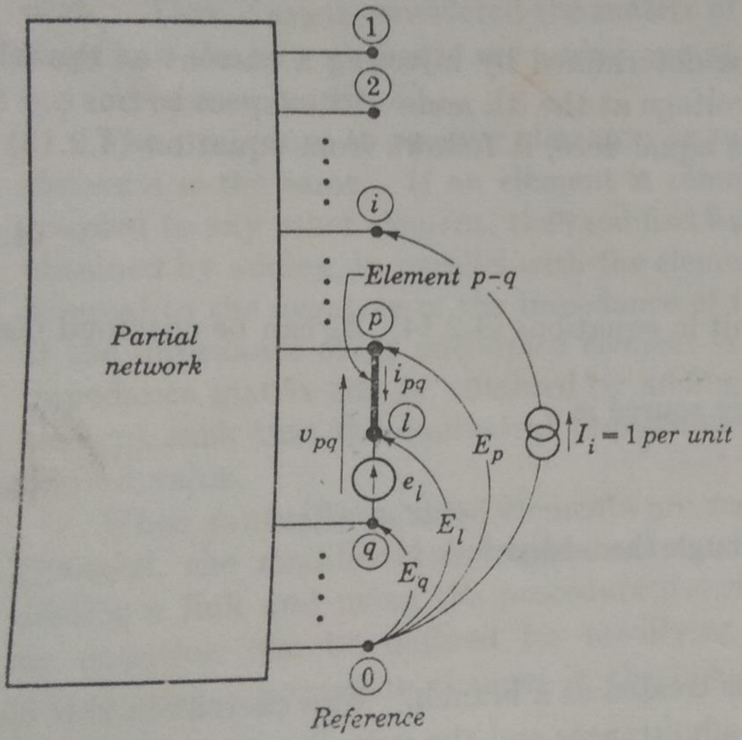


Fig. 4.4 Injected current voltage source in series with added link and bus voltages for calculation of Z_{li}.

fictitious node l which will be eliminated later. The voltage source e_l is selected such that the current through the added link is zero.

The performance equation for the partial network with the added element p–l and the series voltage source e_l is

$$
\begin{array}{c}
\\
\begin{bmatrix} E_1 \\ E_2 \\ \cdots \\ E_p \\ \cdots \\ E_m \\ e_l \end{bmatrix}
\end{array}
=
\begin{array}{c}
\\
\begin{array}{c} 1 \\ \\ \\ p \\ \\ m \\ l \end{array}
\end{array}
\begin{array}{c}
\begin{array}{ccccccc} 1 & & & p & & m & l \end{array} \\
\begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1l} \\
Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2l} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pl} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{ml} \\
Z_{l1} & Z_{l2} & \cdots & Z_{lp} & \cdots & Z_{lm} & Z_{ll}
\end{bmatrix}
\end{array}
\begin{array}{c}
\\
\begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ I_p \\ \cdots \\ I_m \\ I_l \end{bmatrix}
\end{array}
\qquad (4.2.13)
$$

Since

$$e_l = E_l - E_q$$

the element Z_{li} can be determined by injecting a current at the ith bus and calculating the voltage at the lth node with respect to bus q. Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$
\begin{aligned}
E_k &= Z_{ki} I_i \qquad k = 1, 2, \ldots, m \\
e_l &= Z_{li} I_i
\end{aligned}
\qquad (4.2.14)
$$

Letting $I_i = 1$ per unit in equations (4.2.14), Z_{li} can be obtained directly by calculating e_l.

The series voltage source is

$$e_l = E_p - E_q - v_{pl} \qquad (4.2.15)$$

Since the current through the added link is

$$i_{pq} = 0$$

the element p–l can be treated as a branch. The current in this element in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma}$$

where

$$i_{pl} = i_{pq} = 0$$

Therefore

$$v_{pl} = -\frac{\bar{y}_{pl,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pl,pl}}$$

Since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \quad \text{and} \quad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = -\frac{\bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pq,pq}} \tag{4.2.16}$$

Substituting in order from equations (4.2.16), (4.2.6), and (4.2.14) with $I_i = 1$ into equation (4.2.15) yields

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq l \end{matrix} \tag{4.2.17}$$

The element Z_{ll} can be calculated by injecting a current at the lth bus with bus q as reference and calculating the voltage at the lth bus with respect to bus q. Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$E_k = Z_{kl}I_l \qquad k = 1, 2, \ldots, m \tag{4.2.18}$$

$$e_l = Z_{ll}I_l$$

Letting $I_l = 1$ per unit in equation (4.2.18), Z_{ll} can be obtained directly by calculating e_l.

