DSP IAT 3 SCHEME AND SOLUTION

1 a. The transfer function of an analog filter is given as

$$H_a(s) = \frac{1}{(s+1)(s+2)}$$

Obtain H(z) using Impulse Invariant Method. Take sampling frequency of 5 samples/s.

1. a. Given $H_a(s) = \dfrac{1}{(s+1)(s+2)}$

$$\frac{1}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$A(s+1) + B(s+2) = 1$$

let $s+1=0$
$s=-1$

$A(-1+1)+B(-1+2)=1$

$\boxed{B=1}$

let $s+2=0$
$s=-2$

$A(-2+1)+B(-2+2)=1$

$-A=1$

$\boxed{A=-1}$

$$H_a(s) = \frac{-1}{s+1} + \frac{1}{s+2}$$

Upon z-transformation

$$H(z) = \frac{-1}{1-e^{-T_s}z^{-1}} + \frac{1}{1-e^{-2T_s}z^{-1}}$$

Given $f_s = 5$ samples/s

$\therefore T_s = \frac{1}{5} = 0.2$ s

$$\therefore H(z) = \frac{-1}{1-e^{-0.2}z^{-1}} + \frac{1}{1-e^{-0.4}z^{-1}}$$

$$= \frac{-1}{1-0.818z^{-1}} + \frac{1}{1-0.670z^{-1}}$$

$$H(z) = \frac{-(1-0.670z^{-1}) + (1-0.818z^{-1})}{(1-0.818z^{-1})(1-0.670z^{-1})}$$

$$= \frac{\cancel{-1} + 0.670z^{-1} \cancel{+1} - 0.818z^{-1}}{1 - 0.670z^{-1} - 0.818z^{-1} + 0.548z^{-2}}$$

$$= \frac{-0.148z^{-1}}{1 - 1.488z^{-1} + 0.548z^{-2}}$$

$$\boxed{H(z) = \frac{-0.148z}{z^{2} - 1.488z + 0.548}}$$

1.

b. Convert the analog filter into a digital filter whose system function is

$$H(s) = \frac{2}{(s+1)(s+3)}$$

using Bilinear Transformation, with T=0.1 s.

1.b. Given : $H(s) = \frac{2}{(s+1)(s+3)}$

$T_s = 0.1 s$

In Bilinear transformation,

$$s \to \frac{2}{T_s} \frac{z-1}{z+1}$$

$$s \to \frac{2}{0.1} \frac{z-1}{z+1} = 20\left(\frac{z-1}{z+1}\right)$$

$$H(s) = \frac{2}{(s+1)(s+3)}$$

$$= \frac{2}{s^2+4s+3}$$

$$H(z) = \frac{2}{\left(20\left(\frac{z-1}{z+1}\right)\right)^2 + 4\left(20\,\frac{z-1}{z+1}\right) + 3}$$

$$= \frac{2}{400\left(\frac{z-1}{z+1}\right)^2 + 80\,\frac{z-1}{z+1} + 3}$$

$$= \frac{2}{\frac{400(z-1)^2 + 80(z-1)(z+1) + 3(z+1)^2}{(z+1)^2}}$$

$$= \frac{2(z+1)^2}{400(z^2+1-2z) + 80(z^2-1) + 3(z^2+1+2z)}$$

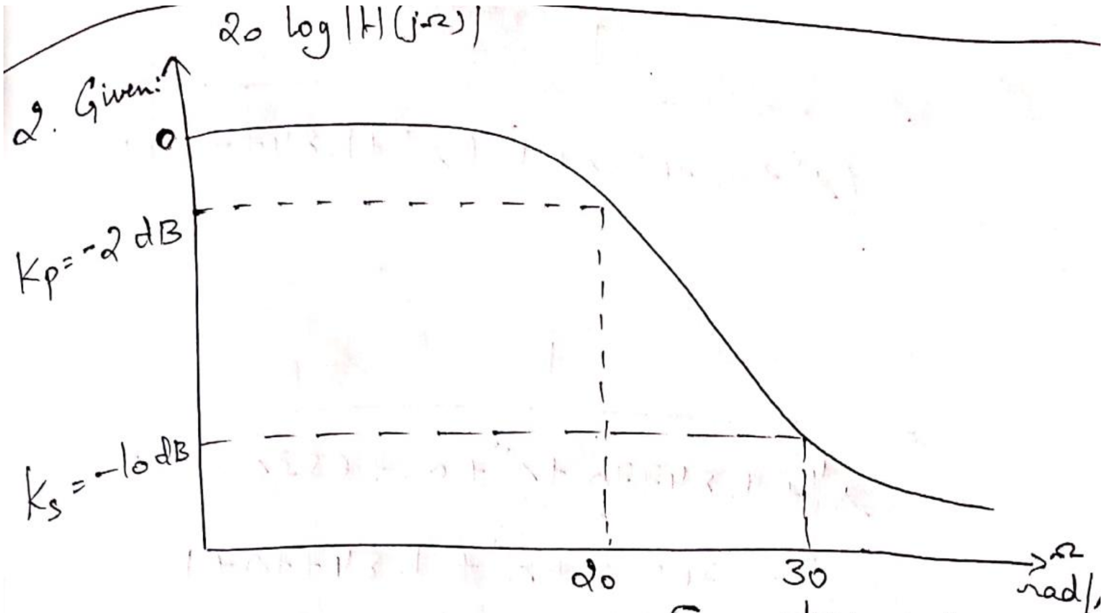
$$= \frac{2z^2+2z+2}{400z^2+400-800z+80z^2-80+3z^2+3+6z}$$

