CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test - III

Sub:	SIGNALS AND SYSTEMS							Code:	17EE54
Date:	19/11/2019	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Solve the following difference equation for y(n) using z-transform and the specified initial conditions. y(n)-(1/9) y(n-2) = x(n-1) where x(n)=3u(n) ;y(-1)=0 and y(-2)=1	10	CO6	L2
2	Prove the linearity and frequency shift property of Fourier Transform	10	CO5	L1
3	Obtain the Fourier transform of the following a)x(t)= $e^{-a\|t\|}$;a>0 b)x(t)=sin(πt) e^{-2t}u(t) c)x(t)= $\frac{2}{t^2+1}$	10	CO5	L2
4	Find the frequency response and the impulse response of the system described by the difference equation y(n) +½ y(n-1) = x(n) - 2x(n-1)	10	CO5	L2
5	Compute DTFT for the signals a)x(n) = $(1/2)^n$u(n-2) b) $a^{\|n\|}$	10	CO5	L2
6	Prove the summation and Parsevals theorem property of DTFT	10	CO5	L1
7	Determine x[n] if X(z)= $\frac{1-z^{-1}+z^{-2}}{(1-½z^{-1})(1-2z^{-1})(1-z^{-1})}$ for the following ROCs (i)|z|< ½ (ii) |z| > 2 (iii)1< |z|< ½	10	CO6	L2

1.

Given ; $y(n) - \frac{1}{9} y(n-2) = x(n-1)$

Taking unilateral Z-transform on both the sides, we get,

$$Y(z) - \frac{1}{9}[y(-2)+y(-1)z^{-1}+z^{-2}Y(z)] = x(-1) + z^{-1}X(z)$$

Substitute $y(-2)=1$; $y(-1)=0$ and $x(-1)=0$

$$Y(z) - \frac{1}{9}[1+z^{-2}Y(z)] = z^{-1}X(z)$$

$$Y(z)[1-\frac{1}{9}z^{-2}] = \frac{1}{9}+\frac{3z^{-1}}{1-z^{-1}} \quad [\therefore X(z) = \frac{3}{1-z^{-1}}]$$

$$= \frac{(1-z^{-1}+27z^{-1})}{9(1-z^{-1})}$$

$$= \frac{\frac{1}{9}(1+26z^{-1})}{(1-z^{-1})}$$

$$\therefore Y(z) = \frac{\frac{1}{9}(1+26z^{-1})}{(1-\frac{1}{9}z^{-2})(1-z^{-1})}$$

$$= \frac{\frac{1}{9}(1+26z^{-1})}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})(1-z^{-1})}$$

By partial fraction expansion, we get,

$$Y(z) = \frac{-79/36}{1-\frac{1}{3}z^{-1}} + \frac{-77/72}{1+\frac{1}{3}z^{-1}} + \frac{27/8}{1-z^{-1}}$$

Taking inverse Z-transform, we get,

$$y(n) = -\frac{79}{36}(\tfrac{1}{3})^n u(n) - \frac{77}{72}(-\tfrac{1}{3})^n u(n) + \frac{27}{8}u(n)$$

$$y(n) = \left[\frac{-79}{36}(\tfrac{1}{3})^n - \frac{77}{72}(-\tfrac{1}{3})^n + \frac{27}{8}\right]u(n)$$

2.

then $z(t) = ax(t)+by(t) \xleftrightarrow{FT} Z(j\omega) = a\,X(j\omega) + b\,Y(j\omega)$

Proof: We have $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

$$Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$\therefore Z(j\omega) = \int_{-\infty}^{\infty} z(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} [ax(t) + by(t)]\, e^{-j\omega t}\, dt$$

$$= a\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} + b\int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}$$

$$Z(j\omega) = a\,X(j\omega) + b\,Y(j\omega)$$

Hence the proof.

(c) Frequency shift :

If $x(t) \xleftrightarrow{FT} X(j\omega)$

then $y(t) = e^{j\beta t}x(t) \xleftrightarrow{FT} Y(j\omega) = X(j(\omega-\beta))$

Proof : We have,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$\therefore Y(j\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{j\beta t} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} x(t)\, e^{-j(\omega-\beta)t}\, dt$$

$$\therefore Y(j\omega) = X(j(\omega-\beta))$$

Hence the proof.