The current in the element p–l is

$$i_{pl} = -I_l = -1$$

This current in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,\rho\sigma}\bar{v}_{\rho\sigma} = -1$$

Again, since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \quad \text{and} \quad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = -\frac{1 + \bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pq,pq}} \tag{4.2.19}$$

Substituting in order from equations (4.2.19), (4.2.6), and (4.2.18) with $I_l = 1$ into (4.2.15) yields

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho l} - \bar{Z}_{\sigma l})}{y_{pq,pq}} \tag{4.2.20}$$

If there is no mutual coupling between the added element and other elements of the partial network, the elements of $\bar{y}_{pq,\rho\sigma}$ are zero and

$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from equation (4.2.17) that

$$Z_{li} = Z_{pi} - Z_{qi} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq l \end{matrix}$$

and from equation (4.2.20),

$$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq,pq}$$

Furthermore, if there is no mutual coupling and p is the reference node,

$$Z_{pi} = 0 \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq l \end{matrix}$$

and

$$Z_{li} = -Z_{qi} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq l \end{matrix}$$

Also

$$Z_{pl} = 0$$

and therefore,

$$Z_{ll} = -Z_{ql} + z_{pq,pq}$$

The elements in the lth row and column of the bus impedance matrix for the augmented partial network are found from equations (4.2.17) and (4.2.20). It remains to calculate the required bus impedance matrix to include the effect of the added link. This can be accomplished by modifying the elements Z_{ij}, where $i, j = 1, 2, \ldots, m$, and eliminating the lth row and column corresponding to the fictitious node.

The fictitious node l is eliminated by short circuiting the series voltage source e_l. From equation (4.2.13),

$$\bar{E}_{BUS} = Z_{BUS}\bar{I}_{BUS} + \bar{Z}_{il}I_l \tag{4.2.21}$$

and

$$e_l = \bar{Z}_{lj}\bar{I}_{BUS} + Z_{ll}I_l = 0 \tag{4.2.22}$$

where $i, j = 1, 2, \ldots, m$. Solving for I_l from equation (4.2.22) and substituting into (4.2.21),

$$\bar{E}_{BUS} = \left(Z_{BUS} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}} \right) \bar{I}_{BUS}$$

which is the performance equation of the partial network including the link p–q. It follows that the required bus impedance matrix is

$$Z_{BUS\,(\text{modified})} = Z_{BUS\,(\text{before elimination})} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}$$

where any element of $Z_{BUS\,(\text{modified})}$ is

$$Z_{ij\,(\text{modified})} = Z_{ij\,(\text{before elimination})} - \frac{Z_{il}Z_{lj}}{Z_{ll}}$$

A summary of the equations for the formation of the bus impedance matrix is given in Table 4.1.

5

18.9 Fast-decoupled Load Flow

This is an extension of Newton-Raphson method formulated in polar coordinates with certain approximation which results into a fast algorithm for load flow solution. Before we discuss this method, we derive load flow equations in polar coordinates.

We know that

$$P_p - jQ_p = V_p^* I_p \text{ and } I_p = \sum_{q=1}^{n} Y_{pq} V_q$$

$$\therefore \quad P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q \tag{18.48}$$

The voltage and admittance in polar coordinates are expressed as

$$V_p = |V_p| \exp(j\delta_p) \text{ and } Y_{pq} = |Y_{pq}| \exp(-j\theta_{pq})$$

Substituting these values in equation (18.48), we obtain

$$P_p - jQ_p = |V_p| \exp(-j\delta_p) \sum_{q=1}^{n} |Y_{pq}| \exp(-j\theta_{pq}) |V_q| \exp(j\delta_q)$$

$$= \sum_{q=1}^{n} |V_p| |V_q| |Y_{pq}| \exp\{-j(\theta_{pq} + \delta_p - \delta_q)\}$$

$$P_p = \sum_{q=1}^{n} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \tag{18.49}$$

and

$$Q_p = \sum_{q=1}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \tag{18.50}$$

$$p = 1, 2, \ldots n$$

Equations (18.49) and (18.50) are rewritten as

$$P_p = |V_p V_p Y_{pp}| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \tag{18.51}$$

$$Q_p = |V_p V_p Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \tag{18.52}$$

These equations after linearization can be rewritten in matrix form as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|E|/|E| \end{bmatrix} \tag{18.53}$$

Here H, N, M and L are elements of Jacobian matrix.

The first assumption under decoupled load flow method is that real power changes (ΔP) are less sensitive to changes in voltage magnitude and are mainly sensitive to angle. Similarly, the reactive power changes are less sensitive to change in angle but mainly sensitive to change in voltage magnitude. With these assumptions, equation (18.53) reduce to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & O \\ O & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|E|/|E| \end{bmatrix} \tag{18.54}$$

The equation (18.54) is decoupled equation which can be expanded as

$$[\Delta P] = [H][\Delta \delta] \tag{18.54a}$$

and

$$[\Delta Q] = [L][\Delta |E|/|E|] \tag{18.54b}$$

Using equations (18.51) and (18.52) the elements of the Jacobian matrices H and L are obtained as follows:

Off-diagonal element of H is

$$H_{pq} = \frac{\partial P_p}{\partial \delta_q} = |V_p V_q Y_{pq}| \sin [\theta_{pq} + \delta_p - \delta_q]$$

$$= |V_p V_q Y_{pq}|[\sin \theta_{pq} \cos (\delta_p - \delta_q) + \sin (\delta_p - \delta_q) \cos \theta_{pq}]$$

$$= |V_p V_q|[|Y_{pq}| \sin \theta_{pq} \cos (\delta_p - \delta_q) + |Y_{pq}| \cos \theta_{pq} \sin (\delta_q - \delta_q)]$$

$$= |V_p V_q|[-B_{pq} \cos (\delta_p - \delta_q) + G_{pq} \sin (\delta_p - \delta_q)] \tag{18.55}$$