$$= \frac{2z^2+2z+2}{483z^2-794z+323}$$

$$\boxed{H(z) = \frac{2(z^2 + z + 1)}{483z^2 - 794z + 323}}$$

2	Design an analog filter with maximally flat response in the passband and an acceptable attenuation of -2 dB at 20 rad/s. The attenuation in the stopband should be more than 10 dB beyond 30 rad/s.

2. Given:



$$\text{Filter order, } N = \frac{\log\left[\left(\cancel{\log}\,10^{-k_p/10}-1\right)\Big/\left(10^{-k_s/10}-1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$N = \frac{\log\left[\left(10^{2/10}-1\right)\Big/\left(10^{10/10}-1\right)\right]}{2\log\left(\frac{20}{30}\right)}$$

$$= \frac{\log\left(0.5848/9\right)}{2\log\left(2/3\right)} = \frac{\log 0.0649}{2\log 0.6666}$$

$$N = \frac{-1.1877}{-0.3522} = 3.37$$

$$\boxed{N \simeq 4}$$

$$H_4(s) = \frac{1}{\prod\limits_{\substack{LHP \\ \text{only}}} (s - s_k)}$$

$$= \frac{1}{(s^2 + 0.7653s + 1)(s^2 + 1.8477s + 1)}$$

$$= \frac{1}{s^4 + 1.8477s^3 + s^2 + 0.7653s^3 + 1.414s^2 + 0.7653s + s^2 + 1.8477s + 1}$$

$$H_4(s) = \frac{1}{s^4 + 2.613s^3 + 3.414s^2 + 2.613s + 1}$$

Cut-off frequency,

$$\Omega_c = \frac{\Omega_p}{\left(10^{-kp/10} - 1\right)^{1/2N}}$$

$$= \frac{20}{\left(10^{2/10} - 1\right)^{1/2 \times 4}}$$

$$\Omega_c = \frac{20}{(0.5848)^{1/8}} = \frac{20}{0.935}$$

$$\boxed{\Omega_c = 21.38 \text{ rad/s}}$$

$$H_a(s) = H_4(s)\Big|_{s \to \frac{s}{\Omega_c}}$$

$$= \frac{1}{s^4 + 2.613s^3 + 3.414s^2 + 2.613s + 1}\Bigg|_{s \to \frac{s}{21.38}}$$

$$= \frac{1}{\left(\frac{s}{21.38}\right)^4 + 2.613\left(\frac{s}{21.38}\right)^3 + 3.414\left(\frac{s}{21.38}\right)^2 + 2.613\left(\frac{s}{21.38}\right) + 1}$$

$$\boxed{H_a(s) = \frac{208944.4}{s^4 + 55.8659s^3 + 1560.5544s^2 + 25536.5669s + 208944.4}}$$

3	Transform the below function into a digital filter using Impulse Invariant Technique. $H(s) = \frac{s+a}{(s+a)^2 + b^2}$

he poles of $H_a(s)$ are obtained from

$$\begin{aligned} (s+a)^2 + b^2 &= 0 \\ \Rightarrow \qquad s &= -a \pm jb \end{aligned}$$