3.

Solution : Given, $x(t) = \sin(\pi t)\, e^{-2t}\, u(t)$

$$= \left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right] e^{-2t}\, u(t)$$

$$= \frac{e^{-2t} \cdot e^{j\pi t}\, u(t)}{2j} - \frac{e^{-2t} \cdot e^{-j\pi t}\, u(t)}{2j}$$

We know that,

$$e^{-2t}\, u(t) \xleftrightarrow{FT} \frac{1}{2+j\omega}$$

Using frequency shifting property, we get,

$$e^{j\pi t}\, e^{-2t}\, u(t) \xleftrightarrow{FT} \frac{1}{2+j(\omega-\pi)}$$

Using Linearity property, we get,

$$\frac{1}{2j}\, e^{j\pi t}\, e^{-2t}\, u(t) \xleftrightarrow{FT} \frac{1}{2j} \cdot \frac{1}{2+j(\omega-\pi)}$$

Similarly,

$$\frac{1}{2j}\, e^{-j\pi t}\, e^{-2t}\, u(t) \xleftrightarrow{FT} \frac{1}{2j} \cdot \frac{1}{2+j(\omega+\pi)}$$

$$\therefore x(t) = \sin(\pi t)e^{-2t}\, u(t) \xleftrightarrow{FT} \frac{1}{j2}\left[\frac{1}{2+j(\omega-\pi)} - \frac{1}{2+j(\omega+\pi)}\right]$$

$$\therefore X(j\omega) = \frac{1}{j2}\left[\frac{1}{2+j(\omega-\pi)} - \frac{1}{2+j(\omega+\pi)}\right]$$

a) $x(t)=e^{-a|t|}; a>0$ b) $x(t)=\sin(\pi t)\, e^{-2t}u(t)$ c) $x(t)= \frac{2}{t^2+1}$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$X(j\omega) = \frac{2a}{a^2+\omega^2}$$

Solution: (a) $x(t) = \frac{2}{t^2+1}$

From the duality property of FT, we have

if $x(t) \xleftrightarrow{FT} X(j\omega)$, the $X(jt) \xleftrightarrow{FT} 2\pi x(-\omega)$

we have the FT pair;

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$$

$$\therefore \quad x(t) = e^{-|t|} \xleftrightarrow{FT} \frac{2}{1+\omega^2} = X(j\omega)$$

Using duality property, the above in modified as

$$X(jt) = \frac{2}{t^2+1} \xleftrightarrow{FT} 2\pi x(-\omega) = 2\pi e^{-|-\omega|} = 2\pi e^{-|\omega|}$$

Hence the FT of $\frac{2}{t^2+1}$ is $\boxed{2\pi e^{-|\omega|}}$

4.

Given ; $y(n) + ½\, y(n-1) = x(n) - 2x(n-1)$

Taking DTFT on both side, we get,

$$Y(e^{j\Omega}) + ½\, e^{-j\Omega}\, Y(e^{j\Omega}) = X(e^{j\Omega}) - 2e^{-j\Omega}\, X(e^{j\Omega})$$

$\therefore$ The frequency response

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1 - 2e^{-j\Omega}}{1 + ½\, e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{1+½\, e^{-j\Omega}} - \frac{2\, e^{-j\Omega}}{1+½\, e^{-j\Omega}}$$

Taking inverse DTFT, we get,

the impulse response $h(n) = \left(\frac{-1}{2}\right)^n u(n) - 2.\left(\frac{-1}{2}\right)^{n-1} u(n-1)$

5.