Similarly off-diagonal element of L is

$$L_{pq} = \frac{\partial Q_p |V_q|}{\partial V_q} = |V_p V_q Y_{pq}| \sin (\theta_{pq} + \delta_p - \delta_q)$$

$$= |V_p V_q|[-B_{pq} \cos (\delta_p - \delta_q) + G_{pq} \sin (\delta_p - \delta_q)] \tag{18.56}$$

From equations (18.55) and (18.56), it is seen that

$$H_{pq} = L_{pq} = |V_p V_q|[G_{pq} \sin (\delta_p - \delta_q) - B_{pq} \cos (\delta_p - \delta_q)]$$

The diagonal elements of H are given as

$$H_{pp} = \frac{\partial P_p}{\partial \delta_p} = -\sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin (\theta_{pq} + \delta_p - \delta_q)$$

$$= -Q_p + |V_p V_p Y_{pp}| \sin \theta_{pp}$$

$$= -Q_p - V_p^2 B_{pp} \tag{18.57}$$

Similarly diagonal elements for the matrix are given by:

$$L_{pp} = \frac{\partial Q_p |V_p|}{\partial V_p} = |2V_p^2 Y_{pp}| \sin \theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin (\theta_{pq} + \delta_p - \delta_q)$$

$$= |2V_p^2 Y_{pp}| \sin \theta_{pp} + Q_p - |V_p^2 Y_{pp}| \sin \theta_{pp}$$

$$= Q_p + |V_p^2 Y_{pp}| \sin \theta_{pp}$$

$$= Q_p - V_p^2 B_{pp} \tag{18.58}$$

In the case of fast decoupled load flow method following approximations are further made for evaluating Jacobian element.

$$\cos (\delta_p - \delta_q) \simeq 1$$

$$G_{pq} \sin (\delta_p - \delta_q) \leq B_{pq}$$

and

$$Q_p \ll B_{pp} V_p^2$$

∴ The Jacobian elements now become

$$L_{pq} = H_{pq} = -|V_p V_q| B_{pq} \text{ for } q \neq p$$

and

$$L_{pp} = H_{pp} = -B_{pp}|V_p|^2$$

With these Jacobian elements equations 18.54(a) and 18.54(b) become

$$[\Delta P_p] = [V_p][V_q][B'_{pq}][\Delta\delta_q] \quad (18.59)$$

and

$$[\Delta Q_p] = [V_p][V_q][B''_{pq}] \frac{\Delta|E_q|}{E_q} \quad (18.60)$$

where B'_{pq} and B''_{pq} are the elements of $[-B_{pq}]$ matrix.

Further decoupling is obtained as follows:

(i) Omit from B'' the angle shifting effects of phase shifters.

(ii) Omit from B' the representation of those network elements that affect MVAr flows i.e. shunt reactors and off-nominal in phase transformer taps.

(iii) Divide equations (18.59) and (18.60) by V_p and assuming $V_q = 1$ p.u. and also neglecting the series resistance in calculating the elements of B'.

With these assumptions, equations (18.59) and (18.60) for the load flow solution take the form

$$\left[\frac{\Delta P_p}{E_p}\right] = [B'][\Delta\delta] \quad (18.61)$$

and

$$\left[\frac{\Delta Q_p}{E_p}\right] = [B''][\Delta E] \quad (18.62)$$

It is to be noted that $[B']$ and $[B'']$ are real and sparse and have similar structures as those of H and L respectively. Since the two matrices are constant and do not change during successive iterations for solution of the load flow problem, they need be evaluated only once and inverted once during the first iteration and then used in all successive iterations. It is because of the nature of Jacobian matrices $[B']$ and $[B'']$ and the sparsity of these matrices that the method is fast.

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Similarly V_4 can be evaluated.

18.5 Newton-Raphson Method

Development of load flow equations: The load flow problem can also be solved by using Newton Raphson method. The equations for the method are derived as follows:

We know that at any bus p,

$$P_p - jQ_p = V_p^* I_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q$$

Let

$$V_p = e_p + jf_p$$

and

$$Y_{pq} = G_{pq} - jB_{pq}$$

$$P_p - jQ_p = (e_p + jf_p)^* \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q)$$

$$= (e_p - jf_p) \sum_{q=1}^{n} (G_{pq} - jB_{pq})(e_q + jf_q)$$

Separating the real and imaginary parts we have

$$P_p = \sum_{q=1}^{n} \{e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})\} \qquad (18.26)$$

and

$$Q_p = \sum_{q=1}^{n} \{f_p(e_q G_{pq} + f_q B_{pq}) - e_p(f_q G_{pq} - e_q B_{pq})\} \qquad (18.25)$$

also

$$|V_p|^2 = e_p^2 + f_p^2 \qquad (18.27)$$

These three sets of equations are the load flow equations and it can be seen that they are non-linear equations in terms of the real and imaginary components of nodal voltages. Here the left hand quantities i.e. P_p, Q_p (for a load bus) and P_p and $|V_p|$ for generator bus are specified and e_p and f_p are unknown quantities. For an n-bus system, the number of unknowns are $2(n-1)$ because the voltage at the slack bus is known and is kept fixed both in magnitude and phase. Therefore, if bus 1 is taken as the slack, the unknown variables are $(e_2, e_3, \ldots, e_{n-1}, e_n, f_2, f_3, \ldots, f_{n-1}, f_n)$. Thus, to solve the problem for $2(n-1)$ variables we need to solve $2(n-1)$ set of equations.