Let $\quad s_1 = -a + jb$ and $s_2 = -a - jb$

The analog transfer function $H_a(s)$ is written in the factored form as

$$\begin{aligned} H_a(s) &= \frac{s+a}{(s+a-jb)(s+a+jb)} \\ &= \frac{C_1}{s+a-jb} + \frac{C_2}{s+a+jb} \end{aligned}$$

where

$$C_1 = \left.\frac{s+a}{s+a+jb}\right|_{s=-a+jb} = \frac{1}{2}$$

and

$$C_2 = C_1^* = \frac{1}{2}$$

We know that

$$H(z) = \sum_{i=1}^{N} \frac{C_i\, z}{z - e^{s_i T}}$$

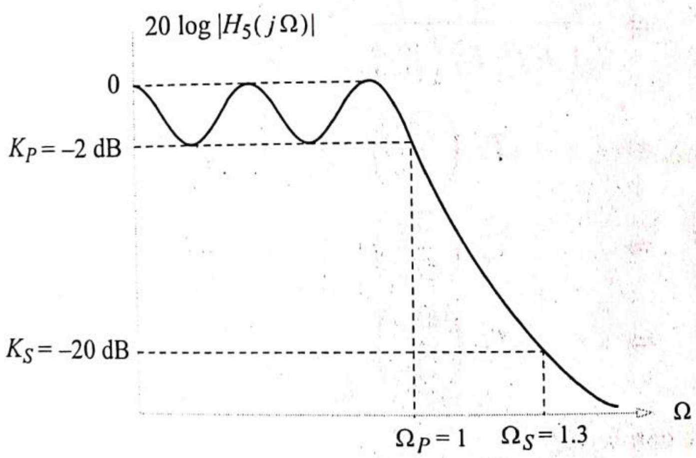
$$\Rightarrow \qquad H(z) = \sum_{i=1}^{2} \frac{C_i\, z}{z - e^{s_i T}}$$

Hence,

$$\begin{aligned} H(z) &= C_1 \frac{z}{z - e^{s_1 T}} + C_2 \frac{z}{z - e^{s_2 T}} \\ &= \frac{1}{2}\left(\frac{z}{z - e^{(-a+jb)T}} + \frac{z}{z - e^{(-a-jb)T}}\right) \\ &= \frac{1}{2}\left(\frac{z^2 - ze^{-aT}e^{-jbT} + z^2 - ze^{-aT}e^{jbT}}{z^2 - ze^{-aT}e^{-jbT} - ze^{-aT}e^{jbT} + e^{-2aT}}\right) \end{aligned}$$

$$\begin{aligned} \Rightarrow \qquad H(z) &= \frac{z^2 - ze^{-aT}\cos bT}{z^2 - 2z\cos bT\ e^{-aT} + e^{-2aT}} \\ &= \frac{1 - e^{-aT}\cos bT\ z^{-1}}{1 - 2\cos bT\ e^{-aT} z^{-1} + e^{-2aT} z^{-2}} \end{aligned}$$

4 Design a Chebyshev I filter to meet the following specifications,
Passband Ripple: ≤ 2 dB
Passband Edge: 1 rad/s
Stopband Attenuation: ≥ 20 dB
Stopband Edge: 1.3 rad/s



ig. Ex.4.9 Magnitude frequency response of the specified lowpass Chebyshev I filter.

we know that,

$$K_P = 20\log\left[\frac{1}{\sqrt{1+\epsilon^2}}\right] = -2$$

$$\Rightarrow \quad \epsilon = 0.76478$$

$$\delta_P = 1 - \frac{1}{\sqrt{1+\epsilon^2}} = 0.20567$$

$20 \log \left[\frac{1}{\sqrt{1+\epsilon^2}}\right] = K_P$

Also

From Fig. Ex.4.9,

$$K_S = 20\log\delta_S = -20$$

$$\Rightarrow \quad \delta_S = 0.1$$

Discrimination factor,

$$d = \sqrt{\frac{(1-\delta_P)^{-2}-1}{\delta_S^{-2}-1}} = 0.077$$

Selectivity factor,

$$K = \frac{\Omega_P}{\Omega_S} = \frac{1}{1.3} = 0.769$$

Minimum value of the filter order is

$$N = \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} = 4.3$$

Rounding off to the next larger integer, we get $N = 5$.

Let us now proceed to find the transfer function of the fifth-order normalized lowpass Chebyshev I filter.