We have,

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$\therefore X(e^{j\Omega}) = \sum_{n=-\infty}^{0} 2^{n}\, e^{-j\Omega n}$$

Put m = -n, then

$$\therefore X(e^{j\Omega}) = \sum_{\infty}^{0} 2^{-m}\, e^{j\Omega m}$$

$$= \sum_{m=0}^{\infty} 2^{-m}\, e^{j\Omega m}$$

$$= \sum_{m=0}^{\infty} (2^{-1} e^{j\Omega})^{m}$$

$$= \frac{1}{1-2^{-1}e^{j\Omega}}$$

$$X(e^{j\Omega}) = \frac{2}{2-e^{j\Omega}}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} a^{|n|}\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} a^{-n} e^{-j\Omega n} + \sum_{n=0}^{\infty} a^{n} e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} (ae^{j\Omega})^{-n} + \sum_{n=0}^{\infty} (ae^{-j\Omega})^{n}$$

$$= \sum_{n=1}^{\infty} (ae^{j\Omega})^{n} + \sum_{n=0}^{\infty} (ae^{-j\Omega})^{n}$$

$$= \frac{ae^{j\Omega}}{1-ae^{j\Omega}} + \frac{1}{1-ae^{-j\Omega}}$$

$$= \frac{1-a^{2}}{1-2a\cos\Omega + a^{2}}$$

6.

(f) Summation :

$$\text{If } x(n) \xleftrightarrow{\text{DTFT}} X(e^{j\Omega})$$

$$y(n) = \sum_{k=-\infty}^{n} x(k) \xleftrightarrow{\text{DTFT}} Y(e^{j\Omega}) = \frac{X(e^{j\Omega})}{1-e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k\,2\pi)$$

Proof : We know that summation is the reverse process of differencing. The summation operation on $x(n)$ yields $y(n)$ whereas the difference operation on $y(n)$ yields $x(n)$.

i.e. $x(n) = y(n) - y(n-1)$

Taking DTFT on both the sides, we get,

$$X(e^{j\Omega}) = Y(e^{j\Omega}) - e^{-j\Omega}\, Y(e^{j\Omega})$$

$$\therefore Y(e^{j\Omega}) = \frac{X(e^{j\Omega})}{1-e^{-j\Omega}} \qquad \text{.......................................} \qquad (3.15)$$

From eqn. 3.15, we cannot determine $Y(e^{j0})$. Therefore we add an impulse to account for a non zero average value in $x(k)$, to get the exact relationship as,

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$$Y(e^{j\Omega}) = \frac{X(e^{j\Omega})}{1-e^{-j\Omega}} + \pi X(e^{j0})\, \delta(\Omega) \qquad ; -\pi < \Omega < \pi.$$

where the first term assumed to be zero for $\Omega = 0$. Since $Y(e^{j\Omega})$ is periodic 2π, we have,

$$Y(e^{j\Omega}) = \frac{X(e^{j\Omega})}{1-e^{-j\Omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\Omega - k2\pi)$$

7.

$$X(z) = \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1-2z^{-1}} + \frac{A_3}{1-z^{-1}}$$

$$A_1 = \frac{1-z^{-1}+z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)(1-2z^{-1})(1-z^{-1})} \times \left(1-\frac{1}{2}z^{-1}\right)\Bigg|_{z^{-1}=2}$$

$$= 1$$

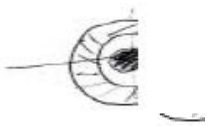
$$A_2 = \frac{1-z^{-1}+z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)(1-2z^{-1})(1-z^{-1})} \times (1-2z^{-1})\Bigg|_{z^{-1}=\frac{1}{2}}$$

$$= 2$$

$$A_3 = \frac{1-z^{-1}+z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)(1-2z^{-1})(1-z^{-1})} \times (1-z^{-1})\Bigg|_{z^{-1}=1}$$

$$= -2$$

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{2}{1-2z^{-1}} - \frac{2}{1-z^{-1}}$$



oles are at $z = \frac{1}{2}$, 2 and 1.

(i) ROC: $|z| < \frac{1}{2}$

The radius of ROC is less than all the poles. So all terms are left-sided gnals. Using (8.41)

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - 2[2]^n u[-n-1] + 2u[-n-1]$$

(ii) ROC: $|z| > 2$

The radius of ROC is greater than all the poles. So all are right-sided verse transforms. Using (8.40)

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2(2)^n u[n] - 2u[n]$$

(iii) ROC: $1 < |z| < 2$

The terms corresponding to $d_k = \frac{1}{2}$ and 1 are right sided and $d_k = 2$ is left ided.

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 2u[n] - 2(2)^n u[-n-1]$$