Newton-Raphson method is an iterative method which approximates the set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first approximation.

The mathematical background of this method is explained as follows:

Let the unknown variables be $(x_1, x_2, \ldots, x_n)$ and the specified quantities $y_1, y_2, \ldots, y_n$. These are related by the set of non-linear equations:

$$\begin{aligned} y_1 &= f_1(x_1, x_2, \ldots, x_n) \\ y_2 &= f_2(x_1, x_2, \ldots, x_n) \\ &\vdots \\ y_n &= f_n(x_1, x_2, \ldots, x_n) \end{aligned} \tag{18.28}$$

To solve these equations we start with an approximate solution $(x_1^0, x_2^0, \ldots, x_n^0)$. Here superscript zero means the zeroth iteration in the process of solving the above non-linear equations (18.28). It is to be noted that the initial solution for the equations should not be very far from the actual solution. Otherwise, there are chances of the solution diverging rather than converging and it may not be possible to achieve a solution whatever be the computer time utilized. At first glance it may appear to be a great drawback for the Newton-Raphson technique but the problem of initial guess for a power system is not at all difficult. A flat voltage profile i.e. $V_p = 1.0 + j0.0$ for $p = 1, 2, \ldots, n$ except the slack bus has been found to be satisfactory for almost all practical systems.

The equations are linearized about the initial guess. We will expand first equation $y_1 = f_1$ and the result for the other equations will follow.

Assume $\Delta x_1^0, \Delta x_2^0, \ldots, \Delta x_n^0$ are the corrections required for $x_1^0, x_2^0, \ldots, x_n^0$ respectively for the next better solution. The equation $y_1 = f_1$ will be

$$\begin{aligned} y_1 &= f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \ldots, x_n^0 + \Delta x_n^0) \\ &= f_1(x_1^0, x_2^0, \ldots, x_n^0) + \Delta x_1^0 \left.\frac{\partial f_1}{\partial x_1}\right|_{x^0} + \Delta x_2^0 \left.\frac{\partial f_1}{\partial x_2}\right|_{x^0} + \cdots + \Delta x_n^0 \left.\frac{\partial f_1}{\partial x_n}\right|_{x^0} + \phi_1 \end{aligned}$$

where ϕ_1 is function of higher order of Δx^s and higher derivatives which are neglected according to Newton-Raphson method. In fact it is this assumption which needs the initial solution to be close to the final solution. If all the [illegible] linearized and arranged in a matrix form, we have

$$\begin{bmatrix} y_1 - f_1(x_1^0, x_2^0, \ldots, x_n^0) \\ y_2 - f_2(x_1^0, x_2^0, \ldots, x_n^0) \\ \vdots \\ y_n - f_n(x_1^0, x_2^0, \ldots, x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \qquad (18.29)$$

$$B = J \cdot C$$

Here J is the first derivative matrix known as the Jacobian matrix. The solution of the equations requires calculation of left hand vector B which is the difference of the specified quantities and calculated quantities at $(x_1^0, \ldots, x_n^0)$. Similarly J is calculated at this guess. Solution of the matrix equation gives $(\Delta x_1^0, \Delta x_2^0, \ldots, \Delta x_n^0)$ and the next better solution is obtained as follows:

$$x_1^1 = x_1^0 + \Delta x_1^0$$

$$x_2^1 = x_2^0 + \Delta x_2^0$$

$$x_n^1 = x_n^0 + \Delta x_n^0$$

The better solution is now available and is

$$(x_1^1, x_2^1, x_3^1, \ldots, x_n^1)$$

With these values the process is repeated till (i) the largest (in magnitude) element in the left column of the equations is less than a prespecified value or (ii) the largest element in the column vector $(\Delta x_1, \Delta x_2, \ldots, \Delta x_n)$ is less than a prespecified value.

When referred to a power system problem (assuming there is only one generator bus which is taken as slack bus and all other buses are load buses), the above set of linearized equations become

$$\underset{2(n-1)\times 1}{\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix}} = \underset{2(n-1)\times 2(n-1)}{\begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \cdots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \cdots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \cdots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \cdots & \frac{\partial P_3}{\partial f_n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \cdots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \cdots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \cdots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \cdots & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \cdots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \cdots & \frac{\partial Q_3}{\partial f_n} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \cdots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \cdots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix}} \underset{2(n-1)\times 1}{\begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix}} \qquad (18.30)$$

In short form it can be written as

$$\begin{bmatrix} \Delta P \\ -- \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ -- \\ \Delta f \end{bmatrix}$$

In case the system contains all types of buses, the set of equations can be written as

$$\begin{bmatrix} \Delta P \\ --- \\ \Delta Q \\ --- \\ |\Delta V_p|^2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ -- \\ \Delta f \end{bmatrix}$$

The elements of the Jacobian matrix can be derived from the three load flow equations (18.25)–(18.27).