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} - \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} = 0.21830398$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{1}{N}} + \frac{1}{2}\left(\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right)^{\frac{-1}{N}} = 1.0235520$$

$$\sigma_k = -a\sin\left[(2k-1)\frac{\pi}{2N}\right]$$

$$\Omega_k = b\cos\left[(2k-1)\frac{\pi}{2N}\right], \quad k = 1, 2, \cdots 2N$$

Here,

$$N = 5$$

$$\sigma_k = -a\sin\left[(2k-1)\frac{\pi}{10}\right]$$

$$\Omega_k = b\cos\left[(2k-1)\frac{\pi}{10}\right], \quad k = 1, \cdots 10$$

values of k from 1 to 5, we get the poles of $H_5(s)H_5(-s)$ that lie to the left-half of the These poles are assigned to $H_5(s)$.

k	σ_k	Ω_k
1	−0.0674610	0.9734557
2	−0.1766151	0.6016287
3	−0.2183083	0
4	−0.1766151	−0.6016287
5	−0.0674610	−0.9734557

$$\begin{aligned}
\text{Hence,}\quad H_5(s) &= \frac{K_N}{(s-s_1)(s-s_5)(s-s_2)(s-s_4)(s-s_3)} \\
&= \frac{K_N}{[(s+0.0674610-j0.9734557)(s+0.0674610+j0.9734557)(s+0.1766151-j0.6016287)(s+0.1766151+j0.6016287)(s+0.2183083)]} \\
&= \frac{K_N}{(s+0.2183083)(s^2+0.134922s+0.95215)(s^2+0.35323s+0.393115)} \\
&= \frac{K_N}{s^5+0.70646s^4+1.4995s^3+0.6934s^2+0.459349s+0.08172}
\end{aligned}$$

Since N is odd, $K_N = b_0 = 0.08172$.

$$\text{Hence,}\quad H_5(s) = \frac{0.08172}{s^5+0.70646s^4+1.4995s^3+0.6934s^2+0.459349s+0.08172}$$

5	Using Impulse Invariant Transformation, design a Chebyshev I filter that satisfies the following constraints. $$0.8 \le \lvert H(\omega)\rvert \le 1, \qquad 0 \le \omega \le 0.2\pi$$ $$\lvert H(\omega)\rvert \le 0.2, \qquad 0.6\pi \le \omega \le \pi.$$

are given the following digital specifications:

Passband ripple: $\delta_P = 1 - 0.8 = 0.2$.

Passband-edge frequency: $\omega_P = 0.2\pi$.

Stopband tolerance: $\delta_S = 0.2$.

Stopband-edge frequency: $\omega_S = 0.6\pi$.

Step 1: Convert the above edge-band digital frequencies into analog frquencies using the formula
$\Omega = \frac{\omega}{T}$ with $T = 1$ sec.

Hence,

$$\begin{aligned}
\Omega_P &= 0.2\pi, \quad \delta_P = 1 - 0.8 = 0.2 \\
\Rightarrow \quad K_P &= 20\log(1 - \delta_P) = -1.94 \text{ dB} \\
\Omega_S &= 0.6\,\pi, \quad \delta_S = 0.2 \\
\Rightarrow \quad K_S &= 20\log\delta_S = -14 \text{ dB}
\end{aligned}$$

Step 2: Design a chebyshev I lowpass analog prototype filter to meet the specifications listed in step 1.

$$\begin{aligned}
d &= \sqrt{\frac{(1-\delta_P)^{-2} - 1}{\delta_S^{-2} - 1}} = 0.153 \\
K &= \frac{\Omega_P}{\Omega_S} = 0.33
\end{aligned}$$

Filter order:

$$\begin{aligned}
N &\geqslant \frac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)} \\
\Rightarrow \quad N &\geqslant 1.446
\end{aligned}$$

Hence, the minimum filter order is $N = 2$.

We know that

$$\begin{aligned}
1 - \delta_P &= \frac{1}{\sqrt{1+\epsilon^2}} \\
\Rightarrow \quad \epsilon &= 0.75
\end{aligned}$$

$$\begin{aligned}
a &= \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{1}{N}} - \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{-1}{N}} \\
&= 0.57735 \\
b &= \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{1}{N}} + \frac{1}{2}\left[\frac{1+\sqrt{1+\epsilon^2}}{\epsilon}\right]^{\frac{-1}{N}} \\
&= 1.1547 \\
\sigma_k &= -a\sin\left[(2k-1)\frac{\pi}{2N}\right], \quad k = 1, 2, 3, 4 \\
\Omega_k &= b\cos\left[(2k-1)\frac{\pi}{2N}\right], \quad k = 1, 2, 3, 4
\end{aligned}$$

k	σ_k	Ω_k	s_k
1	−0.4082481	0.8164962	$-0.4082481 + j0.8164962$
2	−0.4082481	−0.8164962	$-0.4082481 - j0.8164962$
3	0.4082481	−0.8164962	$0.4082481 - j0.8164962$
4	0.4082481	0.8164962	$0.4082481 + j0.8164962$

Hence,
$$H_2(s) = \frac{K_N}{\prod\limits_{\substack{\text{LHP}\\\text{only}}} (s - s_k)} = \frac{K_N}{(s - s_1)(s - s_2)}$$

$$= \frac{K_N}{(s + 0.4082481 - j0.8164962)(s + 0.4082481 + j0.8164962)}$$

$$= \frac{K_N}{(s + 0.4082481)^2 + (0.8164962)^2}$$

$$= \frac{K_N}{s^2 + 0.8164962s + 0.833333}$$

where
$$K_N = \frac{b_0}{\sqrt{1+\epsilon^2}} = \frac{0.833333}{\sqrt{1+(0.75)^2}} = 0.667$$

Thus,
$$H_2(s) = \frac{0.667}{s^2 + 0.8164962s + 0.833333}$$

Since, we want the cutoff at $\Omega_P = 0.2\pi$, let us apply lowpass-to-lowpass transformation on $H_2(s)$ and get $H_a(s)$.

That is,
$$H_a(s) = H_2(s)\Big|_{s \longrightarrow \frac{s}{0.2\pi}}$$

$$= \frac{0.667}{\left(\frac{s}{0.2\pi}\right)^2 + 0.8164962\left(\frac{s}{0.2\pi}\right) + 0.833333}$$

$$= \frac{0.263321}{s^2 + 0.51302s + 0.32899}$$

$$= \frac{0.263321}{(s + 0.25651)^2 + (0.51302)^2}$$

$$= \frac{0.263321}{0.51302} \times \frac{0.51302}{(s + 0.25651)^2 + (0.51302)^2}$$

$$= 0.513276 \times \frac{0.51302}{(s + 0.25651)^2 + (0.51302)^2}$$

3: Design $H(z)$ using IIT with $T = 1$ sec.

Recall :
$$\frac{b}{(s+a)^2 + b^3} \xRightarrow{\text{IIT}} \frac{e^{-aT}\sin bTz^{-1}}{1 - 2e^{-aT}\cos bTz^{-1} + e^{-2aT}z^{-2}}$$

Hence,
$$H(z) = \frac{0.513276 \times e^{-0.25651}\sin(0.51302)z^{-1}}{1 - 2e^{-0.25651}\cos(0.51302)z^{-1} + e^{-2\times 0.25651}z^{-2}}$$

$$= \frac{0.19492z^{-1}}{1 - 1.34828z^{-1} + 0.598685z^{-2}}$$

6 Explain transforming an analog normalized LPF into analog LPF, HPF, BPF and BSF filters using Frequency Transformation Methods.

Frequency Transformations/Spectral Transformations

Let $H(s)$ denote the transfer function of a lowpass analog filter with a passband edge frequency equal to 1 rad/sec. This filter is known as *analog lowpass normalised protype*. If Ω_u is the desired passband edge frequency of the new lowpass filter, then the transfer function of this new lowpass filter is obtained by using the transformation:

$$s \longrightarrow \frac{s}{\Omega_u}$$

$\longrightarrow$ is read as *is replaced by*.

Let $H'(s)$ be the transfer function of the new lowpass filter. Then,

$$H'(s) = H(s)\big|_{s \longrightarrow \frac{s}{\Omega_u}}$$

$$\Rightarrow \quad H'(s) = H\left(\frac{s}{\Omega_u}\right)$$

The frequency response of the new lowpass filter is obtained by letting $s = j\Omega$ in $H'(s)$.