The off-diagonal elements of J_1 are

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}, \quad q \neq p \tag{18.31}$$

and the diagonal elements of J_1 are

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + f_p B_{pp} - f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$= 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}) \tag{18.32}$$

The off-diagonal elements of J_2 are

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \tag{18.33}$$

and the diagonal elements of J_2 are

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq}) \tag{18.34}$$

The off-diagonal elements of J_3 are

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \tag{18.35}$$

and the diagonal elements are

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{n} (f_q G_{pq} - e_q B_{pq}) \tag{18.36}$$

The off-diagonal and diagonal elements of J_4 respectively are

$$\frac{\partial Q_p}{\partial f_q} = -e_p G_{pq} + f_p B_{pq}, \quad q \neq p \tag{18.37}$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_q G_{pq} + f_q B_{pq}) \tag{18.38}$$

The off-diagonal and diagonal elements of J_5 are

$$\frac{\partial |V_p|^2}{\partial e_q} = 0, \quad q \neq p \tag{18.39}$$

and

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p \tag{18.40}$$

The off-diagonal and diagonal elements of J_6 are

$$\frac{\partial |V_p|^2}{\partial f_q} = 0, \quad q \neq p \tag{18.41}$$

and

$$\frac{\partial |V_p|^2}{\partial f_p} = 2f_p \tag{18.42}$$

Next, we calculate the residual column vector consisting of ΔP, ΔQ and $|\Delta V|^2$. Let P_{sp}, Q_{sp}, and $|V_{sp}|$ be the specified quantities at the bus p. Assuming a suitable value of the solution (flat voltage profile in our case) the value of P, Q and $|V|$ at the various buses are calculated. Then

$$\Delta P_p = P_{sp} - P_p^0$$

$$\Delta Q_p = Q_{sp} - Q_p^0$$

$$|\Delta V_p|^2 = |V_{sp}|^2 - |V_p^0|^2 \tag{18.43}$$

where the superscript zero means the value calculated corresponding to initial guess i.e. zeroth iteration.

Having calculated the Jacobian matrix and the residual column vector corresponding to the initial guess (initial solution) the desired increment voltage vector $\begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$ can be calculated by using any standard technique (preferably Gauss elimination with sparsity techniques). The next better solution will be

$$e_p^1 = e_p^0 + \Delta e_p^0$$

$$f_p^1 = f_p^0 + \Delta f_p^0$$

These values of voltages will be used in the next iteration. The process will be repeated and in general the new better estimates for bus voltages will be

$$e_p^{k+1} = e_p^k + \Delta e_p^k$$

$$f_p^{k+1} = f_p^k + \Delta f_p^k$$

The process is repeated till the magnitude of the largest element in the residual column vector is less than the prespecified value. The sequence of steps for the solution of load flow problem using Newton-Raphson method is explained as follows (flow chart in Fig. 18.9):

1. Assume a suitable solution for all buses except the slack bus. Let $V_p = 1 + j0.0$ for $p = 1, 2, \ldots n, p \neq s$, $V_s = a + j0.0$.
2. Set convergence criterion $= \epsilon$ i.e. if the largest of absolute of the residues exceeds ϵ the process is repeated, otherwise it is terminated.
3. Set iteration count $K = 0$
4. Set bus count $p = 1$.
5. Check if p is a slack bus. If yes, go to step 10.
6. Calculate the real and reactive powers P_p and Q_p respectively using equations (18.26) and (18.25).
7. Evaluate $\Delta P_p^k = P_{sp} - P_p^k$.
8. Check if the bus in question is a generator bus. If yes, compare the Q_p^k with the limits. If it exceeds the limit, fix the reactive power generation to the corresponding limit and treat the bus as a load bus for that iteration and go to next step. If the lower limit is violated set $Q_{psp} = Q_{p\,min}$. If the limit is not violated evaluate the voltage residue.

$$|\Delta V_p|^2 = |V_p|^2_{spec} - |V_p^k|^2$$

and go to step 10.

9. Evaluate $\Delta Q_p^k = Q_{ps} - Q_p^k$.
10. Advance the bus count by 1, i.e. $p = p + 1$ and check if all the buses have been accounted. If not, go to step 5.
11. Determine the largest of the absolute value of the residue.
12. If the largest of the absolute value of the residue is less than ϵ, go to step.17.
13. Evaluate elements for Jacobian matrix.
14. Calculate voltage increments Δe_p^k and Δf_p^k.
15. Calculate new bus voltages $e_p^{k+1} = e_p^k + \Delta e_p^k$ and $f_p^{k+1} = f_p^k + \Delta f_p^k$. Evaluate cos δ and sin δ of all voltages.
16. Advance iteration count $K = K + 1$ and go to step 4.
17. Evaluate bus and line powers and print the results.