That is,

$$H'(j\Omega) = H\left(\frac{j\Omega}{\Omega_u}\right)$$

Evaluating the magnitude response at $\Omega = \Omega_u$, we get

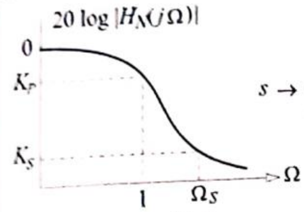
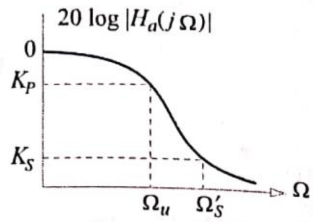
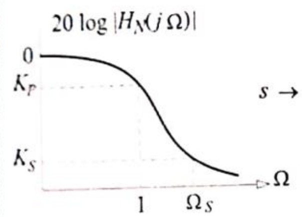
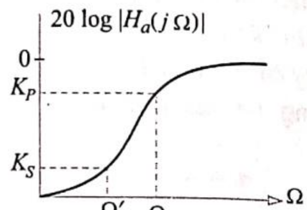
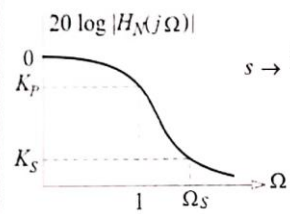
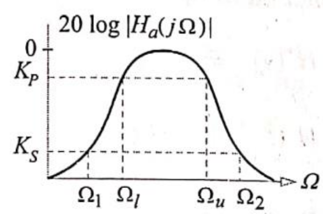
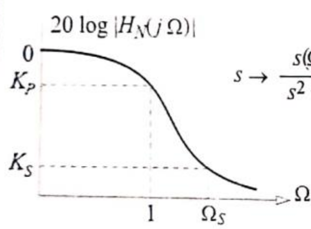
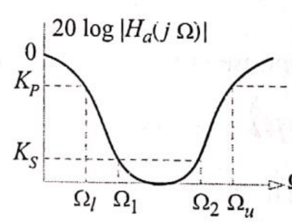
$$|H'(j\Omega_u)| = H(j1)$$

The above equation means that the frequency response of the new lowpass filter evaluated at $\Omega = \Omega_u$ is equal to the value of the prototype transfer function at $\Omega = 1$. In a way, we have translated the cutoff frequency from 1 rad/sec to Ω_u rad/sec. Thus, it justifies the correctness of lowpass-to-lowpass transformation, $s \longrightarrow \frac{s}{\Omega_u}$.

The lowpass-to-highpass transformation is simply achieved by replacing s by $\frac{1}{s}$. If the desired highpass filter has a passband edge frequency Ω_u, then the transformation is

$$s \longrightarrow \frac{\Omega_u}{s}$$

Table 4.2 Analog-to-Analog transformations

Prototype frequency response	Transformed frequency response	Backward design equations
 $s \to \frac{s}{\Omega_u}$ Lowpass, $H_N(s)$	 Lowpass, $H_a(s)$	$\Omega_S = \frac{\Omega'_S}{\Omega_u}$
 $s \to \frac{\Omega_u}{s}$ Lowpass, $H_N(s)$	 Highpass, $H_a(s)$	$\Omega_S = \frac{\Omega_u}{\Omega'_S}$
 $s \to \frac{s^2 + \Omega_u\Omega_l}{s(\Omega_u - \Omega_l)}$ Lowpass, $H_N(s)$	 Bandpass, $H_a(s)$	$\Omega_S = \text{Min}\{\|A\|, \|B\|\}$ $A = \frac{-\Omega_1^2 + \Omega_l\Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$ $B = \frac{\Omega_2^2 - \Omega_l\Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$
 $s \to \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u\Omega_l}$ Lowpass, $H_N(s)$	 Bandstop, $H_a(s)$	$\Omega_S = \text{Min}\{\|A\|, \|B\|\}$ $A = \frac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_l\Omega_u}$ $B = \frac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + \Omega_u\Omega_l}$

For a lowpass-to-bandpass transformation, it should be understood that a bandpass filter is essentially a combination of a lowpass filter and a highpass filter. The transformation is given by

$$s \longrightarrow \frac{s^2 + \Omega_u\Omega_l}{s(\Omega_u - \Omega_l)}$$

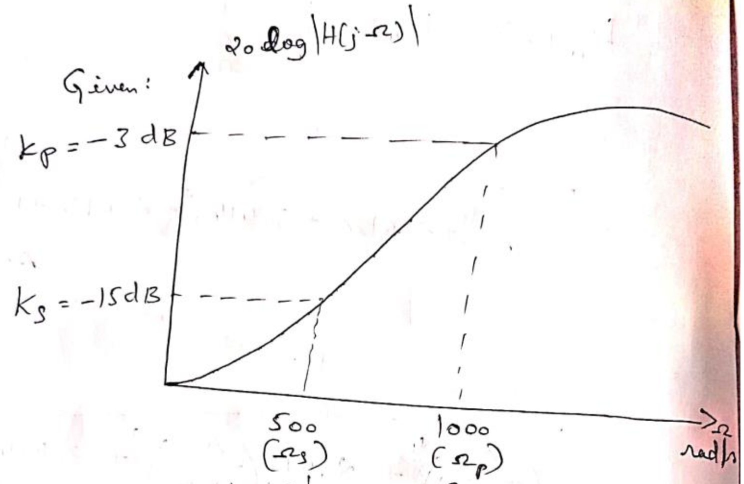
Finally, the lowpass-to-bandstop transformation is

$$s \longrightarrow \frac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u \Omega_l}$$