Example 18.4: The load flow data for the sample power system are given below. The voltage magnitude at bus 2 is to be maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generator at bus 2 are 0.35 and 0.0 p.u. respectively. Determine the set of load flow equa- ... Newton-Raphson method.

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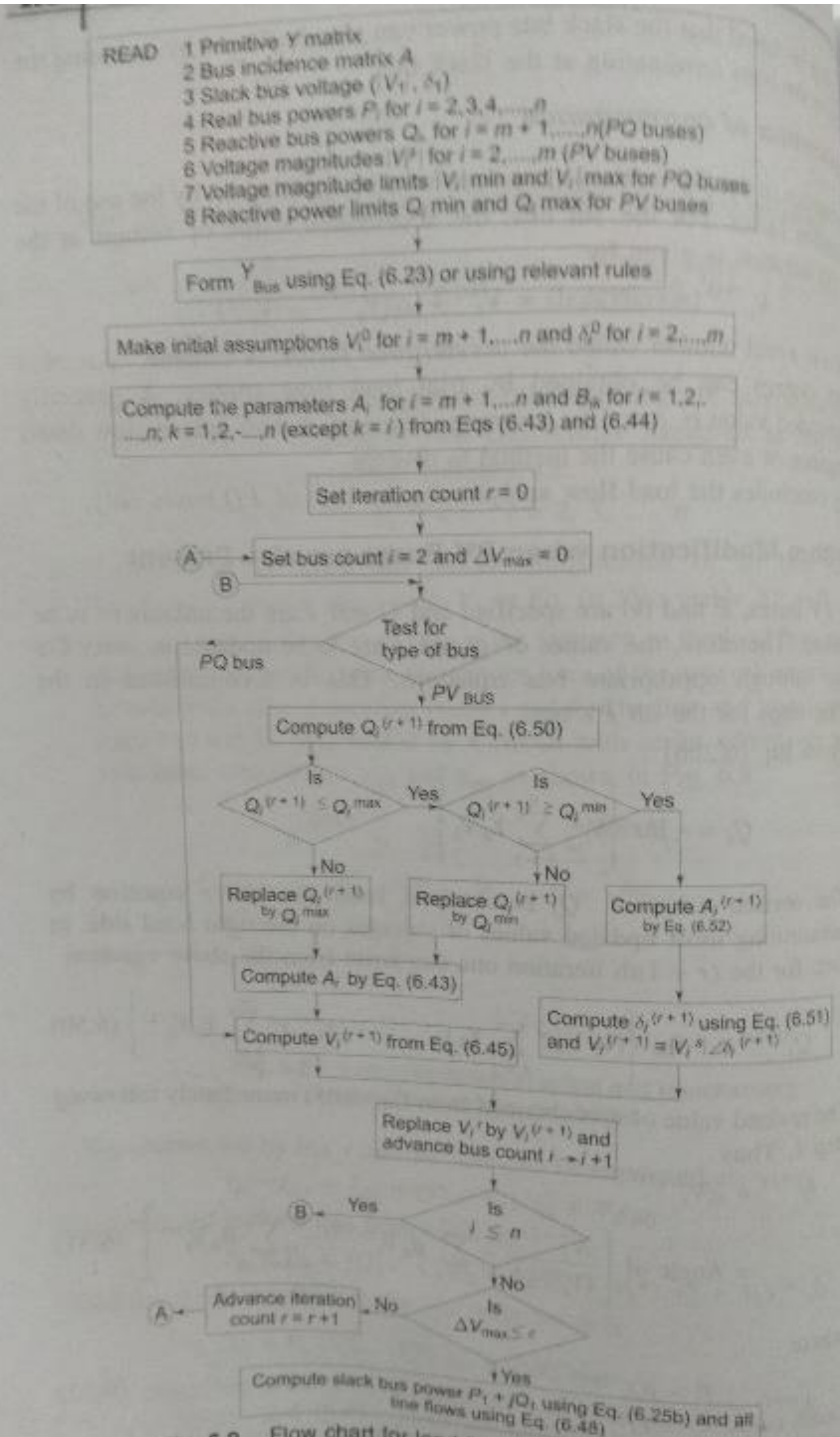

Fig. 6.9 Flow chart for load flow solution by the Gauss-Seidel iterative method using Y_{BUS}

Development of load flow equations.

Nodal current equations

$$I_p = \sum_{q=1}^{n} Y_{pq} V_q \quad p = 1, 2, \dots n$$

$$I_p = Y_{pp} V_p + \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q$$

$$I_p - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q = Y_{pp} V_p$$

$$V_p = \frac{I_p}{Y_{pp}} - \frac{1}{Y_{pp}} \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q$$

$$V_p I_p^* = P_p + jQ_p \qquad V_p^* I_p = P_p - jQ_p$$

$$I_p = \frac{P_p - jQ_p}{V_p^*}$$

∴ substituting.

$$V_p = \frac{1}{Y_{pp}} \left[\frac{P_p - jQ_p}{V_p^*} - \sum_{\substack{q=1 \\ q \neq p}}^{n} Y_{pq} V_q \right] \quad p = 1, 2, \dots n$$

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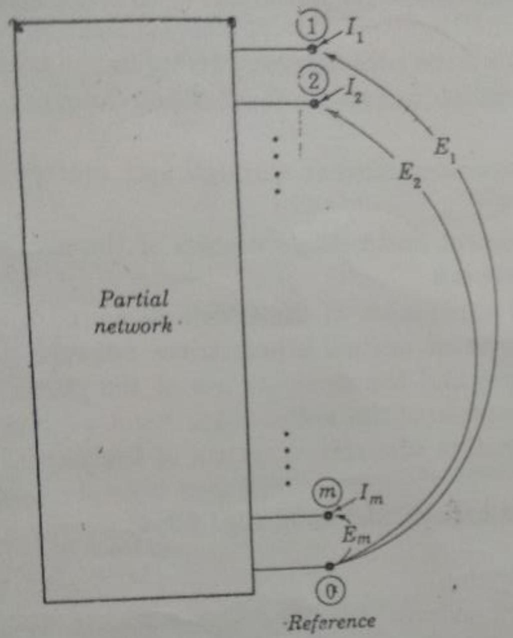
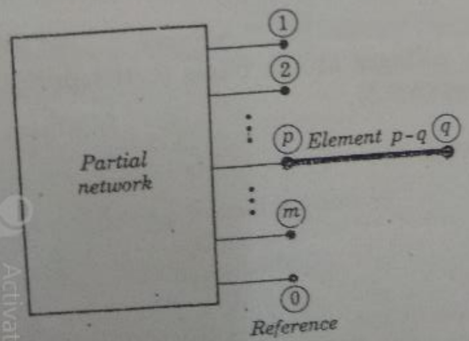


Fig. 4.1 Representation of partial network.



(a)

where $\bar{E}_{BUS}$ = an $m \times 1$ vector of bus voltages measured with respect to the reference node

$\bar{I}_{BUS}$ = an $m \times 1$ vector of impressed bus currents

When an element p–q is added to the partial network it may be a branch or a link as shown in Fig. 4.2.

If p–q is a branch, a new bus q is added to the partial network and the resultant bus impedance matrix is of dimension $(m+1) \times (m+1)$. The new voltage and current vectors are of dimension $(m+1) \times 1$. To determine the new bus impedance matrix requires only the calculation of the elements in the new row and column.

If p–q is a link, no new bus is added to the partial network. In this case, the dimensions of the matrices in the performance equation are unchanged, but all the elements of the bus impedance matrix must be recalculated to include the effect of the added link.

Addition of a branch

The performance equation for the partial network with an added branch p–q is

$$\begin{bmatrix} E_1 \\ E_2 \\ \cdots \\ E_p \\ \cdots \\ E_m \\ E_q \end{bmatrix} = \begin{array}{c} \\ 1 \\ \\ \\ p \\ \\ m \\ q \end{array} \overset{\begin{array}{ccccccc} 1 & & & p & & m & q \end{array}}{\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix}} \begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ I_p \\ \cdots \\ I_m \\ I_q \end{bmatrix} \quad (4.2.1)$$

It is assumed that the network consists of bilateral passive elements. Hence $Z_{qi} = Z_{iq}$ where $i = 1, 2, \ldots, m$ and refers to the buses of the partial network, not including the new bus q. The added branch p–q is assumed to be mutually coupled with one or more elements of the partial network.

The elements Z_{qi} can be determined by injecting a current at the ith bus and calculating the voltage at the qth bus with respect to the reference

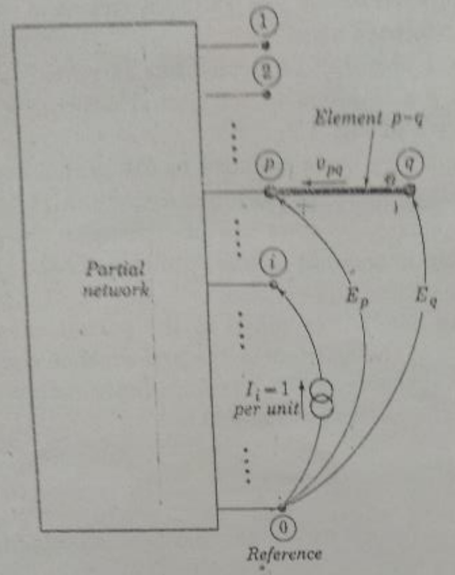


Fig. 4.3 Injected current and bus voltages for calculation of Z_{qi}.

node as shown in Fig. 4.3. Since all other bus currents equal zero, it follows from equation (4.2.1) that

$$
\begin{aligned}
E_1 &= Z_{1i} I_i \\
E_2 &= Z_{2i} I_i \\
&\cdots\cdots \\
E_p &= Z_{pi} I_i \qquad (4.2.2) \\
&\cdots\cdots \\
E_m &= Z_{mi} I_i \\
E_q &= Z_{qi} I_i
\end{aligned}
$$

Letting $I_i = 1$ per unit in equations (4.2.2), Z_{qi} can be obtained directly by calculating E_q.