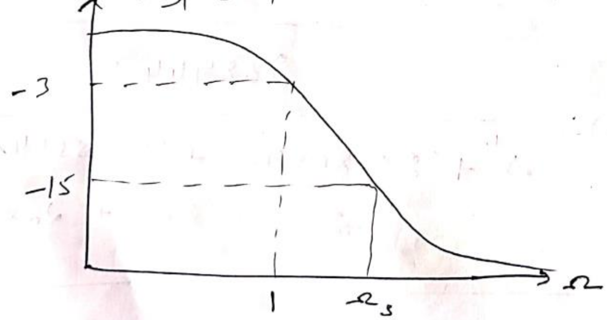
The analog-to-analog transformations are summarized in Table 4.2. The table also supplies the design equations for backward development. For example, if Ω'_S is the desired critical stopband frequency of the transformed lowpass filter, the backward design equation gives the value of that must be used in the design of the normalized lowpass filter such that going through the transformation, $s \longrightarrow \frac{s}{\Omega_u}$ to the normalized lowpass filter results in the required Ω'_S. The backward design equations are needed for designing a normalized lowpass filter, which can then be mapped into a desired filter by applying the appropriate transformations.

7	For the given specifications, K_p= 3 dB, K_s= 15 dB, Ω_p=1000 rad/s, Ω_s=500 rad/s. Design analog Butterworth high pass filter.

7. Given:



The first step is to design a normalized lowpass Butterworth Filter.



$$\Omega_s = \frac{\Omega_u}{\Omega_s'} = \frac{1000}{500} = 2 \text{ rad/s}$$

$$\boxed{\Omega_s = 2 \text{ rad/s}}$$

Normalized LPF Butterworth Filter,

$\Omega_p = 1$, $k_p = -3$ dB

$\Omega_s = 2$, $k_s = -15$ dB

$$N = \frac{\log\left[\left(10^{-k_p/10} - 1\right) \Big/ \left(10^{-k_s/10} - 1\right)\right]}{2 \log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left[\left(10^{3/10} - 1\right) \Big/ \left(10^{15/10} - 1\right)\right]}{2 \log\left(1/2\right)}$$

$$= \frac{-1.4881}{-0.6020} = 2.4719$$

$$\boxed{N \simeq 3}$$

$$H_N(s) = H_3(s) = \frac{1}{(s^2+s+1)(s+1)}$$

$$H_3(s) = \frac{1}{s^3+s^2+s^2+s+s+1}$$

$$\boxed{H_3(s) = \frac{1}{s^3+2s^2+2s+1}}$$

Cut-off Frequency, $\Omega_c = \dfrac{\Omega_p}{\left(10^{-kp/10}-1\right)^{1/2N}}$

$$\Omega_c = \frac{1}{\left(10^{3/10}-1\right)^{1/2\times 3}}$$

$$= \frac{1}{(0.9952)^{0.1666}} = 1.0008$$

$$\boxed{\Omega_c = 1.008 \text{ rad/s}}$$

$$H_a(s) = H_3(s)\Big|_{s \to \frac{s}{\Omega_c}}$$

$$= \frac{1}{s^3 + 2s^2 + s + 1}\Bigg|_{s \to \frac{s}{1.008}}$$

$$= \frac{1}{\left(\frac{s}{1}\right)^3 + 2\left(\frac{s}{1}\right)^2 + \left(\frac{s}{1}\right) + 1}$$

$$\boxed{H_a(s) = \frac{1}{s^3 + 2s^2 + s + 1}}$$

8

a. Draw the cascade form structure for the system given by

$$H(z) = \frac{\left(1 - \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2}\right)\left(1 - \frac{1}{5}z^{-1} + \frac{1}{6}z^{-2}\right)}$$

b. A digital system is given by

$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{\left(1 - \frac{1}{3}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$$