The bus voltages associated with the added element and the voltage across the element are related by

$$E_q = E_p - v_{pq} \qquad (4.2.3)$$

The currents in the elements of the network in Fig. 4.3 are expressed in terms of the primitive admittances and the voltages across the elements by

$$\begin{bmatrix} i_{pq} \\ i_{\rho\sigma} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & y_{pq,\rho\sigma} \\ y_{\rho\sigma,pq} & y_{\rho\sigma,\rho\sigma} \end{bmatrix} \begin{bmatrix} v_{pq} \\ v_{\rho\sigma} \end{bmatrix} \qquad (4.2.4)$$

In equation (4.2.4) pq is a fixed subscript and refers to the added element and $\rho\sigma$ is a variable subscript and refers to all other elements. Then,

i_{pq} and v_{pq}	are, respectively, current through and voltage across the added element
$\bar{i}_{\rho\sigma}$ and $\bar{v}_{\rho\sigma}$	are the current and voltage vectors of the elements of the partial network
$y_{pq,pq}$	is the self-admittance of the added element
$\bar{y}_{pq,\rho\sigma}$	is the vector of mutual admittances between the added element p–q and the elements ρ–σ of the partial network
$\bar{y}_{\rho\sigma,pq}$	is the transpose of the vector $\bar{y}_{pq,\rho\sigma}$
$[y_{\rho\sigma,\rho\sigma}]$	is the primitive admittance matrix of the partial network

The current in the added branch, shown in Fig. 4.3, is

$$i_{pq} = 0 \qquad (4.2.5)$$

However v_{pq} is not equal to zero since the added branch is mutually coupled to one or more of the elements of the partial network. Moreover,

$$\bar{v}_{\rho\sigma} = \bar{E}_\rho - \bar{E}_\sigma \qquad (4.2.6)$$

where $\bar{E}_\rho$ and $\bar{E}_\sigma$ are the voltages at the buses in the partial network. From equations (4.2.4) and (4.2.5),

$$i_{pq} = y_{pq,pq}v_{pq} + \bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma} = 0$$

and therefore,

$$v_{pq} = -\frac{\bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pq,pq}}$$

Substituting for $\bar{v}_{\rho\sigma}$ from equation (4.2.6),

$$v_{pq} = -\frac{\bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}} \qquad (4.2.7)$$

Substituting for v_{pq} in equation (4.2.3) from (4.2.7),

$$E_q = E_p + \frac{\bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}}$$

Finally, substituting for E_q, E_p, $\bar{E}_\rho$, and $\bar{E}_\sigma$ from equation (4.2.2) with $I_i = 1$,

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq q \end{matrix} \qquad (4.2.8)$$

The element Z_{qq} can be calculated by injecting a current at the qth bus and calculating the voltage at that bus. Since all other bus currents equal zero, it follows from equation (4.2.1) that

$$\begin{aligned} E_1 &= Z_{1q} I_q \\ E_2 &= Z_{2q} I_q \\ &\cdots\cdots \\ E_p &= Z_{pq} I_q \\ &\cdots\cdots \\ E_m &= Z_{mq} I_q \\ E_q &= Z_{qq} I_q \end{aligned} \tag{4.2.9}$$

Letting $I_q = 1$ per unit in equations (4.2.9), Z_{qq} can be obtained directly by calculating E_q.

The voltages at buses p and q are related by equation (4.2.3), and the current through the added element is

$$i_{pq} = -I_q = -1 \tag{4.2.10}$$

The voltages across the elements of the partial network are given by equation (4.2.6) and the currents through these elements by (4.2.4). From equations (4.2.4) and (4.2.10),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma} = -1$$

and therefore,

$$v_{pq} = -\frac{1 + \bar{y}_{pq,\rho\sigma} \bar{v}_{\rho\sigma}}{y_{pq,pq}}$$

Substituting for $\bar{v}_{\rho\sigma}$ from equation (4.2.6),

$$v_{pq} = -\frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}} \tag{4.2.11}$$

Substituting for v_{pq} in equation (4.2.3) from (4.2.11),

$$E_q = E_p + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{E}_\rho - \bar{E}_\sigma)}{y_{pq,pq}}$$

Finally, substituting for E_q, E_p, $\bar{E}_\rho$, and $\bar{E}_\sigma$ from equation (4.2.9) with $I_q = 1$,

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho q} - \bar{Z}_{\sigma q})}{y_{pq,pq}} \tag{4.2.12}$$

If there is no mutual coupling between the added branch and other elements of the partial network, then the elements of $\bar{y}_{pq,\rho\sigma}$ are zero and

$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from equation (4.2.8) that

$$Z_{qi} = Z_{pi} \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq q \end{array}$$

and frcm equation (4.2.12) that

$$Z_{qq} = Z_{pq} + z_{pq,pq}$$

Furthermore, if there is no mutual coupling and p is the reference node,

$$Z_{pi} = 0 \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq q \end{array}$$

and

$$Z_{qi} = 0 \qquad \begin{array}{l} i = 1, 2, \ldots, m \\ i \neq q \end{array}$$

Also

$$Z_{pq} = 0$$

and therefore,

$$Z_{qq} = z_{pq,pq}$$