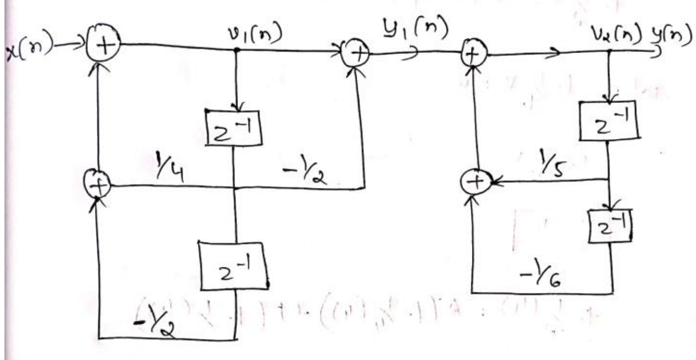
Obtain the parallel form structure.

8. a. Given: $H(z) = \dfrac{(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2})(1 - \frac{1}{5}z^{-1} + \frac{1}{6}z^{-2})}$

Let $x(n) \rightarrow H_1(z) = \left(\dfrac{1 - \frac{1}{2}z^{-1}}{1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2}}\right) \xrightarrow{y_1(n)} H_2(z) = \dfrac{1}{1 - \frac{1}{5}z^{-1} + \frac{1}{6}z^{-2}}$

$\downarrow$

$y(n)$



Cascade Realization of H(z)

8.b. Given: $H(z) = \dfrac{1-\frac{1}{2}z^{-1}}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})}$

let $z^{-1} = v$

$$H(z) = \frac{1-\frac{1}{2}v}{(1-\frac{1}{3}v)(1-\frac{1}{4}v)} = \frac{A}{(1-\frac{1}{3}v)} + \frac{B}{(1-\frac{1}{4}v)}$$

$$1-\frac{1}{2}v = A(1-\frac{1}{4}v) + B(1-\frac{1}{3}v)$$

when $1-\frac{1}{4}v = 0$

$1 = \frac{1}{4}v$

$\boxed{v = 4}$

$$1-\frac{1}{2}(4) = A(1-\frac{1}{4}(4)) + B(1-\frac{1}{3}(4))$$

$$1-2 = A(1-1) + B(-\frac{1}{3})$$

$$-1 = 0 - \frac{B}{3}$$

$\boxed{B = 3}$

$$-1 = 0 - B/3$$

$$\boxed{B = 3}$$

when $1 - \frac{1}{3} v = 0$

$$\boxed{v = 3}$$

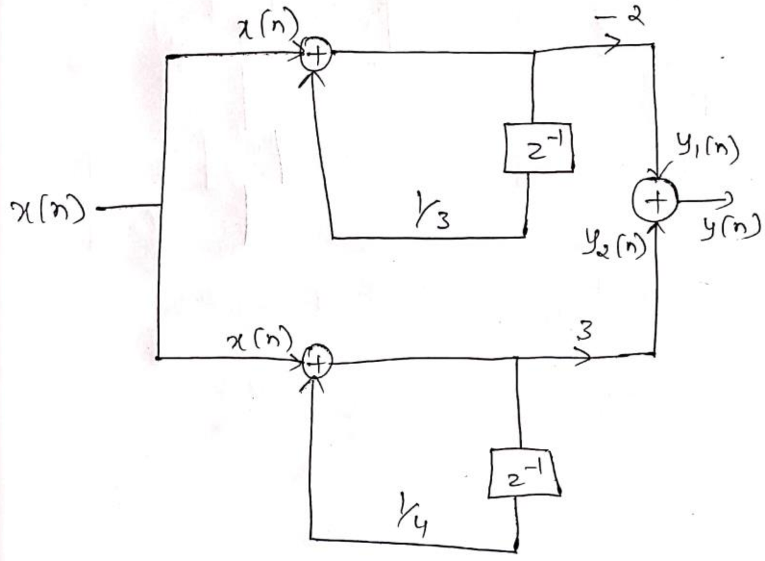
$$1 - \frac{1}{2}(3) = A\left(1 - \frac{1}{4}(3)\right) + B\left(1 - \frac{1}{3}(3)\right)$$

$$1 - \frac{3}{2} = A\left(\frac{4-3}{4}\right) + B(1-1)$$

$$-\frac{1}{\cancel{2}} = A\left(\frac{1}{\cancel{4}_{2}}\right)$$

$$\boxed{A = -2}$$

$$\therefore H(z) = \frac{-2}{1 - \frac{1}{3} v} + \frac{3}{1 - \frac{1}{4} v}$$



Parallel Realization of H(z)